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Evening

Answers & Solutions

for

JEE (Main)-2023 (Online) Phase-2

(Mathematics, Physics and Chemistry)

Time : 3 hrs. **M.M. : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Mathematics, Physics** and **Chemistry** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Let

$A = \left\{\theta \in (0,\ 2\pi) : \dfrac{1+2i\sin\theta}{1-i\sin\theta} \text{ is purely imaginary}\right\}$.

Then the sum of the elements in A is

(1) 4π (2) 3π

(3) π (4) 2π

Answer (1)

Sol. $\dfrac{1+2i\sin\theta}{1-i\sin\theta}$ is purely imaginary

So, $\dfrac{1-2\sin^2\theta}{1+\sin^2\theta} = 0$

$\Rightarrow \sin^2\theta = \dfrac{1}{2}$

$\therefore \quad \theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$, for $\theta \in (0, 2\pi)$

$\therefore$ Sum of all values = $\dfrac{16\pi}{4} = 4\pi$

2. Let the vectors

$\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}$, $\vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}$, $\vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)k$ are also coplanar, then 6 ($a + b + c$) is equal to

(1) 0 (2) 4

(3) 12 (4) 6

Answer (3)

Sol. Given : $\vec{u_1} = \hat{i} + \hat{j} + a\hat{k}, \vec{u_2} = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u_3} = c\hat{i} + \hat{j} + \hat{k}$

and $\vec{v_1} = (a+b)\hat{i} + c\hat{j} + c\hat{k}$, $\vec{v_2} = a\hat{i} + (b+c)\hat{j} + a\hat{k}$

and $\vec{v_3} = b\hat{i} + b\hat{j} + (c+a)\hat{k}$

Now $\begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0 \Rightarrow (b-1) - (1-c) + a(1-bc) = 0$

$a + b + c = 2 + abc$ …(i)

and $\begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ -2a & -2c & 0 \end{vmatrix} = 0 \quad (R_3 \to R_3 - (R_1 + R_2))$

$\Rightarrow \quad abc = 0$

$\therefore \quad a + b + c = 2$

$\therefore \quad 6(a+b+c) = 12$

3. Let a_n be n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 + …. and $S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to

(1) 11310 (2) 11260

(3) 11290 (4) 11280

Answer (3)

Sol. $S_n = 5 + 8 + 14 + 23 + 35 + 50 + \ldots\ldots + a_n$

$S_n = 5 + 8 + 14 + 23 + 35 + \ldots\ldots + a_n$

$0 = 5 + \underbrace{3 + 6 + 9 + 12 + \ldots\ldots - a_n}_{(n-1) \text{ terms}}$

$\therefore \quad a_n = 5 + \dfrac{n-1}{2}[6 + (n-2)3]$

$= 5 + \dfrac{n-1}{2}[3\sum n] = 5 + \dfrac{3}{2}n^2 - \dfrac{3}{2}n$

$= \dfrac{1}{2}[3n^2 - 3n + 10] = 2$

$\therefore \quad a_{40} = 2345$

$S_n = \dfrac{1}{2}[3\sum n^2 - 3\sum n + 10\sum 1] = \dfrac{1}{2}3x$

$S_{30} = \dfrac{1}{2}\left[\dfrac{3\times 30\times 31\times 61}{6} - 3\times\dfrac{30\times 31}{2} + 10\times 30\right] = 13{,}635$

$\therefore \quad S_{30} - a_{40} = 11{,}290$

4. The value of $36(4\cos^2 9° - 1)(4\cos^2 27° - 1)(4\cos^2 81° - 1)(4\cos^2 243° - 1)$ is

(1) 54 (2) 18

(3) 27 (4) 36

Answer (4)

Sol. $36(4\cos^2 9° - 1)(4\cos^2 27° - 1)(4\cos^2 81° - 1)(4\cos^2 243° - 1)$

$= 36[(4\cos^2 9° - 1)(4\sin^2 9° - 1)(4\cos^2 27° - 1)(4\sin^2 27° - 1)]$

$= 36[(4\sin^2 18° - 4 + 1)(4\sin^2 54° - 4 + 1)]$

$= 36[(4\sin^2 18° - 3)(4\sin^2 54° - 3)]$

$$= 36\left|\left\{\frac{\left(\sqrt{5}-1\right)^2}{4} - 3\right\}\left\{\frac{\left(\sqrt{5}+1\right)^2}{4} - 3\right\}\right|$$

$$= 36\left|\frac{\left(6-2\sqrt{5}-12\right)}{4}\frac{\left(6+2\sqrt{5}-12\right)}{4}\right|$$

$$= -36\left|\frac{\left(2\sqrt{5}\right)^2 - 36}{16}\right| = 36$$

5. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then $m + n$ are coprime, then $m + n$ is equal to

(1) 315 (2) 316

(3) 314 (4) 317

Answer (4)

Sol. $\bar{x} = \frac{x_1 + x_2 + ... + 9 + 10 + ... + x_{12}}{12} = \frac{9}{2}$

$x_1 + x_2 ..... + x_{12} + 19 = 54$

$x_1 + x_2 + ... + x_{12} + 7 + 14 = 54 - 19 + 7 + 14$

$\frac{x_1 + x_2 + ... + x_{12} + 7 + 14}{12} = \frac{56}{12}$

New $\bar{x} = \frac{14}{3}$

$\frac{x_1^2 + x_2^2 + ... + x_{12}^2 + (9)^2 + (10)^2}{12} - \frac{9}{2} = 4$

$x_1^2 + x_2^2 + ... + x_{12}^2 + 7^2 + 14^2 = 355$

New variance $= \frac{\sum x_i^2}{N} - (\bar{x})^2$

$= \frac{355}{12} - \left(\frac{14}{3}\right)^2$

$= \frac{281}{36}$

$\Rightarrow$ $m = 281$

$n = 36$

$\therefore$ $m + n = 317$

6. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Then the relation $R = \{(x, y) \in A \times A : x + y = 7\}$ is

(1) an equivalence relation

(2) symmetric but neither reflexive nor transitive

(3) transitive but neither symmetric nor reflexive

(4) reflexive but neither symmetric nor transitive

Answer (2)

Sol. $x + y = 7$

$y = 7 - x$

$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$\because$ $(a, b) \in R \Rightarrow (b, a) \in R$

Relation is symmetric but not reflexive and not transitive

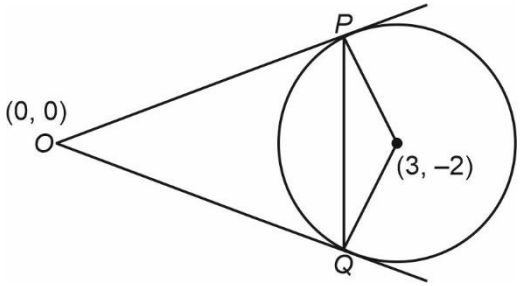
7. Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha, \frac{1}{2}\right)$, then *a* value of α is

(1) $\frac{3}{2}$ (2) $-\frac{1}{2}$

(3) $\frac{5}{2}$ (4) 1

Answer (3)

Sol.



(0, 0) and (3, –2) are the diametric end points of the circumcircle

$\therefore$ $(x - 0)(x - 3) + (y - 0)(y + 2) = 0$

$x^2 + y^2 - 3x + 2y = 0$

Put $\left(\alpha, \frac{1}{2}\right)$ in the above equation

$\alpha^2 + \frac{1}{4} - 3\alpha + 1 = 0$

$\alpha^2 - 3\alpha + \frac{5}{4} = 0$

$4\alpha^2 - 12\alpha + 5 = 0$

$4\alpha^2 - 10\alpha - 2\alpha + 5 = 0$

$2\alpha(2\alpha - 5) - 1(2\alpha - 5) = 0$

$\alpha = \frac{1}{2}, \frac{5}{2}$

8. Let $A(0, 1)$, $B(1, 1)$ and $C(1, 0)$ be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $\left(\alpha + \beta\sqrt{2}, 0\right)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to

(1) 8 (2) 12

(3) 6 (4) $\frac{9}{2}$

Answer (1)

Sol. Mid-point is (0, 1), (1, 0) and (1, 1)

In center (I) = $\left(\frac{4}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$

$y^2 = 4ax$

$\left(\frac{4}{4+2\sqrt{2}}\right)^2 = 4a\left(\frac{4}{4+2\sqrt{2}}\right) \Rightarrow a = \frac{1}{4+2\sqrt{2}}$

Focus (a, 0)

$= \left(\frac{1}{4+2\sqrt{2}}, 0\right)$

$= \left(\frac{4-2\sqrt{2}}{8}, 0\right)$

$\alpha = \frac{4}{8} = \frac{1}{2}, \beta = -\frac{1}{4}$

$\frac{\alpha}{\beta^2} = \frac{\frac{1}{2}}{\left(\frac{-1}{4}\right)^2} = \frac{16}{2} = 8$

9. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to

(1) $13^3 - 13$ (2) $11^3 - 11$

(3) $10^3 - 10$ (4) $12^3 - 12$

Answer (4)

Sol. $T_{r+1} = {}^{11}C_r \left(2x^2\right)^{11-r} (2x)^{-r}$

For coefficient of x^7

$\therefore \quad 22 - 2r - r = 7$

$\therefore \quad r = 5$

For coefficient of x^{10}

$22 - 3r = 10$

$\Rightarrow \quad r = 4$

Absolute difference = $\left|{}^{11}C_5 \cdot \frac{2^6}{2^5} - {}^{11}C_4 \cdot \frac{2^7}{2^4}\right|$

$= |924 - 2640| = 1716$

$= 12^3 - 12$

Option (4) is correct

10. For $a, b \in Z$ and $|a - b| \le 10$, let the angle between the plane $P : ax + y - z = b$ and the line $l : x - 1 = a - y = z + 1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point (6, –6, 4) from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to

(1) 32 (2) 85

(3) 25 (4) 48

Answer (1)

