## Answers \& Solutions

Time : 3 hrs.
M.M. : 300

JEE (Main)-2023 (Online) Phase-2
(Mathematics, Physics and Chemistry)

## IMPORTANT INSTRUCTIONS:

(1) The test is of $\mathbf{3}$ hours duration.
(2) The Test Booklet consists of 90 questions. The maximum marks are 300 .
(3) There are three parts in the question paper consisting of Mathematics, Physics and Chemistry having 30 questions in each part of equal weightage. Each part (subject) has two sections.
(i) Section-A: This section contains 20 multiple choice questions which have only one correct
answer. Each question carries $\mathbf{4}$ marks for correct answer and $\mathbf{- 1}$ mark for wrong answer.
(ii) Section-B: This section contains 10 questions. In Section-B, attempt any five questions out
of $\mathbf{1 0}$. The answer to each of the questions is a numerical value. Each question carries $\mathbf{4}$ marks for correct answer and $\mathbf{- 1}$ mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

## MATHEMATICS

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer:

1. Let $\mu$ be the mean and $\sigma$ be the standard deviation of the distribution

| $x_{i}$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{i}$ | $k+2$ | $2 k$ | $k^{2}-1$ | $k^{2}-1$ | $k^{2}+1$ | $k-3$ |

where $\sum f_{i}=62$. If $[x]$ denotes the greatest integer $\leq x$, then $\left[\mu^{2}+\sigma^{2}\right]$ is equal to
(1) 9
(2) 8
(3) 7
(4) 6

Answer (2)
Sol.

| $x_{i}$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f_{i}$ | $k+2$ | $2 k$ | $k^{2}-1$ | $k^{2}-1$ | $k^{2}+1$ | $k-3$ |

$$
\mu=\frac{2 k+2 k^{2}-2+3 k^{2}-3+4 k^{2}+4+5 k-15}{62}=\frac{9 k^{2}+7 k-16}{62}
$$

and $\sum f_{i}=62 \Rightarrow 3 k^{2}+4 k-2=62$

$$
\begin{aligned}
& \Rightarrow 3 k^{2}+4 k-64=0 \\
& \Rightarrow(3 k+16)(k-4)=0 \\
& \therefore k=4
\end{aligned}
$$

$$
\therefore \quad \mu=\frac{156}{62}
$$

$$
\sigma^{2}=\frac{1 \cdot(2 k)+4\left(k^{2}-1\right)+9\left(k^{2}-1\right)+16\left(k^{2}+1\right)+25(k-3)}{62}-\left(\frac{156}{62}\right)^{2}
$$

$$
=\frac{29 k^{2}+27 k-72}{62}-\mu^{2}
$$

$$
=\frac{500}{62}
$$

$\therefore \quad \sigma^{2}+\mu^{2}=\frac{500}{62} \Rightarrow\left[\sigma^{2}+\mu^{2}\right]=8$
2. Let the image of the point $P(1,2,6)$ in the plane passing through the points $A(1,2,0)$ and $B(1,4,1)$ $C(0,5,1)$ be $Q(\alpha, \beta, \gamma)$. Then $\left(\alpha^{2}+\beta^{2}+\gamma^{2}\right)$ is equal to
(1) 65
(2) 62
(3) 76
(4) 70

Answer (1)
Sol. Plane passes through the points $A(1,2,0), B(1,4,1)$ and $C(0,5,1)$

Normal vector $=\left|\begin{array}{ccc}i & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ -1 & 3 & 1\end{array}\right|=\hat{i}(-1)-\hat{j}(1)+2 \hat{k}$

$$
\begin{align*}
& =<-1,-1,2> \\
\therefore & -1(x-1)-1(y-2)+2 z=0 \\
\therefore & x+y-2 z=3 \quad \ldots \text { (i) }
\end{align*}
$$

Now image of $P(1,2,6)$ in (i)

$$
\begin{aligned}
& \frac{x-1}{1}=\frac{y-2}{1}=\frac{z-6}{-2}=\frac{-2(-9-3)}{6}=4 \\
& \therefore \quad x=5, y=6, z=-2 \\
& \therefore \quad \alpha=5, \beta=6, \gamma=-2 \\
& \therefore \quad \alpha^{2}+\beta^{2}+\gamma^{2}=65
\end{aligned}
$$

3. Let the number $(22)^{2022}+(2022)^{22}$ leave the remainder $\alpha$ when divided by 3 and $\beta$ when divided by 7 . Then $\left(\alpha^{2}+\beta^{2}\right)$ is equal to
(1) 20
(2) 13
(3) 5
(4) 10

Answer (3)
Sol. For $\alpha:(21+1)^{2022}+\underbrace{(2022)^{22}}_{\text {divisible by } 3}$

$$
=3 K_{1}+1
$$

For $\beta$ : $(21+1)^{2022}+(2023-1)^{22}$
$=7 \lambda+1+7 \mu+1$
$=7 K_{2}+2$
So, $\alpha=1, \beta=2$
$\alpha^{2}+\beta^{2}=5$
4. Eight persons are to be transported from city $A$ to city $B$ in three cars of different makes. If each car can accommodate at most three persons, then the number of ways, in which they can be transported, is
(1) 1120
(2) 3360
(3) 1680
(4) 560

Answer (2)
Sol. Total ways $={ }^{8} C_{3} \times{ }^{5} C_{3} \times{ }^{2} C_{2} \times 3$ !

$$
=3360
$$

5. Let $\vec{a}=2 \hat{i}+7 \hat{j}-\hat{k}, \hat{b}=3 \hat{i}+5 \hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+2 \hat{k}$. Let $\vec{d}$ be a vector which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d}=12$. Then $(-\hat{i}+\hat{j}-\hat{k}) \cdot(\vec{c} \times \vec{d})$ is equal to
(1) 24
(2) 44
(3) 42
(4) 48

Answer (2)
Sol. $\vec{d}=\lambda(\vec{a} \times \vec{b})$

$$
\begin{aligned}
& \vec{a} \times \vec{b}=\left|\begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
2 & 7 & -1 \\
3 & 0 & 5
\end{array}\right| \\
& =\hat{i}(35)-13 \hat{j}-21 \hat{k} \\
& \vec{d}=\lambda(35 \hat{i}-13 \hat{j}-21 \hat{k}) \\
& \vec{c} \cdot \vec{d}=\lambda(35+13-42)=12 \\
& \Rightarrow \lambda=2 \\
& \vec{c} \times \vec{d}=2 \times\left|\begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
1 & -1 & 2 \\
35 & -13 & -21
\end{array}\right| \\
& =2(47 \hat{i}+91 \hat{j}+22 \hat{k}) \\
& (-\hat{i}+\hat{j}-\hat{k}) \cdot(\vec{c} \times \vec{d})=2(-\hat{i}+\hat{j}-\hat{k})(47 \hat{i}+91 \hat{j}+22 \hat{k})=44
\end{aligned}
$$

6. Let $f$ be a continuous function satisfying $\int_{0}^{t^{2}}\left(f(x)+x^{2}\right) d x=\frac{4}{3} t^{3}, \forall t>0$. Then $f\left(\frac{\pi^{2}}{4}\right)$ is equal to
(1) $\pi^{2}\left(1-\frac{\pi^{2}}{16}\right)$
(2) $-\pi\left(1+\frac{\pi^{3}}{16}\right)$
(3) $\pi\left(1-\frac{\pi^{3}}{16}\right)$
(4) $-\pi^{2}\left(1+\frac{\pi^{2}}{16}\right)$

## Answer (3)

Sol. $\left(f\left(t^{2}\right)+t^{4}\right) 2 t=4 t^{2}$
$f\left(t^{2}\right)+t^{4}=2 t$
$f\left(x^{2}\right)=-x^{4}+2 x$
Let $x^{2}=u$
$f(u)=-u^{2}+2 \sqrt{u}$
$f(x)=-x^{2}+2 \sqrt{x}$
$f\left(\frac{\pi^{2}}{4}\right)=-\frac{\pi^{4}}{4^{2}}+2 \times \frac{\pi}{2}$
$=\frac{-\pi^{4}}{16}+\pi$
$=\pi\left(1-\frac{\pi^{3}}{16}\right)$
7. For $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if $\int\left(\left(\frac{x}{e}\right)^{2 x}+\left(\frac{e}{x}\right)^{2 x}\right) \log _{e} x$ $d x=\frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x}-\frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x}+C$, where $e=\sum_{n=0}^{\infty} \frac{1}{n!}$ and $C$ is constant of integration, then $\alpha+2 \beta+3 \gamma-4 \delta$ is equal to
(1) 1
(2) 4
(3) -4
(4) -8

Answer (2)
Sol. Let $\left(\frac{x}{e}\right)^{2 x}=t$
$2 x(\ln x-1)=\ln t$

$$
\left(2(\ln x-1)+2 x \times \frac{1}{x}\right) d x=\frac{1}{t} d t
$$

$\ln x d x=\frac{1}{2 t} d t$

$$
\begin{aligned}
I & =\int\left(t+\frac{1}{t}\right) \times \frac{1}{2 t} d t=\frac{1}{2} \int\left(1+\frac{1}{t^{2}}\right) d t \\
& =\frac{1}{2}\left(t-\frac{1}{t}\right)+C \\
& =\frac{1}{2}\left(\frac{x}{e}\right)^{2 x}-\frac{1}{2}\left(\frac{e}{x}\right)^{2 x}+C \\
\Rightarrow & \alpha=2, \beta=2, \gamma=2, \delta=2 \\
\therefore & \alpha+2 \beta+3 \gamma-4 \delta=2+4+6-8=4
\end{aligned}
$$

8. Let $g(x)=f(x)+f(1-x)$ and $f^{\prime \prime}(x)>0, x \in(0,1)$. If $g$ is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha, 1)$, then $\tan ^{-1}(2 \alpha)+$ $\tan ^{-1}\left(\frac{1}{\alpha}\right)+\tan ^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to
(1) $\pi$
(2) $\frac{5 \pi}{4}$
(3) $\frac{3 \pi}{4}$
(4) $\frac{3 \pi}{2}$

## Answer (1)

Sol. $g(x)=f(x)+f(1-x)$

$$
\begin{aligned}
& g^{\prime}(x)=f^{\prime}(x)-f^{\prime}(1-x) \\
& g^{\prime \prime}(x)=f^{\prime \prime}(x)+f^{\prime \prime}(1-x)>0
\end{aligned}
$$

$g^{\prime}(x)$ is increasing
$g^{\prime}(0)<g^{\prime}(1)$
$f^{\prime}(0)-f^{\prime}(1)<f^{\prime}(1)-f^{\prime}(0)$
$\Rightarrow \quad f^{\prime}(0)<f^{\prime}(1)$

$$
g^{\prime}(x)=0
$$

$$
\Rightarrow \quad f^{\prime}(x)=f^{\prime}(1-x)
$$

$$
x=1-x
$$

$$
x=\frac{1}{2}
$$

$g^{\prime}(x)$ is positive for $x \in\left(0, \frac{1}{2}\right)$
$g^{\prime}(x)$ is negative for $x \in\left(\frac{1}{2}, 1\right)$
$\therefore \quad \alpha=\frac{1}{2}$

$$
\begin{aligned}
& \tan ^{-1}(2 \alpha)+\tan ^{-1}\left(\frac{1}{\alpha}\right)+\tan ^{-1}\left(\frac{\alpha+1}{\alpha}\right) \\
& =\tan ^{-1}(1)+\tan ^{-1}(2)+\tan ^{-1}(3) \\
& =\pi
\end{aligned}
$$

9. The statement $\sim\lfloor p \vee(\sim(p \wedge q))\rfloor$ is equivalent to
(1) $\sim(p \wedge q)$
(2) $(p \wedge q) \wedge(\sim p)$
(3) $(\sim(p \wedge q)) \wedge q$
(4) $\sim(p \vee q)$

Answer (4)

Sol. $\sim\lfloor p \vee(\sim(p \wedge q))\rfloor$
$=\sim p \wedge \sim(\sim p \wedge q)$
$=\sim p \wedge(p \vee \sim q)$
$=(\sim p \wedge p) \vee(\sim p \wedge \sim q)$
$=\sim p \wedge \sim q$
$=\sim(p \vee q)$
10. Let a circle of radius 4 be concentric to the ellipse $15 x^{2}+19 y^{2}=285$. Then the common tangents are inclined to the minor axis of the ellipse at the angle
(1) $\frac{\pi}{3}$
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{6}$
(4) $\frac{\pi}{12}$

Answer (1)
Sol.