Sol. $\theta = \cos^{-1}\left(\frac{1}{3}\right) = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

$\cos(90^\circ - \theta) = \frac{a + 1(-1) + (-1)(1)}{\sqrt{a^2+2} \cdot \sqrt{3}} = \frac{2\sqrt{2}}{3}$

$\Rightarrow \quad \frac{a-2}{\sqrt{a^2+2}} = \frac{2\sqrt{2}}{\sqrt{3}}$

After solving we get $a = -\frac{2}{5}$, $a = -2$

$\therefore \quad a = -2 \qquad (\because a \in Z)$

Distance = $3\sqrt{6}$

$$\left|\frac{6a-6-4-b}{\sqrt{a^2+2}}\right| = 3\sqrt{6}$$

$= |22+b| = 18$

$\Rightarrow$ $b = -4, -18 - 22 = -40$

$\therefore$ $b = -4$ $\quad (\because |a-b| \le 10)$

$\therefore$ $a^4 + b^2 = 16 + 16 = 32$

Option (1) is correct

11. The integral $\int\left(\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$ is equal to

(1) $\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x + C$ (2) $\left(\frac{x}{2}\right)^x - \left(\frac{2}{x}\right)^x + C$

(3) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{x}{2}\right) + C$ (4) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{2}{x}\right) + C$

Answer (*)

Sol. $I = \int\left(\left(\frac{x}{2}\right)^x + \left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$

Let $\left(\frac{x}{2}\right)^x = t$

$\Rightarrow$ $x\log_2\left(\frac{x}{2}\right) = \log_2 t$

$\Rightarrow$ $x\log_e\left(\frac{x}{2}\right)\cdot\log_2 e = \log_e t.\log_2 e$

On differentiating both sides we get :

$$\log_e\left(\frac{x}{2}\right) + x\cdot\frac{2}{x}\cdot\frac{1}{2} = \frac{1}{t}\frac{dt}{dx}$$

$\Rightarrow$ $\log_e\left(\frac{x}{2}\right) + 1 = \frac{1}{t}\frac{dt}{dx}$

Then solution is not possible.

12. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is (6!)k, then k is equal to

(1) 2835 (2) 5670

(3) 1890 (4) 945

Answer (2)

Sol. Total words = $\frac{11!}{2!2!2!}$

C, S comes together in = $\frac{10!}{2!2!2!}\times 2!$

$\therefore$ required number of words

$= \frac{11!}{(2!)^3} - \frac{10!}{(2!)^2}$

$= \frac{10!}{(2!)^2}\left(\frac{11}{2} - 1\right)$

$= \frac{10!}{8}\times 9$

$= 6!\times\frac{10\times 9\times 8\times 7\times 9}{8}$

$= 6!\,(5670)$

$\therefore$ $k = 5670$

13. The negation of $(p\wedge(\sim q))\vee(\sim p)$ is equivalent to

(1) $p\wedge(\sim q)$ (2) $p\wedge q$

(3) $p\vee(q\vee(\sim p))$ (4) $p\wedge(q\wedge(\sim p))$

Answer (2)

Sol. $\sim\lfloor(p\wedge(\sim q))\vee(\sim p)\rfloor$

$\sim\lfloor(p\wedge\sim q)\wedge(p)\rfloor$

$(\sim p\vee q)\wedge(p)$

$= (\sim p\wedge p)\vee(q\wedge p)$

$= C\vee(q\wedge p)$

$= (p\wedge q)$

14. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and

$$\lim_{x\to\frac{1}{\alpha}}\left(\frac{1-\cos\left(x^2+bx+a\right)}{2(1-\alpha x)^2}\right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$$, then k is equal to

(1) 2β (2) α

(3) 2α (4) β

Answer (3)

Sol. Equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}$ is

$$a+bx+x^2=\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)$$

$$\lim_{x\to\frac{1}{\alpha}}\left(\frac{1-\cos\left(\left(x-\frac{1}{\alpha}\right)\left(x-\frac{1}{\beta}\right)\right)}{2\alpha^2\left(x-\frac{1}{\alpha}\right)^2\left(x-\frac{1}{\beta}\right)^2}\right)^{\frac{1}{2}}\times\left(x-\frac{1}{\beta}\right)$$

$$=\frac{1}{2\alpha}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$$

$\Rightarrow \quad k = 2\alpha$

15. The area of the quadrilateral *ABCD* with vertices *A*(2, 1, 1), *B*(1, 2, 5), *C*(–2, –3, 5) and *D*(1, –6, –7) is equal to

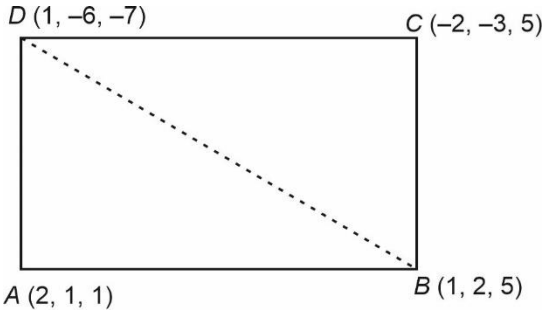
(1) 48 (2) $8\sqrt{38}$

(3) 54 (4) $9\sqrt{38}$

Answer (2)

Sol. $\therefore \quad \overrightarrow{AB} = -\hat{i}+\hat{j}+4\hat{k}$

$$\overrightarrow{AD} = -\hat{i}-7\hat{j}-8\hat{k}$$



$\therefore$ Area of ΔABD

$$=\frac{1}{2}\left|\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&1&4\\-1&-7&8\end{vmatrix}\right|$$

$$=\frac{1}{2}\left|10\hat{i}-6\hat{j}+4\hat{k}\right|=2\sqrt{38}$$

and $\overrightarrow{CB} = 3\hat{i}+5\hat{j}$ and $\overrightarrow{CD} = 3\hat{i}-3\hat{j}-12\hat{k}$

$\therefore$ Area of $\Delta CBD = \frac{1}{2}\left|\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-3&-12\\3&5&0\end{vmatrix}\right|$

$$=\left|6\left(5\hat{i}-3\hat{j}-2\hat{k}\right)\right|$$

$$=6\sqrt{38}$$

$\therefore$ Area of $\square ABCD = 8\sqrt{38}$ square units

16. If the probability that the random variable *X* takes values *x* is given by $P(X = x) = k(x + 1)3^{-x}$, $x = 0, 1, 2, 3, \ldots\ldots$, where *k* is a constant, then $P(X \geq 2)$ is equal to

(1) $\frac{7}{27}$ (2) $\frac{7}{18}$

(3) $\frac{11}{18}$ (4) $\frac{20}{27}$

Answer (1)

Sol. $P(X=0)+P(x=1)+P(x=2)+\ldots\ldots\ldots=1$

$$\frac{k}{3^0}+\frac{2k}{3^1}+\frac{3k}{3^2}+\ldots\ldots=1$$

$$k\left(1+\frac{2}{3}+\frac{3}{3^2}+\ldots\ldots\right)=1$$

$$S=1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\ldots\ldots$$

$$\frac{S}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^2}+\ldots\ldots$$

$$\Rightarrow \quad \frac{2S}{3}=1+\frac{1}{3}+\frac{1}{3^2}+\ldots\ldots$$

$$S=\frac{9}{4}$$

$$k=\frac{4}{9}$$

Now $P(X\geq 2)=P(2)+P(3)+\ldots\ldots$

$$=1-P(0)-P(1)$$

$$=1-\left(\frac{k}{1}+\frac{2k}{3}\right)=1-\frac{20}{27}=\frac{7}{27}$$

17. Let *P* be the plane passing through the line $\frac{x-1}{1}=\frac{y-2}{-3}=\frac{z+5}{7}$ and the point (2, 4, –3). If the image of the point (–1, 3, 4) in the plane *P* is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(1) 10 (2) 12

(3) 9 (4) 11

Answer (1)

Sol. Equation of plane is

$$\begin{vmatrix}x-2&y-4&z+3\\1&-3&7\\1&2&2\end{vmatrix}=0$$

$$(x-2)(-20)-(y-4)(-5)+(z+3)(5)=0$$

$$4x-y-z-7=0$$

$$\frac{x+1}{4}=\frac{y-3}{-1}=\frac{z-4}{-1}=-2\left(\frac{-4-3-4-7}{16+1+1}\right)=2$$

$$x=7=\alpha, y=1=\beta, z=2=\gamma$$

$$\alpha+\beta+\gamma=10$$

18. $25^{190}-19^{190}-8^{190}+2^{190}$ is divisible by

(1) neither 14 nor 34 (2) 14 but not by 34

(3) 34 but not by 14 (4) both 14 and 34

Answer (3)

Sol. $\left(25^{190}-19^{190}\right), 8^{190}, 2^{190}$ are divisible by 2

$\left(25^{190}-8^{190}\right)$ & $\left(-19^{190}+2^{190}\right)$ are divisible by 17

So given number is divisible by 34

$25^{190} \equiv 4^{190} \equiv 16^{95} \equiv 2^{95} \equiv 4\cdot(8)^{31} \equiv 4(1)^{31} \equiv 4 \pmod 7$

$(19)^{190} \equiv (-2)^{190} \equiv 2^{190} \equiv 2(8)^{63} \equiv 2(1)^{63} \equiv 2 \pmod 7$

$8^{190} \equiv 1^{190} \equiv 1 \pmod 7$

$2^{190} \equiv 2(8)^{63} \equiv 1 \pmod 7$

$25^{190}-19^{190}-8^{190}+2^{190} \equiv 4-2-1+1 \equiv 5 \pmod 7$

$\Rightarrow$ Not divisible by 7

19. Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$x+y+\sqrt{3}z=0$

$-x+(\tan\theta)y+\sqrt{7}z=0$

$x+y+(\tan\theta)z=0$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta\in S}\theta$ is equal to

(1) 20 (2) 40

(3) 30 (4) 10

Answer (1)