Let the equation of tangent of ellipse be
$y=m x \pm \sqrt{19 m^{2}+15}$
If it is tangent to circle $x^{2}+y^{2}=16$

$$
\begin{aligned}
& \frac{\sqrt{19 m^{2}+15}}{\sqrt{1+m^{2}}}=4 \\
\Rightarrow & 19 m^{2}+15=16+16 m^{2} \\
\Rightarrow & 3 m^{2}=1 \\
& m= \pm \frac{1}{\sqrt{3}}
\end{aligned}
$$

$\therefore$ Angle made by tangent with minor axis i.e., with $y$ axis is $\frac{\pi}{3}$
11. Let $S=\left\{x \in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right): 9^{1-\tan ^{2} x}+9^{\tan ^{2} x}=10\right\}$ and $\beta=\sum_{x \in S} \tan ^{2}\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta-14)^{2}$ is equal to
(1) 16
(2) 8
(3) 64
(4) 32

## Answer (4)

Sol. $9^{1-\tan ^{2} x}+9^{\tan ^{2} x}=10$
Let $9^{\tan ^{2} x}=y$
$\frac{9}{y}+y=10$
$\Rightarrow y^{2}-10 y+9=0$
$\Rightarrow \quad y=1, y=9$

$$
9^{\tan ^{2} x}=1 \text { OR } 9^{\tan ^{2} x}=9
$$

$\Rightarrow \tan x=0$ OR $\tan x= \pm 1$

$$
x=0 \quad x=\frac{\pi}{4}, \frac{-\pi}{4}
$$

$\therefore \quad \beta=\sum_{x \in S} \tan ^{2} \frac{x}{3}=\tan ^{2} 0+\tan ^{2} \frac{\pi}{12}+\tan ^{2} \frac{\pi}{12}$
$=2(2-\sqrt{3})^{2}$
$=2(7-4 \sqrt{3})$
$=140-8 \sqrt{3}$
$\therefore \quad \frac{1}{6}(\beta-14)^{2}=\frac{1}{6} \times 64 \times 3=32$
12. Let $A=\{2,3,4\}$ and $B=\{8,9,12\}$. Then the number of elements in the relation
$R=\left\{\left(\left(a_{1}, b_{1}\right),\left(a_{2}, b_{2}\right)\right) \in(A \times B, A \times B): a_{1}\right.$ divides $b_{2}$ and $a_{2}$ divides $\left.b_{1}\right\}$ is
(1) 36
(2) 24
(3) 18
(4) 12

Answer (1)
Sol. $A=\{2,3,4\}, B=\{8,9,12\}$
$a_{1} \in A, b_{2} \in B$
$a_{1}$ divides $b_{2}$
$\left(a_{1}, b_{2}\right) \in\{(2,4),(2,12),(3,9),(3,12),(4,8),(4,12)\}$
$a_{2} \in A, b_{1} \in B$
$a_{2}$ divides $b_{1}$
.... same as above...
$\therefore$ Number of relations $=6 \times 6=36$
13. Let a die be rolled $n$ times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then $k$ is equal to
(1) 60
(2) 15
(3) 90
(4) 30

Answer (1)
Sol. $P($ getting odd 7 time $)=P($ getting odd 9 times $)$
${ }^{n} C_{7}\left(\frac{1}{2}\right)^{7}\left(\frac{1}{2}\right)^{n-7}={ }^{n} C_{9}\left(\frac{1}{2}\right)^{9}\left(\frac{1}{2}\right)^{n-9}$
${ }^{n} C_{7}={ }^{n} C_{9}$
$\Rightarrow n=16$
$P(2$ times even $)={ }^{16} C_{2}\left(\frac{1}{2}\right)^{14}\left(\frac{1}{2}\right)^{2}$
$=\frac{15 \times 4}{2^{15}}=\frac{60}{2^{15}}=\frac{k}{2^{15}}$
$k=60$
14. Let $S=\left\{z=x+i y: \frac{2 z-3 i}{4 z+2 i}\right.$ is a real number $\}$.

Then which of the following is NOT correct?
(1) $y+x^{2}+y^{2} \neq-\frac{1}{4}$
(2) $(x, y)=\left(0,-\frac{1}{2}\right)$
(3) $x=0$
(4) $y \in\left(-\infty,-\frac{1}{2}\right) \cup\left(-\frac{1}{2}, \infty\right)$

Answer (2)
Sol. $\because \quad z=x+i y$ and $\frac{2 z-3 i}{4 z+2 i}$ is real number
$\therefore \frac{2 x+(2 y-3) i}{4 x+(4 y+2) i}$ is real number.
$\therefore \quad \frac{(2 x+(2 y-3) i)(4 x-(4 y+2) i)}{(4 x)^{2}+(4 y+2)^{2}}$ is real number
$\therefore \quad-2 x(4 y+2)+(2 y-3) 4 x=0$
$\therefore \quad x=0$
Here $x=0$ and $y \in \mathbb{R}$ but $y \neq-\frac{1}{2}$ is not acceptable in this case

Denominator will be zero.
Hence, option (2) is correct.
15. If $S_{n}=4+11+21+34+50+\ldots$ to $n$ terms, then $\frac{1}{60}\left(S_{29}-S_{9}\right)$ is equal to
(1) 223
(2) 226
(3) 220
(4) 227

Answer (1)
Sol. $S=4+11+21+34+50+\ldots+T_{n}$
$S=4+11+21+34+\ldots+T_{n-1}+T_{n}$
$T_{n}=4+7+10+13+\ldots$
$T_{n}=4+\frac{1}{2}\left(3 n^{2}+5 n-8\right)$
$\sum T_{n}=S_{n}=\frac{3}{2} \sum n^{2}+\frac{5}{2} \sum n$
$=\frac{3}{2} \frac{n(n+1)(2 n+1)}{6}+\frac{5}{2} \frac{n(n+1)}{2}$
$\frac{S_{29}-S_{9}}{60}=223$
16. If $A=\frac{1}{5!6!7!}\left|\begin{array}{lll}5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9!\end{array}\right|$, then $|\operatorname{adj}(\operatorname{adj}(2 A))|$ is equal to
(1) $2^{20}$
(2) $2^{8}$
(3) $2^{12}$
(4) $2^{16}$

## Answer (4)

Sol. $|A|=\frac{1}{5!6!7!}\left|\begin{array}{lll}5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9!\end{array}\right|$
$=\frac{1}{5!6!7!} 5!\times 6!\times 7!\left|\begin{array}{lll}1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72\end{array}\right|$
$\Rightarrow \quad|A|=2$

$$
\begin{aligned}
|\operatorname{adj}(\operatorname{adj}(2 A))|=|2 A|^{(n-1)^{2}} & =2^{12}|A|^{4} \\
& =2^{12} \cdot 2^{4} \\
& =2^{16}
\end{aligned}
$$

17. Let $A$ be the point $(1,2)$ and $B$ be any point on the curve $x^{2}+y^{2}=16$. If the centre of the locus of the point $P$, which divides the line segment $A B$ in the ratio $3: 2$ is the point $C(\alpha, \beta)$, then the length of the line segment $A C$ is
(1) $\frac{3 \sqrt{5}}{5}$
(2) $\frac{4 \sqrt{5}}{5}$
(3) $\frac{2 \sqrt{5}}{5}$
(4) $\frac{6 \sqrt{5}}{5}$