Sol. $\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$

$\left(\tan^2\theta-\sqrt{7}\right)-\left(-\tan\theta-\sqrt{7}\right)+\sqrt{3}\left(-1-\tan\theta\right)=0$

$\tan^2\theta+\left(1-\sqrt{3}\right)\tan\theta-\sqrt{3}=0$

$\left(\tan\theta+1\right)\left(\tan\theta-\sqrt{3}\right)=0$

$\tan\theta=\sqrt{3}, -1$

$\theta=\frac{\pi}{3}, \frac{-2\pi}{3}, \frac{-\pi}{4}, \frac{3\pi}{4}$

$\frac{120}{\pi}\sum\theta = 120\left(\frac{1}{3}-\frac{2}{3}-\frac{1}{4}+\frac{3}{4}\right)=20$

20. If $A=\begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1}=\alpha A+\beta I$ and $\alpha+\beta=-2$, then $4\alpha^2+\beta^2+\lambda^2$ is equal to :

(1) 12 (2) 19

(3) 14 (4) 10

Answer (3)

Sol. $A=\begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$

$|A-kI|=0 \Rightarrow \begin{vmatrix} 1-k & 5 \\ \lambda & 10-k \end{vmatrix}=0$

$\Rightarrow k^2-11k+10-5\lambda=0$

$\Rightarrow A^2-11A+(10-5\lambda)I=0$

$\Rightarrow A^{-1}=\frac{1}{10-5\lambda}(-A+11I)$

$\Rightarrow \alpha=-\frac{1}{10-5\lambda}, \beta=\frac{11}{10-5\lambda}$

$\because \alpha+\beta=-2 \Rightarrow \lambda=3, \alpha=\frac{1}{5}, \beta=\frac{-11}{5}$

$\Rightarrow 4\alpha^2+\beta^2+\lambda^2=14$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If domain of the function

$\log_e\left(\frac{6x^2+5x+1}{2x-1}\right)+\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha, \beta) \cup (\gamma, \delta]$, then $18(\alpha^2+\beta^2+\gamma^2+\delta^2)$ is equal to _____.

Answer (20)

Sol. $\frac{6x^2+5x+1}{2x-1}>0$ ...(1), $-1 \le \frac{2x^2-3x+4}{3x-5} \le 1$ ...(2)

From (1)

$\frac{6x^2+5x+1}{2x-1}>0 \Rightarrow \frac{(3x+1)(2x+1)}{(2x-1)}>0$

$\Rightarrow x \in \left(-\frac{1}{2}, -\frac{1}{3}\right) \cup \left(\frac{1}{2}, \infty\right)$ ...(a)

From (2)

$$-1 \le \frac{2x^2-3x+4}{3x-5} \le 1 \Rightarrow \frac{2x^2-3x+4}{3x-5} \ge -1 \quad \ldots(3)$$

and $$\frac{2x^2-3x+4}{3x-5} \le 1 \quad \ldots(4)$$

From (3)

$$\frac{2x^2-3x+4}{3x-5}+1 \ge 0 \Rightarrow x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \cup \left(\frac{5}{3}, \infty\right) \quad \ldots(b)$$

From (4)

$$\frac{2x^2-3x+4}{3x-5}-1 \le 0 \Rightarrow \frac{2x^2-6x+9}{3x-5} \le 0$$

$$\Rightarrow \quad \frac{1}{3x-5} \le 0 \quad (\because \ 2x^2-6x+9>0 \ \forall \ x \in R)$$

$$\Rightarrow \quad x \in \left(-\infty, \frac{5}{3}\right) \quad \ldots(c)$$

$\Rightarrow$ Intersection of (a), (b) and (c)

$$\left(-\frac{1}{2}, -\frac{1}{3}\right) \cup \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$$

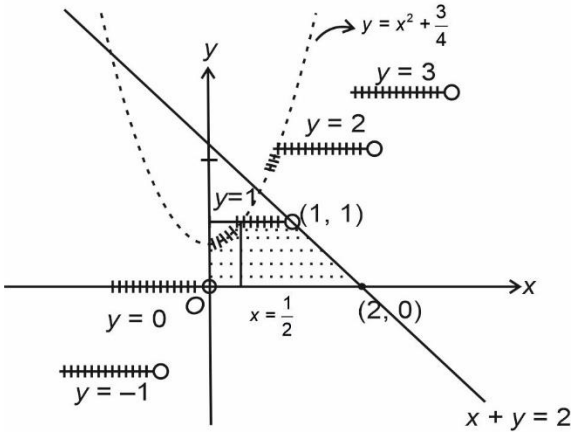
$$\Rightarrow \quad \alpha^2+\beta^2+\gamma^2+\delta^2 = \frac{10}{9}$$

$$\Rightarrow \quad 18(\alpha^2+\beta^2+\gamma^2+\delta^2) = 20$$

22. Let the area enclosed by the lines $x + y = 2$, $y = 0$, $x = 0$ and the curve $f(x) = \min\left\{x^2+\frac{3}{4}, 1+[x]\right\}$ where $[x]$ denotes the greatest integer $\le x$, be A. Then the value of $12A$ is _____.

Answer (17)

Sol.



$$A = \int_0^{\frac{1}{2}} \left(x^2+\frac{3}{4}\right)dx + \frac{1}{2}\left(\frac{1}{2}+\frac{3}{2}\right)\times 1 = \frac{5}{12}+1$$

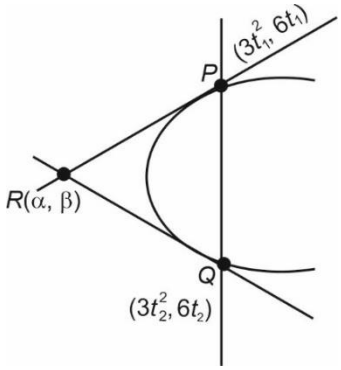
$$\Rightarrow \quad 12A = 17$$

23. The ordinates of the points P and Q on the parabola with focus (3, 0) and directrix $x = -3$ are in the ratio 3 : 1. If $R(\alpha, \beta)$ is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to _____.

Answer (16)

Sol. Focus (3, 0), Directrix, $x = -3$

$\Rightarrow$ Parabola $y^2 = 12x$



Equation of chord of contact w.r.to $R(\alpha, \beta)$

$\beta y = 6(x + \alpha)$ $\quad \ldots(1)$

Equation of chord PQ is

$$\left(\frac{t_1+t_2}{2}\right)y = x + 3t_1t_2 \quad \ldots(2)$$

From (1) & (2)

$$\frac{t_1+t_2}{2} = \frac{\beta}{6}, \ \alpha = 3t_1t_2$$

$$\Rightarrow \quad \frac{\beta^2}{\alpha} = \frac{9(t_1+t_2)^2}{3t_1t_2} = 16 \qquad \left(\because \ \frac{6t_1}{6t_2} = 3, \text{ given}\right)$$

24. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points (2, –1, 0), (2, 0, –1), and (5, 1, 1). If the foot of the perpendicular drawn from the point (7, 4, –1) on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____.

Answer (11.00)

Sol. $$\begin{vmatrix} x-2 & y+1 & z \\ 0 & 1 & -1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$3(x - 2) + 3(-y - 1 - z) = 0$

$x - 2 - y - 1 - z = 0$

$x - y - z = 3$

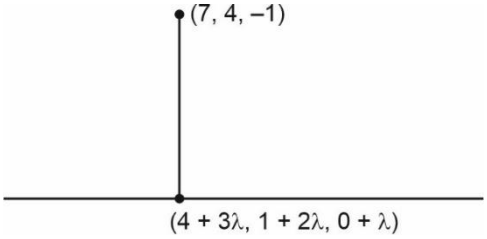
Direction of line

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -7 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= \hat{i}(1-7) - \hat{j}(-3+7) + \hat{k}(-3+1)$$

$$= -6\hat{i} - 4\hat{j} - 2\hat{k}$$

(4, 1, 0) is a point on line of intersection



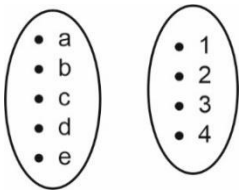
$3(3\lambda - 3) + 2(2\lambda - 3) + 1(\lambda + 1) = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

25. Let $R = \{a, b, c, d, e\}$ and $S = \{1, 2, 3, 4\}$. Total number of onto functions $f : R \to S$ such that $f(a) \neq 1$, is equal to _____.

Answer (180*)

Sol. If $f(a) = 1$



If $f(a) = 1$ & one of $f(b)$, $f(c)$, $f(d)$, $f(e) = 1$ then total such cases $= 4 \cdot 3! = 24$

If only $f(a) = 1$, then

Total cases $= 3^4 - \left({}^3C_1 \cdot 2^4\right) + \left({}^3C_2 \cdot 1\right)$

$= 36$

No of onto functions when $f(a) = 1$ is $24 + 36 = 60$

Total no. of onto functions

$$= 4^5 - \left({}^4C_1 \cdot 3^5\right) + \left({}^4C_2 \cdot 2^5\right) - \left({}^4C_3 \cdot 1\right)$$

$= 1024 - 973 + 192 - 4$

$= 240$

Number of required functions $= 240 - 60$

$= 180$

26. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x + 2| - 4 = 0$ respectively, where $[x]$ denotes the greatest integer $\leq x$. Then $m^2 + mn + n^2$ is equal to _____.

Answer (09.00)

Sol. $x^2 - 12x + [x] + 31 = 0$

$$\underbrace{x^2 - 12x + 31}_{\geq -5} + [x] = 0$$

It could have its solution in [5, 6] but it does not exist as at $x = 5$ and 6

LHS = 1

$\boxed{m = 0}$

$x^2 - 5|x + 2| - 4 = 0$

Case 1

$x \geq -2$

$x^2 - 5(x + 2) - 4 = 0$

$x^2 - 5x - 14 = 0 \Rightarrow x = 7, -2$

Case 2

$x < -2$

$x^2 + 5x + 10 - 4 = 0$

$x = -2, -3$

3 solution i.e., $x = -3, -2, 7$

$n = 3$

27. Let $[t]$ denote the greatest integer function. If $\int_0^{2.4} \left[x^2\right] dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$, then $\alpha + \beta + \gamma + \delta$ is equal to _____.