Answer (1)

Sol. $A(1,2), B(4 \cos \theta, 4 \sin \theta)$
$P\left(\frac{12 \cos \theta+2}{5}, \frac{12 \sin \theta+4}{5}\right) \equiv(h, k)$
$\left(\frac{5 h-2}{12}\right)^{2}+\left(\frac{5 k-4}{12}\right)^{2}=1$
$\left(h-\frac{2}{5}\right)^{2}+\left(k-\frac{4}{5}\right)^{2}=\left(\frac{12}{5}\right)^{2}$
$C\left(\frac{2}{5}, \frac{4}{5}\right)$
$A C=\sqrt{\left(\frac{3}{5}\right)^{2}+\left(\frac{6}{5}\right)^{2}}=\frac{3 \sqrt{5}}{5}$
18. If the coefficients of $x$ and $x^{2}$ in $(1+x)^{p}(1-x)^{q}$ are 4 and -5 respectively, then $2 p+3 q$ is equal to
(1) 60
(2) 69
(3) 66
(4) 63

Answer (4)
Sol. $(1+x)^{p}(1-x)^{q}$

$$
\begin{align*}
& =\left(1+p x+\frac{p(p-1)}{2} x^{2}+\ldots .\right)\left(1-q x+\frac{q(q-1)}{2} x^{2}+\ldots .\right) \\
& =1+(p-q) x+\left(\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-p q\right) x^{2}+\ldots . . \\
& p-q=4 \Rightarrow p=q+4 \quad \ldots \text { (i) } \tag{i}
\end{align*}
$$

$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-p q=-5$
$\frac{(q+4)(q+3)}{2}+\frac{q(q-1)}{2}-(q+4) q=-5$
$\Rightarrow 6-q=-5$
$\Rightarrow q=11$
$\therefore \quad p=15$

$$
2 p+3 q=63
$$

19. If the points $P$ and $Q$ are respectively the circumcenter and the orthocentre of a $\triangle A B C$, then $\overrightarrow{P A}+\overrightarrow{P B}+\overrightarrow{P C}$ is equal to
(1) $2 \overrightarrow{Q P}$
(2) $2 \overrightarrow{P Q}$
(3) $\overrightarrow{P Q}$
(4) $\overrightarrow{Q P}$

## Answer (3)

Sol. Let $G$ be centroid
$\overrightarrow{P Q}=3 \overrightarrow{P G}$
Also $\overrightarrow{P G}=\frac{\overrightarrow{P A}+\overrightarrow{P B}+\overrightarrow{P C}}{3}$
$\Rightarrow \overrightarrow{P A}+\overrightarrow{P B}+\overrightarrow{P C}=3 \overrightarrow{P G}=\overrightarrow{P Q}$
20. Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points $A$ and $B$ respectively. Then the distance of the mid-point of the line segment $A B$ from the plane $2 x-2 y+z=14$ is
(1) 3
(2) $\frac{11}{3}$
(3) 4
(4) $\frac{10}{3}$

Answer (3)

Sol.

$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1} \quad \frac{x+3}{6}=\frac{y-3}{-3}=\frac{z-6}{1}$
For point $A,(\lambda,-2 \lambda+6,5 \lambda-8) \equiv(4 t+5,3 t+7, t-2)$

$$
\Rightarrow t=-1, \lambda=1
$$

$\Rightarrow A(1,4,-3)$
For point $B,(\mu,-2 \mu+6,5 \mu-8)$

$$
\equiv(6 k-3,-3 k+3, k+6) \Rightarrow k=1, \mu=3
$$

$\Rightarrow B(3,0,7)$
mid-point of $A B=M(2,2,2)$
$\Rightarrow$ distance of $M$ from the plane $2 x-2 y+z-14=0$ is 4

## SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10 . The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., $06.25,07.00,-00.33,-00.30,30.27,-27.30$ ) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
21. Suppose $a_{1}, a_{2}, 2, a_{3}, a_{4}$ be in an arithmeticogeometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then $a_{4}$ is equal to $\qquad$ -

## Answer (16)

Sol. Here $r=2$, let common difference $=d$

$$
\begin{align*}
& a_{2}=\left(a_{1}+d\right) \times 2, \quad 2=\left(a_{1}+2 d\right) \times 2^{2}  \tag{i}\\
& a_{3}=\left(a_{1}+3 d\right) \times 2^{3}, \quad a_{4}=\left(a_{1}+4 d\right) \times 2^{4} \\
& a_{1}+a_{2}+2+a_{3}+a_{4}=\frac{49}{2} \\
& 31 a_{1}+98 d=\frac{49}{2} \tag{ii}
\end{align*}
$$

From (i) and (ii)
$a_{1}=0, d=\frac{1}{4}$
$a_{4}=16(0+1)=16$
22. In the figure, $\theta_{1}+\theta_{2}=\frac{\pi}{2}$ and $\sqrt{3}(B E)=4(A B)$. If the area of $\triangle C A B$ is $2 \sqrt{3}-3$ unit $^{2}$, when $\frac{\theta_{2}}{\theta_{1}}$ is the largest, then the perimeter (in unit) of $\triangle C E D$ is equal to $\qquad$ .


Answer (6)

Sol.


Let $A B=x, B D=y$
$\Rightarrow \sqrt{3} B E=4 A B$
$\Rightarrow \sqrt{3}(y+D E)=4 x$
$D E=\frac{4 x}{\sqrt{3}}-y$
$\Delta C A B=2 \sqrt{3}-3$
$\Rightarrow \quad \frac{1}{2} x y=2 \sqrt{3}-3 \mathrm{~s}$
$\tan \theta_{2}=\frac{\left(\frac{4 x}{\sqrt{3}}-y\right)}{x}=\frac{4}{\sqrt{3}}-\frac{(4 \sqrt{3}-6)}{x^{2}}$
$\tan \theta_{1}=\frac{y}{x}=\frac{4 \sqrt{3}-6}{x^{2}}$
$\tan \theta_{1} \cdot \tan \theta_{2}=1$
$\Rightarrow\left(\frac{4}{\sqrt{3}}-\frac{4 \sqrt{3}-6}{x^{2}}\right) \cdot\left(\frac{4 \sqrt{3}-6}{x^{2}}\right)=1$
$\Rightarrow \frac{4 \sqrt{3}-6}{x^{2}}=\sqrt{3}$ or $\frac{1}{\sqrt{3}}$
$\frac{\theta_{2}}{\theta_{1}}$ is maximum when $\frac{4 \sqrt{3}-6}{x^{2}}=\frac{1}{\sqrt{3}}$
$\Rightarrow \quad x=3-\sqrt{3}$ and $\theta_{2}=60^{\circ}$
$\Rightarrow \tan 60^{\circ}=\frac{D E}{C D} \Rightarrow D E=x \cdot \sqrt{3}=3 \sqrt{3}-3$
$\cos 60^{\circ}=\frac{C D}{C E} \Rightarrow C E=2 x=6-2 \sqrt{3}$
$\Rightarrow$ Perimeter of $\triangle C E D=C D+D E+C E=6$
23. If the area of the region $\left\{(x, y):\left|x^{2}-2\right| \leq y \leq x\right\}$ is $A$, then $6 A+16 \sqrt{2}$ is equal to $\qquad$ -.
Answer (27)
Sol.