Answer (06.00)

Sol. $\int_0^{2.4} \left[x^2\right] dx \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx$

$$+ \int_{\sqrt{3}}^{2} 3\,dx + \int_2^{\sqrt{5}} 4\,dx + \int_{\sqrt{5}}^{2.4} 5\,dx$$

$$= 0 + (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) + 4(\sqrt{5} - 2) + 5(2.4 - \sqrt{5})$$

$$= 9 - \sqrt{2} - \sqrt{3} - \sqrt{5}$$

$\alpha = 9$, $\beta = -1$, $\gamma = -1$, $\delta = -1$

28. Let the solution curve $x = x(y),\ 0 < y < \frac{\pi}{2},$ of the differential equation $(\log_e(\cos y))^2 \cos y\, dx -$ $(1 + 3x \log_e(\cos y)) \sin y\, dy = 0$ satisfy $x\left(\frac{\pi}{3}\right)$ $= \frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are coprime, then mn is equal to

Answer (12)

Sol. Given equation can be written as,

$$\frac{dx}{dy} + \left(\frac{-3\tan y}{\ln\cos y}\right)x = \frac{\sin y}{(\ln\cos y)^2 \cdot \cos y}$$

Integrating factor (I.F.) =

$$e^{3\int \frac{-\tan y}{\ln\cos y}\,dy} = (\ln\cos y)^3$$

Solution of differential equation

$$x\cdot(\ln\cos y)^3 = \int \frac{\sin y}{\cos y}(\ln\cos y)\,dy$$

$$\Rightarrow\quad x(\ln\cos y)^3 = \frac{-(\ln\cos y)^2}{2} + c$$

$$x\left(\frac{\pi}{3}\right) = \frac{1}{2\ln 2}$$

$\Rightarrow\ c = 0$

Here, $x = \frac{-1}{2\ln\cos y}$

$$x\left(\frac{\pi}{6}\right) = -\frac{1}{2\ln\frac{\sqrt{3}}{2}} = \frac{1}{\ln 4 - \ln 3}$$

$m = 4,\ n = 3$

$\Rightarrow\ m.n = 12$

29. Let k and m be positive real numbers such that the function $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \ge 1 \end{cases}$ is differentiable for all $x > 0$. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to _____.

Answer (309)

Sol. For continuity at $x = 1$

$f(1^-) = f(1^+)$

$$\Rightarrow\quad 3 + k\sqrt{2} = m + k^2 \qquad \ldots(i)$$

Now, $f'(x)\Big|_{x=1^-} = 6x + \frac{k}{2\sqrt{x+1}}\Big|_{x=1^-} = 6 + \frac{k}{2\sqrt{2}}$

Also, $f'(x)\Big|_{x=1^+} = 2mn\Big|_{x=1^+} = 2m$

For differentiability at $x = 1$

$$6 + \frac{k}{2\sqrt{2}} = 2m \qquad \ldots(ii)$$

From (i) and (ii), $k = \frac{7}{4\sqrt{2}}$ and $m = \frac{103}{32}$

Now, $f'(8) = 2mn\big|_{x=8} = 16m = \frac{103}{2}$

And, $f'\left(\frac{1}{8}\right) = \left(6x + \frac{k}{2\sqrt{x+1}}\right)\Big|_{x=\frac{1}{8}} = \frac{4}{3}$

Now, $\frac{8\cdot f'(8)}{f'\left(\frac{1}{8}\right)} = 309$

30. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to _____.

Answer (150)

Sol. As given, $\frac{2}{y} = \frac{1}{x} + \frac{1}{y}$ …(i)

$$2y^2 = xz \qquad \ldots(ii)$$

and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{\sqrt{2}}$ …(iii)

From (i) and (iii) $y = \sqrt{2}$ …(iv)

Now from (ii) $xz = 4$ …(v)

Now using (ii), (iv) and (v)

$$x + z = 4\sqrt{2}$$

Required, $3(x + y + z)^2 = 3\left(\sqrt{2} + 4\sqrt{2}\right)^2$

$= 150$

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. The power radiated from a linear antenna of length l is proportional to

(Given, λ = Wavelength of wave):

(1) $\frac{l}{\lambda}$ (2) $\left(\frac{l}{\lambda}\right)^2$

(3) $\frac{l}{\lambda^2}$ (4) $\frac{l^2}{\lambda}$

Answer (2)

Sol. Power radiated from a linear antenna $\propto \frac{l^2}{\lambda^2}$

$\Rightarrow$ Option (2)

32. The width of fringe is 2 mm on the screen in a double slit experiment for the light of wavelength of 400 nm. The width of the fringe for the light of wavelength 600 nm will be:

(1) 4 mm (2) 2 mm

(3) 1.33 mm (4) 3 mm

Answer (4)

Sol. $\beta = \frac{\lambda D}{d}$

$\Rightarrow \beta' = \frac{\lambda'}{\lambda}.\beta = \frac{600}{400} \times 2 \text{ mm}$

= 3 mm

33. The trajectory of projectile, projected from the ground is given by $y = x - \frac{x^2}{20}$. Where x and y are measured in meter. The maximum height attained by the projectile will be.

(1) 200 m (2) 10 m

(3) 5 m (4) $10\sqrt{2}$ m

Answer (3)

Sol. $\frac{dy}{dx} = 0$

$\Rightarrow 1 - \frac{x}{10} = 0$

$\Rightarrow x = 10$

$\Rightarrow y_{max} = 10 - \frac{10^2}{20}$

= 5 m

34. The orbital angular momentum of a satellite is L, when it is revolving in a circular orbit at height h from earth surface. If the distance of satellite from the earth centre is increased by eight times to its initial value, then the new angular momentum will be

(1) 8L

(2) 9L

(3) 4L

(4) 3L

Answer (4)

Sol. $v = \sqrt{\frac{GM}{r}}$

$\Rightarrow L = mvr = m\sqrt{GMr}$

$\Rightarrow \frac{L'}{L} = \sqrt{9}$

$\Rightarrow L' = 3L$

35. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Electromagnets are made of soft iron.

Reason R : Soft iron has high permeability and low retentivity.

In the light of above statements, choose the **most appropriate** answer from the options given below.

(1) **A** is not correct but **R** is correct

(2) **A** is correct but **R** is not correct

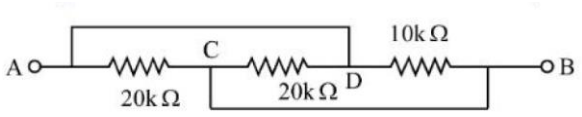
(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(4) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

Answer (3)

Sol. Electromagnets are made of soft iron because soft iron has high permeability and low retentivity.

36. The equivalent resistance between A and B as shown in figure is:



(1) 10 kΩ (2) 5 kΩ

(3) 20 kΩ (4) 30 kΩ

Answer (2)

Sol. All are in parallel

$$\Rightarrow \frac{1}{R}=\frac{1}{20}+\frac{1}{20}+\frac{1}{10}=\frac{1}{5}$$

$\Rightarrow R = 5\ k\Omega$

37. In photo electric effect

A. The photocurrent is proportional to the intensity of the incident radiation.

B. Maximum kinetic energy with which photoelectrons are emitted depends on the intensity of incident light.

C. Max. K.E with which photoelectrons are emitted depends on the frequency of incident light.

D. The emission of photoelectrons require a minimum threshold intensity of incident radiation.

E. Max. K.E of the photoelectrons is independent of the frequency of the incident light.

Choose the correct answer from the options given below:

(1) A and B only

(2) A and E only

(3) A and C only

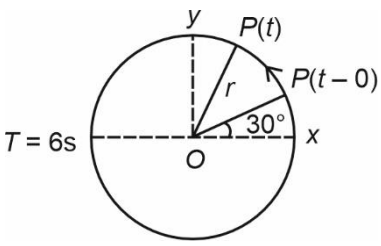
(4) B and C only

Answer (3)

Sol. K_{max} = hf – work function

$\Rightarrow$ B is wrong while C is correct.

38. For particle *P* revolving round the centre *O* with radius of circular path *r* and regular velocity ω, as shown in below figure, the projection of *OP* on the *x*-axis at time *t* is



(1) $x(t)=r\cos\left(\omega t-\frac{\pi}{6}\omega\right)$

(2) $x(t)=r\cos\left(\omega t+\frac{\pi}{6}\right)$

(3) $x(t)=r\sin\left(\omega t+\frac{\pi}{6}\right)$

(4) $x(t)=r\cos(\omega t)$

Answer (2)

Sol. $x(t)=r\cos\left(\omega t+\frac{\pi}{6}\right)$

$$\left[\therefore \cos\theta=\frac{\text{Base}}{\text{Hypotenuse}}\right]$$

39. An emf of 0.08 V is induced in a metal rod of length 10 cm held normal to a uniform magnetic field of 0.4 T, when move with a velocity of:

(1) 0.5 ms^{-1} (2) 20 ms^{-1}

(3) 3.2 ms^{-1} (4) 2 ms^{-1}

Answer (4)

Sol. $\varepsilon = Blv$

$$\Rightarrow 0.08=0.4\times\frac{10}{100}\times v$$

$\Rightarrow v = 2$ m/s

40. The acceleration due to gravity at height *h* above the earth if $h << R$ (Radius of earth) is given by

(1) $g'=g\left(1-\frac{h^2}{2R^2}\right)$ (2) $g'=g\left(1-\frac{h}{2R}\right)$

(3) $g'=g\left(1-\frac{2h}{R}\right)$ (4) $g'=g\left(1-\frac{2h^2}{R^2}\right)$

Answer (3)

Sol. $g'=\frac{GM}{(R+h)^2}=\frac{GM}{R^2}\left[1+\frac{h}{R}\right]^{-2}$

$$\Rightarrow g'\simeq g\left(1-\frac{2h}{R}\right)$$

41. A radio active material is reduced to 1/8 of its original amount in 3 days. If 8×10^{-3} kg of the material is left after 5 days the initial amount of the material is

(1) 40 g (2) 32 g

(3) 64 g (4) 256 g

Answer (4)

Sol. $\frac{1}{8}=\left(\frac{1}{2}\right)^3$

$\Rightarrow 3t_{1/2}=3\,\text{days}\Rightarrow t_{1/2}=1\,\text{day}$

$\Rightarrow$ Initial amount = $2^5 \times 8 \times 10^{-3}$ kg

= 256 grams

42. The waves emitted when a metal target is bombarded with high energy electrons are

(1) Microwaves (2) Infrared rays

(3) X-rays (4) Radio Waves

Answer (3)

Sol. X-rays are emitted when a metal target is bombarded with high energy electrons.