$A=\frac{1}{2}(1+2) \times 1-\int_{1}^{\sqrt{2}}\left(2-x^{2}\right) d x-\int_{\sqrt{2}}^{2}\left(x^{2}-2\right) d x$
$A=\frac{9}{2}-\frac{8 \sqrt{2}}{3}$
$6 A=27-16 \sqrt{2}$
$\Rightarrow 6 A+16 \sqrt{2}=27$
24. Let the tangent at any point $P$ on a curve passing through the points $(1,1)$ and $\left(\frac{1}{10}, 100\right)$, intersect positive $x$-axis and $y$-axis at the points $A$ and $B$ respectively. If $P A: P B=1: k$ and $y=y(x)$ is the solution of the differential equation $e^{\frac{d y}{d x}}=k x+\frac{k}{2}, y(0)=k$, then $4 y(1)-5 \log _{e} 3$ is equal to $\qquad$ $-$

## Answer (*)

Sol.


Equation of tangent at
$P(x, y)$ is :
$Y-y=\frac{d y}{d x}(X-x)$

Coordinate of $A=\left(x-y \frac{d x}{d y}, 0\right)$
Coordinate of $B=\left(0, y-x \frac{d y}{d x}\right)$
$\therefore \quad(x, y)=\left(\frac{k x-k y \frac{d x}{d y}}{k+1}, \frac{y-x \frac{d y}{d x}}{k+1}\right)$
$\therefore \quad y(k+1)=y-x \frac{d y}{d x}$

$$
\begin{align*}
& k y=-x \frac{d y}{d x} \\
& k \frac{d x}{x}=-\frac{d y}{y} \\
& k \ln |x|+\ln |y|=\ln C \\
& |x|^{k}|y|=C \tag{1}
\end{align*}
$$

$\therefore \quad$ Equation (1) passes through (1, 1) and

$$
\begin{aligned}
& \left(\frac{1}{10}, 100\right) \\
\therefore & C=1 \text { and } k=2 \\
\therefore & |y|=x^{-2} \Rightarrow y=\frac{1}{x^{2}}
\end{aligned}
$$

Now $e^{\frac{d y}{d x}}=2 x+1$

$$
\begin{aligned}
& \frac{d y}{d x}=\ln (2 x+1) \\
& y=\frac{1}{2}(2 x+1)(\ln |2 x+1|-1)+C
\end{aligned}
$$

$\therefore \quad$ This passes through (0, 2)
$\therefore \quad C=\frac{5}{2}$
$\therefore \quad 2 y=(2 x+1)(\ln |2 x+1|-1)+5$
$2(y(1))=3(\ln (3)-1)+5$
$2 y(1)=3 \ln (3)+2$
No integral value is possible
25. Let the foot of perpendicular from the point $A(4,3$, 1) on the plane $P: x-y+2 z+3=0$ be $N$. If $B(5$, $\alpha, \beta), \alpha, \beta \in \mathbb{Z}$ is a point on plane $P$ such that the area of the triangle $A B N$ is $3 \sqrt{2}$, then $\alpha^{2}+\beta^{2}+\alpha \beta$ is equal to
Answer (07.00)

Sol. $5-\alpha+2 \beta+3=0$
$\alpha-2 \beta=8$
$\frac{x_{1}-4}{1}=\frac{y_{1}-3}{-1}=\frac{z_{1}-1}{2}=\frac{-(6)}{6}$
$N \equiv\left(x_{1}, y_{1}, z_{1}\right)=(3,4,-1)$
$B \equiv(5, \alpha, \beta)$
$\frac{1}{2} A N \times B N=3 \sqrt{2}$
$A N \cdot B N=6 \sqrt{2}$
$\frac{6}{\sqrt{6}} \cdot B N=6 \sqrt{2}$
$B N=2 \sqrt{3}$
$4+(\alpha-4)^{2}+(\beta+1)^{2}=12$
$(\alpha-4)^{2}+(\beta+1)^{2}=8$
$(2 \beta+4)^{2}+(\beta+1)^{2}=8$
$5 \beta^{2}+18 \beta+9=0$
$\beta=-3,-\frac{3}{5}$
$\beta=-3, \alpha=2$
26. The sum of all the four-digit numbers that can be formed using all the digits $2,1,2,3$ is equal to $\qquad$ .
Answer (26664)
Sol. Sum of all unit place numbers
$=3!\times 2+\frac{3!}{2!} \times 3+\frac{3!}{2!}$
$=24$
Sum of all numbers $=1.24+10.24+100.24+$ $1000 \cdot 24$

$$
=26664
$$

27. If the domain of the function $f(x)=\sec ^{-1}\left(\frac{2 x}{5 x+3}\right)$ is $[\alpha, \beta) \cup(\gamma, \delta]$, then $|3 \alpha+10(\beta+\gamma)+21 \delta|$ is equal to $\qquad$ -.
Answer (24.00)
Sol. $\frac{2 x}{5 x+3} \geq 1$ OR $\leq-1$
$\frac{-3 x-3}{5 x+3} \geq 0 \quad \frac{7 x+3}{5 x+3} \leq 0$
$\frac{x+1}{5 x+3} \quad 0 \quad x \in\left(\frac{-3}{5}, \frac{-3}{7}\right]$
$x \in\left[-1,-\frac{3}{5}\right)$
$3 \alpha=-3,10(\beta+\gamma)=-12,21 \delta=-9$
28. Let the equations of two adjacent sides of a parallelogram $A B C D$ be $2 x-3 y=-23$ and $5 x+4 y$ $=23$. If the equation of its one diagonal $A C$ is $3 x+7 y=23$ and the distance of $A$ from the other diagonal is $d$, then $50 d^{2}$ is equal to $\qquad$ .