43 A bullet of mass 0.1 kg moving horizontally with speed 400 ms^{-1} hits a wooden block of mass 3.9 kg kept on a horizontal rough surface. The bullet gets embedded into the block and moves 20 m before coming to rest. The coefficient of friction between the block and the surface is _____.

(1) 0.90 (2) 0.50

(3) 0.65 (4) 0.25

Answer (4)

Sol. $0.1 \times 400 = (0.1 + 3.9)v$

$\Rightarrow v = 10$ m/s

Also, $0^2 - 10^2 = 2(-\mu g)(s)$

$\Rightarrow \mu=\frac{100}{2\times10\times20}=0.25$

44. Match List I with List II

	List I		List II
A.	Torque	I.	$ML^{-2}T^{-2}$
B.	Stress	II.	$ML^{2}T^{-2}$
C.	Pressure gradient	III.	$ML^{-1}T^{-1}$
D.	Coefficient of viscosity	IV.	$ML^{-1}T^{-2}$

Choose the correct answer from the options given below :

(1) A-II, B-I, C-IV, D-III

(2) A-IV, B-II, C-III, D-I

(3) A-II, B-IV, C-I, D-III

(4) A-III, B-IV, C-I, D-II

Answer (3)

Sol. Torque = Force × distance $\equiv ML^2T^{-2}$

Stress = $\frac{\text{Force}}{\text{Area}}$ = $ML^{-1}T^{-2}$

Pressure Gradient = $\frac{dp}{dx}$ = $ML^{-2}T^{-2}$

$\eta\equiv\frac{F}{6\pi rv}\equiv\frac{MLT^{-2}}{L\cdot LT^{-1}}=ML^{-1}T^{-1}$

45. A hydraulic automobile lift is designed to lift vehicles of mass 5000 kg. The area of cross section of the cylinder carrying to load is 250 cm^2. The maximum pressure the smaller piston would have to bear is [Assume g = 10 m/s^2]

(1) $20 \times 10^{+6}$ Pa

(2) $2 \times 10^{+5}$ Pa

(3) $200 \times 10^{+6}$ Pa

(4) $2 \times 10^{+6}$ Pa

Answer (4)

Sol. $P=\frac{F}{A}$

$=\frac{5000\times10}{250\times10^{-4}}$

= 2×10^6 N/m^2

46. Work done by a Carnot engine operating between temperatures 127°C and 27°C is 2 kJ. The amount of heat transferred to the engine by the reservoir is:

(1) 8 kJ

(2) 2.67 kJ

(3) 2 kJ

(4) 4 kJ

Answer (1)

Sol. $\eta=1-\frac{T_L}{T_H}=1-\frac{300}{400}$

$=\frac{1}{4}$

$\Rightarrow \frac{2\,\text{kJ}}{Q}=\frac{1}{4}\Rightarrow Q = 8$ kJ

47. Given below are two statements:

Statement I: Area under velocity-time graph gives the distance travelled by the body in a given time.

Statement II: Area under acceleration-time graph is equal to the change in velocity-in the given time.

In the light of given statements, choose the **correct** answer from the options given below.

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is correct but Statement II is false

(4) Statement I is incorrect but Statement II is true

Answer (4)

Sol. $\int vdt = \Delta x$ = displacement

$\int adt = \Delta v$ = change in velocity

48. The temperature at which the kinetic energy of oxygen molecules becomes double than its value at 27°C is

(1) 927°C (2) 327°C

(3) 1227°C (4) 627°C

Answer (2)

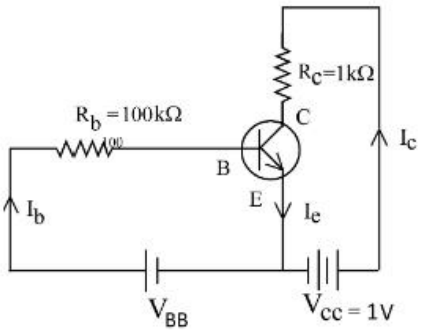
Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$\Rightarrow \sqrt{2} = \sqrt{\frac{T'}{T}} = \sqrt{\frac{T'}{300}}$

$\Rightarrow T'$ = 600 K

= 327°C

49. For a given transistor amplifier circuit in CE configuration V_{CC} = 1 V, R_C = 1 kΩ, R_b = 100 kΩ and β = 100. Value of base current I_b is



(1) I_b = 1.0 μA (2) I_b = 0.1 μA

(3) I_b = 100 μA (4) I_b = 10 μA

Answer (4)

Sol. $I_c = \frac{1}{1000} = 1\text{mA}$

and, $\beta = \frac{I_c}{I_b}$

$\Rightarrow \quad 100 = \frac{1\times10^{-3}}{I_b}$

$\Rightarrow \quad I_b = \frac{1\times10^{-3}}{100} = 10^{-5}$ A

= 10 μA

50. Electric potential at a point '*P*' due to a point charge of 5 × 10^{-9} C is 50 V. The distance of '*P*' from the point charge is:

(Assume, $\frac{1}{4\pi\varepsilon_0} = 9\times10^9 \text{NM}^2\text{C}^{-2}$)

(1) 9 cm

(2) 3 cm

(3) 0.9 cm

(4) 90 cm

Answer (4)

Sol. $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

$\Rightarrow \quad 50 = 9\times10^9 \times \frac{5\times10^{-9}}{r}$

$\Rightarrow \quad r$ = 0.9 m = 90 cm

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. A series combination of resistor of resistance 100 Ω inductor of inductance 1 H and capacitor of capacitance 6.25 μF is connected to an ac source. The quality factor of the circuit will be _____

Answer (4)

Sol. $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{100}\sqrt{\frac{1}{6.25\times10^{-6}}}$

$= \frac{1}{100}\times\frac{1000}{2.5} = 4$

52. A guitar string of length 90 cm vibrates with a fundamental frequency of 120 hz. The length of the string producing a fundamental of 180 Hz will be _____ cm

Answer (60)

Sol. $f_0 = \frac{v}{2L}$

$\Rightarrow \quad 120 = \frac{v}{2(90)}$ …(i)

& $\quad 180 = \frac{v}{2(L)}$ …(ii)

$\Rightarrow \quad \frac{2}{3} = \frac{L}{90} \Rightarrow L$ = 60 cm

53. A 600 pF capacitor is charged by 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. Electrostatic energy lost in the process is_____ μJ

Answer (6)

Sol. $Q_i = CV$ & $U_i = \frac{1}{2}CV^2$

$Q_f = \frac{CV}{2}$ & $U_f = \frac{1}{2}C\left(\frac{V}{2}\right)^2\times2$

$= \frac{CV^2}{4}$

$\Rightarrow$ Loss $= \frac{CV^2}{4} = \frac{600\times10^{-12}\times(200)^2}{4}$ J

= 6 μJ

54. The number density of free electrons in copper is nearly 8×10^{28} m^{-3}. A copper wire has its area of cross section = 2×10^{-6} m^2 and is carrying a current of 3.2 A. The drift speed of the electrons is _____ $\times 10^{-6}$ ms^{-1}

Answer (125)

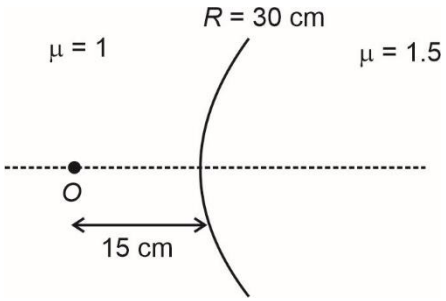
Sol. $I = nAev_d$

$\Rightarrow \quad 3.2 = 8 \times 10^{28} \times 2 \times 10^{-6} \times 1.6 \times 10^{-19} \times v_d$

$\Rightarrow \quad v_d = \frac{3.2}{16\times1.6\times1000}\text{m/s} = 125\times10^{-6}$ m/s

55. Two transparent media having refractive indices 1.0 and 1.5 are separated by a spherical refracting surface of radius of curvature 30 cm. The centre of curvature of surface is towards denser medium and a point object is placed on the principal axis in rarer medium at a distance of 15 cm from the pole of the surface. The distance of image from the pole of the surface is _____ cm.

Answer (30)



$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \quad \frac{1.5}{v} - \frac{1}{-15} = \frac{0.5}{30}$

$\Rightarrow \quad \frac{1.5}{v} = \frac{1}{60} - \frac{1}{15} = \frac{1-4}{60}$

$\Rightarrow \quad v$ = –30 cm

56. A body of mass 5 kg is moving with a momentum of 10 kg ms^{-1}. Now a force of 2 N acts on the body in the direction of its motion for 5 s. The increase in the Kinetic energy of the body is ____ J.

Answer (30)

Sol. $\Delta p = \int F dt$

$\Rightarrow \quad 5v - 10 = 2 \times 5$

$\Rightarrow \quad v$ = 4 m/s

$\Rightarrow \quad \Delta k = \frac{1}{2}(5)\,[4^2 - 2^2]$

= 30 J

57. The ratio of magnetic field at the centre of a current carrying coil of radius r to the magnetic field at distance r from the centre of coil on its axis is $\sqrt{x}:1$. The value of x is ____.

Answer (8)

Sol. $B = \dfrac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$

$\Rightarrow$ Ratio $= \dfrac{\dfrac{\mu_0 I}{2r}}{\dfrac{\mu_0 I r^2}{2(2\sqrt{2}r^3)}}$

$= \dfrac{\dfrac{1}{2}}{\dfrac{1}{4\sqrt{2}}} = 2\sqrt{2} = \sqrt{8}$

58. A steel rod of length 1 m and cross sectional area 10^{-4} m^2 is heated from 0°C to 200°C without being allowed to extend or bend. The compressive tension produced in the rod is _____ × 10^4 N. (Given Young's modulus of steel = 2 × 10^{11} Nm^{-2}, coefficient of linear expansion = 10^{-5}K^{-1})

Answer (4)

Sol. $\Delta L = \dfrac{FL}{Ay} = L \propto \Delta T$

$\Rightarrow$ $F = Ay \propto \Delta T$

$= 10^{-4} \times 2 \times 10^{11} \times 10^{-5} \times 200$ N

$= 4 \times 10^4$ N

59. The ratio of wavelength of spectral lines H_α and H_β in the Balmer series is $\dfrac{x}{20}$. The value of x is _____

Answer (27)

Sol. For Balmer series, $n_f = 2$

$\dfrac{hc}{\lambda_\alpha} = 13.6\left[\dfrac{1}{4} - \dfrac{1}{9}\right]$

and $\dfrac{hc}{\lambda_\beta} = 13.6\left[\dfrac{1}{4} - \dfrac{1}{16}\right]$

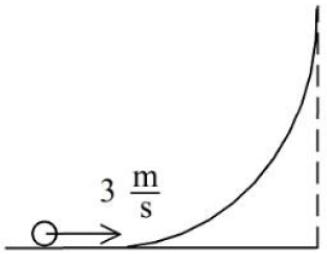
$\Rightarrow$ $\dfrac{\lambda_\alpha}{\lambda_\beta} = \dfrac{\dfrac{1}{4} - \dfrac{1}{16}}{\dfrac{1}{4} - \dfrac{1}{9}}$

$= \dfrac{36}{64} \times \dfrac{12}{5}$

$= \dfrac{27}{20}$

60. A hollow spherical ball of uniform density rolls up a curved surface with an initial velocity 3 m/s (as shown in figure). Maximum height with respect to the initial position covered by it will be ____ cm

(take, g = 10 m/s^2)



Answer (75)

Sol. Energy conservation:

$\Rightarrow$ $\dfrac{1}{2}MV^2 + \dfrac{1}{2} \cdot \dfrac{2}{3}MR^2\omega^2 = Mgh$

$\Rightarrow$ $Mgh = \dfrac{1}{2}MV^2 + \dfrac{1}{3}MV^2 = \dfrac{5}{6}MV^2$

$\Rightarrow$ $h = \dfrac{5V^2}{6g} = \dfrac{5 \times 9}{60}$ metres

$= 75$ cm

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

61. For a good quality cement, the ratio of lime to the total of the oxides of Si, Al and Fe should be as close as to
    (1) 4 (2) 1
    (3) 2 (4) 3

Answer (3)

Sol. For a good quality cement, the ratio of lime to the total of the oxides of Si, Al and Fe should be close to 2.0.

62. Which of the following can reduce decomposition of H_2O_2 on exposure to light
    (1) Urea (2) Alkali
    (3) Glass containers (4) Dust

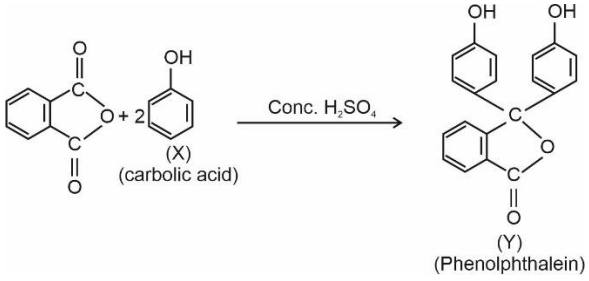
Answer (1)

Sol. Urea acts as a negative catalyst and retardes the decomposition of H_2O_2 to H_2O and O_2 on exposure to light.

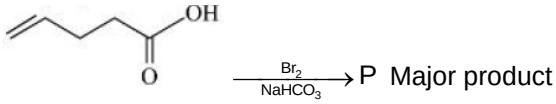
63. A compound 'X' when treated with phthalic anhydride in presence of concentrated H_2SO_4 yields 'Y'. 'Y' is used as an acid/base indicator. 'X' and 'Y' are respectively
    (1) Anisole, methyl orange
    (2) Salicylaldehyde, Phenolphthalein
    (3) Toludine, Phenolphthalein
    (4) Carbolic acid, Phenolphthalein

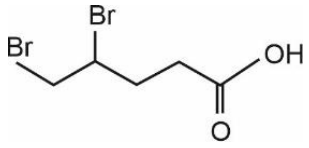
Answer (4)

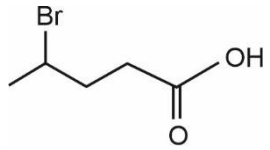
Sol. Carbolic acid or phenol reacts with phthalic anhydride in presence of conc. H_2SO_4 to form phenolphthalein which is used as an acid base indicator.

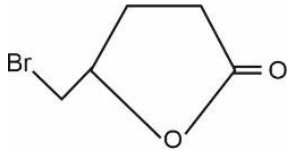


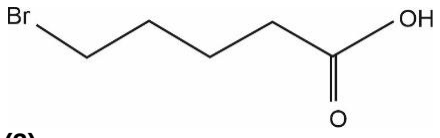
64. Major product 'P' formed in the following reaction is



(1) 

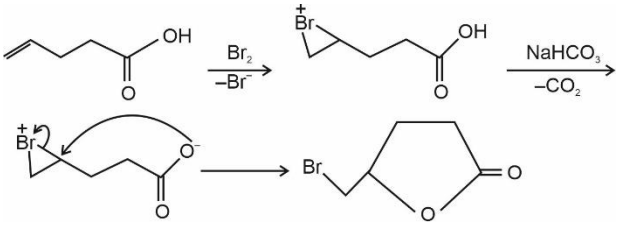
(2) 

(3) 

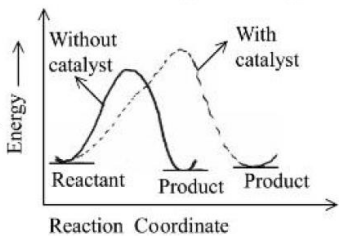
(4) 

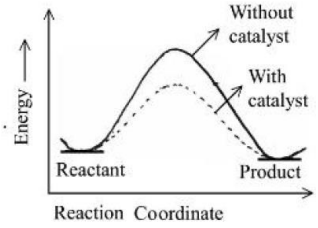
Answer (3)

Sol.

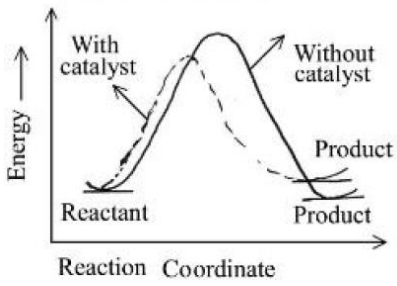


65. The correct reaction profile diagram for a positive catalyst reaction.

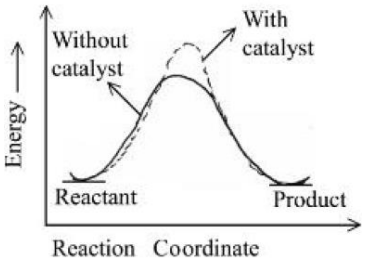
(1) 

(2) 

(3)



(4)



Answer (2)

Sol. Most of the chemical reactions have energy barrier. A positive catalyst in a chemical reaction increases the rate of reaction by lowering the energy barrier. It does not alter the energy of reactant as well as product.

66. Which of the following have same number of significant figures?

(A) 0.00253 (B) 1.0003

(C) 15.0 (D) 163

Choose the correct answer from the options given below

(1) A, B and C only (2) C and D only

(3) B and C only (4) A, C and D only

Answer (4)

Sol. The number of significant figures of the given numbers are as follows:

	Numbers	**No. of significant figures**
(A)	0.00253	3
(B)	1.0003	5
(C)	15.0	3
(D)	163	3

$\therefore$ (A), (C) and (D) have same number of significant figures.

67. The descending order of acidity for the following carboxylic acid is-

(A) CH_3COOH (B) $F_3C–COOH$

(C) $ClCH_2–COOH$ (D) $FCH_2–COOH$

(E) $BrCH_2–COOH$

Choose the correct answer from the options given below:

(1) B > C > D > E > A (2) E > D > B > A > C

(3) B > D > C > E > A (4) D > B > A > E > C

Answer (3)

Sol. The correct order of acidic strength of the given carboxylic acids will be decided by the stability of their conjugate bases. The +I effect of CH_3 group destabilises the carboxylate anion. But the –I effect of halogen substituted methyl group stabilises the carboxylate anion. Higher the –I effect of the halogen substituted methyl group, higher the acidic strength of the corresponding acid.