## Answer (529)

Sol. $A B \equiv 2 x-3 y=-23$
$B C \equiv 5 x+4 y=23$
$A C \equiv 3 x+7 y=23$
Solving the lines pairwise gives
$A(-4,5), B(-1,7), C(3,2)$

$\because \quad A C$ and $B D$ have same mid-point
$\Rightarrow D$ is $(0,0)$
Equation of $B D$ is $7 x+4=0$

$$
d=\left|\frac{7(-4)+5}{\sqrt{7^{2}+1^{2}}}\right|=\frac{23}{\sqrt{50}}
$$

Hence, $50 d^{2}=23^{2}=529$
29. Let the quadratic curve passing through the point $(-1,0)$ and touching the line $y=x$ at $(1,1)$ be $y=f(x)$. Then the $x$-intercept of the normal to the curve at the point $(\alpha, \alpha+1)$ in the first quadrant is $\qquad$ .

## Answer (11)

Sol. Let the curve be $y=a x^{2}+b x+c$
Passes through $(-1,0)$
$\Rightarrow a-b+c=0$
Passes through $(1,1)$
$\Rightarrow a+b+c=1$
Tangent to $y=x$ at $(1,1)$
$\left.\Rightarrow \frac{d y}{d x}\right|_{x=1}=1$
$\Rightarrow 2 a+b=1$
...(iii)

Solving (i), (ii) and (iii)
$a=\frac{1}{4}, b=\frac{1}{2}$ and $c=\frac{1}{4}$
Hence the curve is $y=\frac{x^{2}}{4}+\frac{x}{2}+\frac{1}{4}$
$(\alpha, \alpha+1)$ lies on curve
$\Rightarrow \quad \alpha+1=\frac{\alpha^{2}}{4}+\frac{\alpha}{2}+\frac{1}{4}$
$\Rightarrow(\alpha-3)(\alpha+1)=0$
$\Rightarrow \quad \alpha=3$
Point in first quadrant is $(3,4)$
$\left.\frac{d y}{d x}\right|_{x=3}=\frac{x}{2}+\left.\frac{1}{2}\right|_{x=3}=2$
$\Rightarrow$ Slope of normal $=\frac{-1}{2}$
Equation of normal $y-4=-\frac{1}{2}(x-3)$
$x$-intercept $=11$
30. Let $S$ be the set of values of $\lambda$, for which the system of equations
$6 \lambda x-3 y+3 z=4 \lambda^{2}$,
$2 x+6 \lambda y+4 z=1$,
$3 x+2 y+3 \lambda z=\lambda$ has no solution. Then $12 \sum_{\lambda \in S}|\lambda|$ is equal to $\qquad$ .

## Answer (24)

Sol. For no solution

$$
\begin{aligned}
& \left|\begin{array}{ccc}
6 \lambda & -3 & 3 \\
2 & 6 \lambda & 4 \\
3 & 2 & 3 \lambda
\end{array}\right|=0 \\
& \Rightarrow 9 \lambda^{3}-7 \lambda-2=0 \\
& \Rightarrow \quad(\lambda-1)(3 \lambda+1)(3 \lambda+2)=0 \\
& \Rightarrow 12 \sum_{\lambda \in S}|\lambda|=12 \times\left(1+\frac{1}{3}+\frac{2}{3}\right)=24
\end{aligned}
$$

## PHYSICS

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer:

31. The half life of a radioactive substance is T . The time taken, for disintegrating $\frac{7}{8}$ th part of its original mass will be:
(1) 2 T
(2) $3 T$
(3) T
(4) 8 T

Answer (2)
Sol. $t_{1 / 2}=T$

$$
\begin{aligned}
\text { Remaining } & =\left(\frac{N_{0}}{8}\right) \\
& =\frac{N_{0}}{(2)^{3}}
\end{aligned}
$$

$\Rightarrow 3$ half life
$\Rightarrow t=3 \mathrm{~T}$
32. Two projectiles are projected at $30^{\circ}$ and $60^{\circ}$ with the horizontal with the same speed. The ratio of the maximum height attained by the two projectiles respectively is:
(1) $\sqrt{3}: 1$
(2) $1: \sqrt{3}$
(3) $2: \sqrt{3}$
(4) $1: 3$

Answer (4)
Sol. $\frac{\left(H_{1}\right)_{\max }}{\left(H_{2}\right)_{\max }}=\frac{\left(u_{1}^{2} \sin ^{2} \theta_{1}\right)_{1}}{\left(u_{2}^{2} \sin ^{2} \theta_{2}\right)_{2}}=\left(\frac{\sin 30}{\sin 60}\right)^{2}=\left(\frac{1}{\sqrt{3}}\right)^{2}=\frac{1}{3}$
33. A gas is compressed adiabatically, which one of the following statement is NOT true?
(1) There is no heat supplied to the system
(2) There is no change in the internal energy
(3) The temperature of the gas increases
(4) The change in the internal energy is equal to the work done on the gas
Answer (2)
Sol. If adiabatic process,
$\Delta Q=0$, and $\Delta U \neq 0$
$\Delta U=-\Delta W$
If gas is compressed
$\Delta W<0$
$\Delta U>0$
$\Delta T>0$
34. The amplitude of magnetic field in an electromagnetic wave propagating along $y$-axis is $6.0 \times 10^{-7} \mathrm{~T}$. The maximum value of electric field in the electromagnetic wave is
(1) $2 \times 10^{15} \mathrm{Vm}^{-1}$
(2) $180 \mathrm{Vm}^{-1}$
(3) $6.0 \times 10^{-7} \mathrm{Vm}^{-1}$
(4) $5 \times 10^{14} \mathrm{Vm}^{-1}$

Answer (2)
Sol. $E_{0}=c B_{0}$

$$
\begin{aligned}
& =3 \times 10^{8} \times 6 \times 10^{-7} \\
& =180 \mathrm{Vm}^{-1}
\end{aligned}
$$

35. Given below are two statements:

Statement I : Rotation of the earth shows effect on the value of acceleration due to gravity (g).
Statement II : The effect of rotation of the earth on the value of ' $g$ ' at the equator is minimum and that at the pole is maximum.
In the light of the above statements, choose the correct answer from the options given below
(1) Statement I is false but statement II is true
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is true but statement II is false

Answer (4)
Sol. Rotation shows effect on acceleration due to gravity (g).