$\therefore$ Correct order of acidic strength is

$$\underset{(B)}{F_3C-COOH} > \underset{(D)}{FCH_2-COOH} > \underset{(C)}{ClCH_2-COOH} > \underset{(E)}{BrCH_2-COOH} > \underset{(A)}{CH_3-COOH}$$

68. Match List-I with List-II

LIST-I Coordination Complex		**LIST-II Number of unpaired electrons**	
A.	$[Cr(CN)_6]^{3-}$	I.	0
B.	$[Fe(H_2O)_6]^{2+}$	II.	3
C.	$[Co(NH_3)_6]^{3+}$	III.	2
D.	$[Ni(NH_3)_6]^{2+}$	IV.	4

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

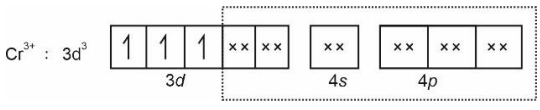
(2) A-III, B-IV, C-I, D-II

(3) A-II, B-I, C-IV, D-III

(4) A-IV, B-III, C-II, D-I

Answer (1)

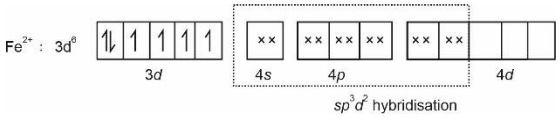
Sol. (A). $[Cr(CN)_6]^{3-}$



d^2sp^3 hybridisation

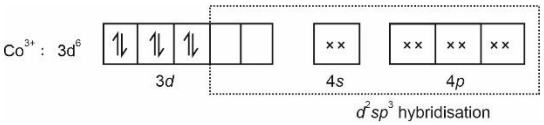
No. of unpaired electrons = 3

(B). $[Fe(H_2O)_6]^{2+}$



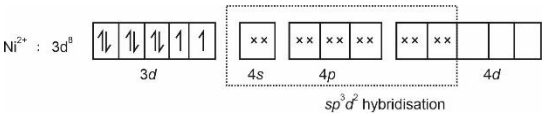
No. of unpaired electrons = 4

(C). $[Co(NH_3)_6]^{3+}$



No. of unpaired electrons = 0

(D) $[Ni(NH_3)_6]^{2+}$



No. of unpaired electrons = 2

A-II : B-IV ; C-I ; D-III

69. Given below are two statements:

Statement-I : Methyl orange is a weak acid.

Statement-II : The benzenoid form of methyl orange is more intense/deeply coloured than the quinonoid form.

In the light of the above statement, choose the ***most appropriate*** answer from the options given below:

(1) Both Statement-I and Statement-II are incorrect

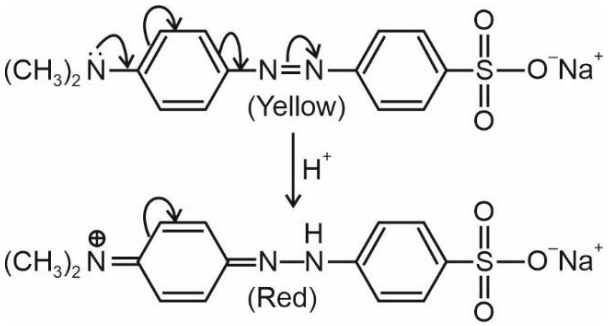
(2) Statement-I is incorrect but Statement-II is correct

(3) Both Statement-I and Statement-II are correct

(4) Statement-I is correct but Statement-II is incorrect

Answer (4)

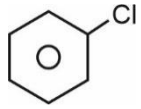
Sol. The Structure of methyl orange is given below. It is a weak acid. It is yellow in basic medium and acquires more intense red colour in acidic medium due to the formation of quinonoid form.

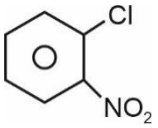


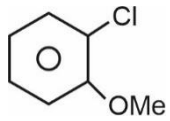
∴ Statement-I is correct but Statement-II is incorrect.

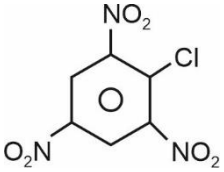
70. The correct order of reactivity of following haloarenes towards nucleophilic substitution with aqueous NaOH is:

Choose the correct answer from the options given below:

(A) 

(B) 

(C) 

(D) 

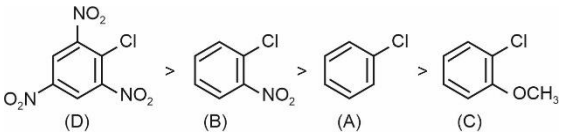
(1) A > B > D > C (2) C > A > D > B

(3) D > C > B > A (4) D > B > A > C

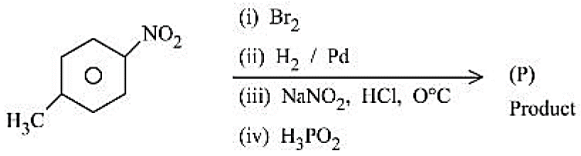
Answer (4)

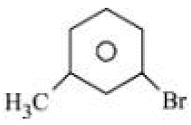
Sol. Aryl halides are less reactive towards nucleophilic aromatic substitution with aqueous NaOH. Introduction of electron releasing group like OCH_3 group further decreases its reactivity towards nucleophilic substitution. But the introduction of electron withdrawing groups like NO_2 group particularly at the ortho and para position increases its reactivity towards nucleophilic substitution. Higher the number of electron withdrawing groups, higher the reactivity.

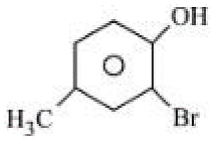
∴ The correct reactivity order is

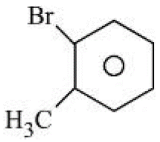


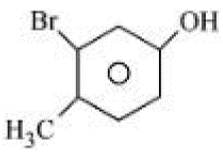
71. The product (P) formed from the following multistep reaction is:



(1) 

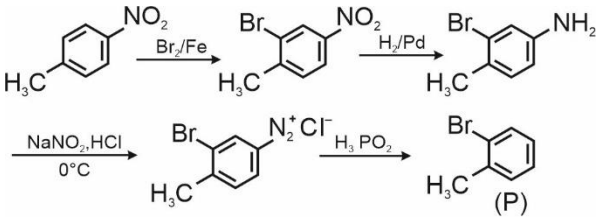
(2) 

(3) 

(4) 

Answer (3)

Sol.



72. Arrange the following gases in increasing order of van der Waals constant 'a'

(A) Ar (B) CH_4

(C) H_2O (D) C_6H_6

Choose the correct option from the following.

(1) D, C, B and A (2) B, C, D and A

(3) C, D, B and A (4) A, B, C and D

Answer (4)

Sol. The values of Vander Waal's constant 'a' for the given gases are:

Gases	Vander Waal's Constant 'a'
A. Ar	1.35 L^2 atm mol^2
B. CH_4	2.25 L^2 atm mol^2
C. H_2O	5.536 L^2 atm mol^2
D. C_6H_6	18.57 L^2 atm mol^2

$\therefore$ Increasing order of 'a' for the given gases

A < B < C < D

73. Which of these reactions is **not** a part of breakdown of ozone in stratosphere?

(1) $2\ \dot{Cl}O \longrightarrow ClO_2\ (g) + \dot{Cl}\ (g)$

(2) $\dot{Cl}O\ (g) + O\ (g) \longrightarrow \dot{Cl}\ (g) + O_2\ (g)$

(3) $\dot{Cl}\ (g) + O_3\ (g) \longrightarrow \dot{Cl}O\ (g) + O_2\ (g)$

(4) $CF_2Cl_2\ (g) \xrightarrow{uv} \dot{Cl}\ (g) + \dot{C}F_2\ Cl\ (g)$

Answer (1)

Sol. The following reaction is not a part of breakdown of ozone in stratosphere

1. $2\dot{Cl}O \longrightarrow \dot{Cl}(g) + O_2(g)$

(Reference NCERT, Class XI, Part II, p413)

74. Given below are two statements:

Statement I: In redox titration, the indicators used are sensitive to change in pH of the solution.

Statement II: In acid-base titration, the indicators used are sensitive to change in oxidation potential.

In the light of the above statements, choose the ***most appropriate*** answer from the options given below

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are correct

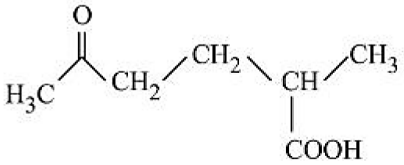
Answer (2)

Sol. In redox titration, the indicators used are sensitive to change in oxidation potential and not change in pH.

In acid base titration, the indicators are sensitive to change in pH and not change in oxidation potential.

$\therefore$ Both the statements are incorrect.

75. The correct IUPAC nomenclature for the following compound is:



(1) 2-Methyl-5-oxohexanoic acid

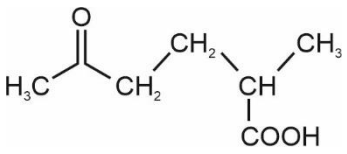
(2) 2-Formyl-5-methylhexan-6-oic acid

(3) 5-Methyl-2-oxohexan-6-oic acid

(4) 5-Formyl-2-methylhexanoic acid

Answer (1)

Sol. IUPAC name of the following compound is



2-Methyl-5-oxo-hexanoic acid

76. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Sodium is about 30 times as abundant as potassium in the oceans.

Reason R: Potassium is bigger in size than sodium.

In the light of the above statements, choose the correct answer from the options given below

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) **A** is true but **R** is false

(3) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(4) **A** is false but **R** is true

Answer (3)

Sol. (**A):** Sodium is about 30 times as abundant as potassium in the oceans. This is true.

(R): Potassium is bigger in size than sodium. This is also true. But this is not the correct explanation of (A). The correct explanation should be higher solubility of sodium salts in water than that of potassium salts.

77. The statement/s which are true about antagonists from the following is/are:

A. They bind to the receptor site.

B. Get transferred inside the cell for their action.

C. Inhibit the natural communication of the body.

D. Mimic the natural messenger.

Choose the correct answer from the options given below:

(1) B only (2) A, C and D

(3) A and C (4) A and B

Answer (3)

Sol. Drugs that bind to the receptor site and inhibit its natural function are called antagonists. (Reference: NCERT, class XII, Part-II, p-451).