Effect of rotation on g is maximum at equator and minimum at pole.
36. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason $\mathbf{R}$
Assertion A : An electric fan continues to rotate for some time after the current is switched off.
Reason R: Fan continues to rotate due to inertia of motion.
In the light of above statements, choose the most appropriate answer from the options given below.
(1) $\mathbf{A}$ is correct but $\mathbf{R}$ is not correct
(2) $\mathbf{A}$ is not correct but $\mathbf{R}$ is correct
(3) Both $\mathbf{A}$ and $\mathbf{R}$ are correct and $\mathbf{R}$ is the correct explanation of $\mathbf{A}$
(4) Both $\mathbf{A}$ and $\mathbf{R}$ are correct but $\mathbf{R}$ is NOT the correct explanation of $\mathbf{A}$

## Answer (3)

Sol. Fan continue to rotate due to inertia of motion.
37. Young's moduli of the material of wires $A$ and $B$ are in the ratio of $1: 4$, while its area of cross sections are in the ratio of $1: 3$. If the same amount of load is applied to both the wires, the amount of elongation produced in the wires $A$ and $B$ will be in the ratio of
[Assume length of wires $A$ and $B$ are same]
(1) $12: 1$
(2) $1: 36$
(3) $36: 1$
(4) $1: 12$

Answer (1)
Sol. $\Delta L=\left(\frac{F L}{A Y}\right)$
$\frac{\Delta L_{1}}{\Delta L_{2}}=\frac{F_{1}}{F_{2}} \times\left(\frac{A_{2}}{A_{1}}\right)\left(\frac{Y_{2}}{Y_{1}}\right)$
$=1 \times\left(\frac{3}{1}\right) \times\left(\frac{4}{1}\right)$
$\frac{\Delta L_{1}}{\Delta L_{2}}=12: 1$
38. A person travels $x$ distance with velocity $v_{1}$ and then $x$ distance with velocity $v_{2}$ in the same direction. The average velocity of the person is $v$, then the relation between $v, v_{1}$ and $v_{2}$ will be
(1) $v=\frac{v_{1}+v_{2}}{2}$
(2) $\frac{1}{v}=\frac{1}{v_{1}}+\frac{1}{v_{2}}$
(3) $v=v_{1}+v_{2}$
(4) $\frac{2}{v}=\frac{1}{v_{1}}+\frac{1}{v_{2}}$

## Answer (4)

Sol. $V_{\mathrm{avg}}=\frac{x+x}{\left(\frac{x}{v_{1}}+\frac{x}{v_{2}}\right)}=\left(\frac{2 v_{1} v_{2}}{v_{1}+v_{2}}\right)$
39. For a periodic motion represented by the equation
$y=\sin \omega t+\cos \omega t$
the amplitude of the motion is
(1) 1
(2) 0.5
(3) 2
(4) $\sqrt{2}$

Answer (4)
Sol. $Y=\sin \omega t+\cos \omega t$
$=\sqrt{2} \sin \left(\omega t+\frac{\pi}{4}\right)$
Amplitude $=\sqrt{2}$
40. A massage signal of frequency 3 kHz is used to modulate a carrier signal of frequency 1.5 MHz . The bandwidth of the amplitude modulated wave is
(1) 6 MHz
(2) 6 kHz
(3) 3 MHz
(4) 3 kHz

Answer (2)
Sol. Bandwidth $=2 \times$ signal frequency

$$
\begin{aligned}
& =2 \times 3 \mathrm{kHz} \\
& =6 \mathrm{kHz}
\end{aligned}
$$

41. The ratio of intensities at two points $P$ and $Q$ on the screen in a Young's double slit experiment where phase difference between two waves of same amplitude are $\pi / 3$ and $\pi / 2$, respectively are
(1) $2: 3$
(2) $1: 3$
(3) $3: 1$
(4) $3: 2$

Answer (4)
Sol. $I=4 I_{0} \cos ^{2}\left(\frac{\phi}{2}\right)$
$\frac{I_{1}}{l_{2}}=\frac{\cos ^{2}\left(\frac{\pi}{6}\right)}{\cos ^{2}\left(\frac{\pi}{4}\right)}=\frac{\left(\frac{3}{4}\right)}{\frac{1}{2}}=\left(\frac{3}{2}\right)$
42. The time period of a satellite, revolving above earth's surface at a height equal to $R$ will be
(Given $g=\pi^{2} \mathrm{~m} / \mathrm{s}^{2}, R=$ radius of earth)
(1) $\sqrt{2 R}$
(2) $\sqrt{8 R}$
(3) $\sqrt{32 R}$
(4) $\sqrt{4 R}$

Answer (3)
Sol. $v=\sqrt{\left(\frac{G M}{2 R}\right)}$
$T=\frac{2 \pi R}{\sqrt{\frac{G M}{2 R}}}=\frac{2 \pi R \times 2}{\sqrt{\frac{g R}{2}}}=\frac{4 \pi R \sqrt{2}}{\pi \sqrt{R}}$
$\Rightarrow T=4 \sqrt{2 R}=\sqrt{32 R}$
43. If each diode has a forward bias resistance of $25 \Omega$ in the below circuit,


Which of the following options is correct?
(1) $\frac{l_{1}}{l_{2}}=1$
(2) $\frac{I_{2}}{I_{3}}=1$
(3) $\frac{l_{1}}{l_{2}}=2$
(4) $\frac{I_{3}}{I_{4}}=1$

Answer (3)
Sol. $I_{1}=\left(\frac{5}{100}\right)$

$$
\frac{l_{1}}{l_{2}}=2
$$

$I_{2}=\left(\frac{5}{200}\right)$
$\frac{I_{2}}{I_{3}}=\infty$
$I_{3}=0$
$I_{4}=\left(\frac{5}{200}\right) \quad \frac{I_{3}}{I_{4}}=0$
44. In a metallic conductor, under the effect of applied electric field, the free electrons of the conductor
(1) Drift from higher potential to lower potential
(2) Move with the uniform velocity throughout from lower potential to higher potential
(3) Move in the straight line paths in the same direction
(4) Move in the curved paths from lower potential to higher potential

## Answer (4)

Sol. Electron drifts from lower potential to higher potential on curved path.
45. The variation of stopping potential $\left(\mathrm{V}_{0}\right)$ as a function of the frequency $(v)$ of the incident light for a metal is shown in figure. The work function of the surface is

(1) 2.98 eV
(2) 2.07 eV
(3) 1.36 eV
(4) 18.6 eV

Answer (2)
Sol. $e V_{0}=\frac{h c}{\lambda}-\phi$
$V_{0}=\frac{h c}{e \lambda}-\left(\frac{\phi}{e}\right)=\frac{h \nu}{e}-\left(\frac{\phi}{e}\right)$

When $V_{0}=0$,

$$
\begin{aligned}
\frac{\phi}{e} & =\left(\frac{h v}{e}\right) \\
\phi & =h v=\frac{6.626 \times 10^{-34} \times 5 \times 10^{14}}{1.6 \times 10^{-19}} \\
& =\frac{6.626 \times 5}{1.6} \times 10^{-1} \\
& =\frac{6.626 \times 5}{16} \approx 2.07 \mathrm{eV}
\end{aligned}
$$

46. A bar magnet is released from rest along the axis of a very long vertical copper tube. After some time the magnet will
(1) move down with an acceleration equal to $g$
(2) oscillate inside the tube
(3) move down with almost constant speed
(4) move down with an acceleration greater than g

## Answer (3)

Sol. Due to change in magnetic field eddy current is generated and this will oppose the motion, so after attaining some speed, magnet will move down with constant speed.
47. The distance between two plates of a capacitor is d and its capacitance is $C_{1}$, when air is the medium between the plates. If a metal sheet of thickness $\frac{2 d}{3}$ and of the same area as plate is introduced between the plates, the capacitance of the capacitor becomes $C_{2}$. The ratio $\frac{C_{2}}{C_{1}}$ is
(1) $3: 1$
(2) $2: 1$
(3) $4: 1$
(4) $1: 1$

Answer (1)
Sol.