78. Match List I with List II

	List I Natural amino acid		**List II One Letter Code**
(A)	Glutamic acid	(I)	Q
(B)	Glutamine	(II)	W
(C)	Tyrosine	(III)	E
(D)	Tryptophan	(IV)	Y

Choose the correct answer from the options given below:

(1) (A)–III, B–IV, (C)–I, (D)–II

(2) (A)–IV, B–III, (C)–I, (D)–II

(3) (A)–III, B–I, (C)–IV, (D)–II

(4) (A)–II, B–I, (C)–IV, (D)–III

Answer (3)

Sol. One letter codes of the given natural amino acids are

Natural amino acid	**One Letter Code**
(A) Glutamic acid	(III) E
(B) Glutamine	(I) Q
(C) Tyrosine	(IV) Y
(D) Tryptophan	(II) W

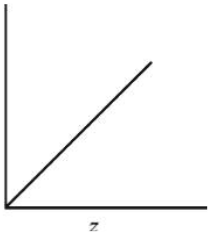
$\therefore$ Correct match is

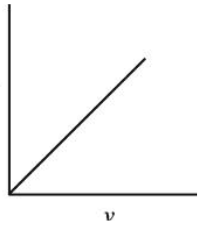
(A)–III, B–I, (C)–IV, (D)–II

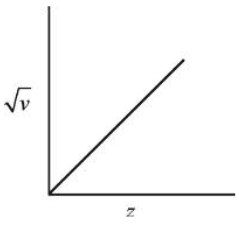
79. Henry Moseley studied characteristic X-ray spectra of elements. The graph which represents his observation correctly is

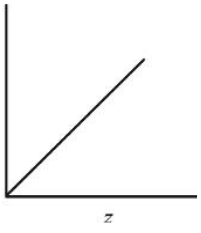
Given ν = frequency of X-ray emitted

Z = atomic number

(1) 

(2) 

(3) 

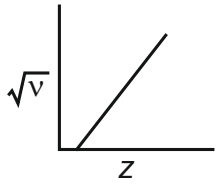
(4) 

Answer (3)

Sol. Moseley's observation on characteristic X-ray spectra of elements relates frequency (ν) of X-rays emitted to the atomic number by the following relation:

$\sqrt{\nu} = a(Z - b)$

The plot of $\sqrt{\nu}$ vs Z is linear with –ve intercept on Y-axis.



Out of the four graphs given, graph (3) can be considered correct but it needs to modified with respect to intercept on Y-axis.

80. In Hall-Heroult process, the following is used for reducing Al_2O_3:-

(1) Magnesium (2) Na_3AlF_6

(3) Graphite (4) CaF_2

Answer (3)

Sol. Graphite anode is used in the electrolytic reduction of Al_2O_3 in Hall-Haroult process.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. If the boiling points of two solvents X and Y (having same molecular weights) are in the ratio 2 : 1 and their enthalpy of vaporizations are in the ratio 1 : 2, then the boiling point elevation constant of X is m times the boiling point elevation constant of Y. The value of m is (nearest integer).

Answer (8)

Sol. The boiling point elevation constant (K_b) of a volatile liquid is given by the following expression

$$K_b = \frac{RT_b^2M}{\Delta H_v}$$

Where T_b = boiling point of the liquid

M = molar mass of the liquid

ΔH_v = Enthalpy of vaporisation

Given : $M_x = M_y$

$(T_b)_x = 2(T_b)_y$

$2(\Delta H_v)_x = (\Delta H_v)_y$

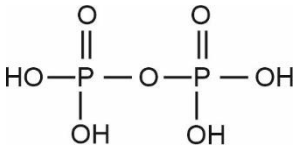
$$\therefore \quad \frac{(K_b)_x}{(K_b)_y} = \left(\frac{2}{1}\right)^2 \times \frac{2}{1} = \frac{8}{1} = m$$

$\therefore$ m = 8

82. The ratio of sigma and π bonds present in pyrophosphoric acid is _____.

Answer (6)

Sol. The structure of pyrophosphoric acid is



$$\frac{\text{No. of } \sigma \text{ bonds}}{\text{No. of } \pi \text{ bonds}} = \frac{12}{2} = 6$$

83. The number of atomic orbitals from the following having 5 radial nodes is _____.

7s, 7p, 6s, 8p, 8d

Answer (3)

Sol. The number of radial nodes in an orbital is given as number of radial nodes = n – l – 1

Number of radial nodes

7s : n = 7, l = 0 7 – 0 – 1 = 6

7p : n = 7, l = 1 7 – 1 – 1 = 5

6s : n = 6, l = 0 6 – 0 – 1 = 5

8p : n = 8, l = 1 8 – 1 – 1 = 6

8d : n = 8, l = 2 8 – 2 – 1 = 5

84. The observed magnetic moment of the complex $[Mn(NCS)_6]^{x-}$ is 6.06 BM. The numerical value of x is

Answer (4)

Sol. $[Mn(NCS)_6]^{x-}$

μ = 6.06 BM

Number of unpaired electrons = 5

Mn^{2+} : $3d^5$

Ligand : NCS^-

$\Rightarrow$ x = 4

85. The sum of oxidation state of the metals in $Fe(CO)_5$, VO^{2+} and WO_3 is

Answer (10)

Sol. $Fe(CO)_5$ O.S. of Fe = 0

VO^{2+} O.S. of V = + 4

WO_3 O.S. of W = + 6

Sum of oxidation state of metals = 10

86. For complete combustion of ethene,

$$C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(l)$$

the amount of heat produced as measured in bomb calorimeter is 1406 kJ mol^{-1} at 300 K. The minimum value of $T\Delta S$ needed to reach equilibrium is (–) _____ kJ. (Nearest integer)

Given: R = 8.3 J K^{-1} mol^{-1}

Answer (1411)

Sol. $C_2H_4(g) + 3O_2(g) \rightarrow 2CO_2(g) + 2H_2O(l)$

$\Delta U = -1406$ kJ mol^{-1}

$\Delta H = \Delta U + \Delta n_g RT$

$$= -1406 + \frac{(-2)\times 8.3\times 300}{1000}$$

$= -1406 - 4.98$

$\simeq -1411$ kJ mol^{-1}

The minimum value of $T\Delta S$ at equilibrium = ΔH

$T\Delta S = -1411$ kJ

87. The number of <u>incorrect</u> statements from the following is _____

A. The electrical work that a reaction can perform at constant pressure and temperature is equal to the reaction Gibbs energy.

B. E^o_{cell} is dependent on the pressure.

C. $\frac{dE^\theta cell}{dT} = \frac{\Delta_r S^\theta}{nF}$

D. A cell is operating reversibly if the cell potential is exactly balanced by an opposing source of potential difference.

Answer (1)

Sol. E^o_{cell} of an electrochemical cell depends on equilibrium constant of the cell reaction and does not depend on the pressure of gas involved in the cell. Rest of the statements are correct.

88. The solubility product of $BaSO_4$ is 1×10^{-10} at 298 K. The solubility of $BaSO_4$ in 0.1 M K_2SO_4(aq) solution is _____ $\times 10^{-9}$ g L^{-1} (nearest integer).

Given: Molar mass of $BaSO_4$ is 233 g mol^{-1}

Answer (233)

Sol. $BaSO_4(s) \rightleftharpoons \underset{S}{Ba^{2+}}(aq) + \underset{S+0.1}{SO_4^{2-}}(aq)$

$K_2SO_4 \longrightarrow \underset{0.1}{K^+}(aq) + \underset{0.1}{SO_4^{2-}}(aq)$

$K_{SP} = \left[Ba^{2+}\right]\left[SO_4^{2-}\right]$

$1\times 10^{-10} = S(S + 0.1)$

$\simeq 0.1S$

$S = 10^{-9}$ M = 233×10^{-9} gL^{-1}

89. Coagulating value of the electrolytes $AlCl_3$ and NaCl for As_2S_3 are 0.09 and 50.04 respectively. The coagulating power of $AlCl_3$ is *x* times the coagulating power of NaCl. The value of *x* is _____.

Answer (556)

Sol. Coagulating value of $AlCl_3$ for As_2S_3 = 0.09

Coagulating value of NaCl for As_2S_3 = 50.04

$$\frac{\text{Coagulating power of } AlCl_3}{\text{Coagulating power of NaCl}} = \frac{50.04}{0.09} = 556$$

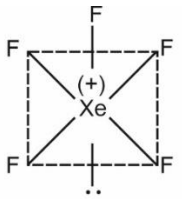
90. The number of species from the following carrying a single lone pair on central atom Xenon is _____.

$XeF_5^+, XeO_3, XeO_2F_2, XeF_5^-, XeO_3F_2, XeOF_4, XeF_4$

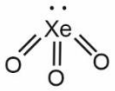
Answer (4)

Sol.

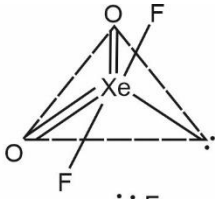
XeF_5^+



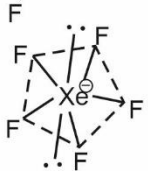
XeO_3



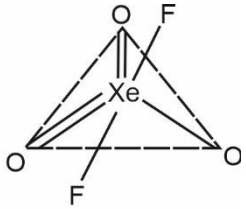
XeO_2F_2



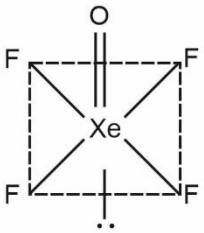
XeF_5^-



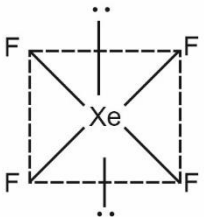
XeO_3F_2



$XeOF_4$



XeF_4



XeF_5^+, XeO_3, XeO_2F_2 and $XeOF_4$ have single lone pair on central atom.

❐ ❐ ❐