$C_{1}=\left(\frac{\varepsilon_{0} A}{d}\right)$

$C_{\text {eq }}=\frac{\frac{\varepsilon_{0} A}{(d / 3)} \times \frac{k \varepsilon_{0} A}{(2 d / 3)}}{\frac{\varepsilon_{0} A}{d / 3}+\frac{k \varepsilon_{0} A}{(2 d / 3)}}$
$C_{\text {eq }}=\frac{9 k \varepsilon_{0} A}{(6+3 k) d}$ as $k \rightarrow \infty$
$C_{2}=C_{\text {eq }}=\frac{9 \varepsilon_{0} A}{\left(\frac{6}{k}+3\right) d}=\left(\frac{3 \varepsilon_{0} A}{d}\right)$
$\Rightarrow \frac{C_{2}}{C_{1}}=3$
48. A gas mixture consists of 2 moles of oxygen and 4 moles of neon at temperature T. Neglecting all vibrational modes, the total internal energy of the system will be,
(1) $11 R T$
(2) $8 R T$
(3) $4 R T$
(4) $16 R T$

Answer (1)
Sol. Internal energy $=n_{1} C V_{1} T+n_{2} C V_{2} T$

$$
\begin{aligned}
& =2 \times\left(\frac{5 R}{2}\right) T+4 \times \frac{3 R}{2} T \\
& =11 R T
\end{aligned}
$$

49. In an experiment with vernier callipers of least count 0.1 mm , when two jaws are joined together the zero of vernier scale lies right to the zero of the main scale and $6^{\text {th }}$ division of vernier scale coincides with the main scale division, While measuring the diameter of a spherical bob, the zero of vernier scale lies in between 3.2 cm and 3.3 cm marks and $4^{\text {th }}$ division of vernier scale coincides with the main scale division. The diameter of bob is measured as
(1) 3.22 cm
(2) 3.18 cm
(3) 3.26 cm
(4) 3.25 cm

## Answer (2)

Sol. L.C. $=0.1 \mathrm{~mm}$
For $6^{\text {th }}$ division $=0.6 \mathrm{~mm}$
Reading $=3.20+0.04-0.06$

$$
=3.20-0.02=3.18 \mathrm{~cm}
$$

50. Given below are two statements:

Statement I: For diamagnetic substance $-1 \leq x<0$, where $x$ is the magnetic susceptibility.
Statement II: Diamagnetic substance when placed in an external magnetic field, tend to move from stronger to weaker part of the field.

In the light of the above statements, choose the correct answer from the options give below.
(1) Both Statement I and Statement II are False
(2) Statement I is correct but Statement II is false
(3) Statement I is incorrect but Statement II is true
(4) Both Statement I and Statement II are true

Answer (4)
Sol. For diamagnetic substance ( $-1<x<0$ )
and diamagnetic is repelled by the magnetic field

## SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10 . The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, $-00.33,-00.30,30.27,-27.30$ ) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
51. A point object, ' $O$ ' is placed in front of two thin symmetrical coaxial convex lenses $L_{1}$ and $L_{2}$ with focal length 24 cm and 9 cm respectively. The distance between two lenses is 10 cm and the object is placed 6 cm away from lens $L_{1}$ as shown in the figure. The distance between the object and the image formed by the system of two lenses is
$\qquad$ cm


Answer (34)
Sol.


For $L_{1}$
For $L_{2}$
$\frac{1}{v}+\frac{1}{6}=\frac{1}{24}$
$u=-18 \mathrm{~cm}$
$\frac{1}{v}=\frac{1}{24}-\frac{1}{6}$
$\frac{1}{v}+\frac{1}{18}=\frac{1}{9}$

$$
\begin{array}{rlrl} 
& =\frac{1-4}{24} & \frac{1}{v} & =\frac{1}{9}-\frac{1}{18} \\
\frac{1}{v} & =\left(\frac{-3}{24}\right) & & =\frac{2-1}{18} \\
v & =-8 \mathrm{~cm} & \frac{1}{v} & =\frac{1}{18} \\
& v & =18 \mathrm{~cm}
\end{array}
$$

Distance between object and distance $=16+18$

## 34 cm

52. A force of $-P \hat{k}$ acts on the origin of the coordinate system. The torque about the point $(2,-3)$ is $P(a \hat{i}+b \hat{j})$, The ratio of $\frac{a}{b}$ is $\frac{x}{2}$. The value of $x$ is

## Answer (3)

Sol. $\vec{F}=-p \hat{k}$

$$
\begin{aligned}
& \vec{r}=(0-2) \hat{i}+(0+3) \hat{j}=-2 \hat{i}+3 \hat{j} \\
& \vec{r} \times \vec{F}=\vec{\tau} \Rightarrow(-2 \hat{i}+3 \hat{j}) \times(-p \hat{k}) \\
& \\
& \quad \vec{\tau}=-2 p \hat{j}-3 p \hat{i}=p(a \hat{i}+b \hat{j}) \\
& \Rightarrow \quad a=-3 \\
& \quad b=-2 \\
& \therefore \quad \frac{a}{b}=\left(\frac{3}{2}\right) \Rightarrow \text { so } x=3
\end{aligned}
$$

53. An electron revolves around an infinite cylindrical wire having uniform linear charge density $2 \times 10^{-8}$ C $\mathrm{m}^{-1}$ in circular path under the influence of attractive electrostatic field as shown in the figure. The velocity of electron with which it is revolving is
$\qquad$ $\times 10^{6} \mathrm{~m} \mathrm{~s}^{-1}$. Given mass of electron $=9$ $\times 10^{-31} \mathrm{~kg}$


Answer (8)
Sol. $E=\left(\frac{2 k \lambda}{r}\right)$
Force $=\left(\frac{m v^{2}}{r}\right)$
$\Rightarrow \quad \frac{2 k \lambda e}{r}=\frac{m v^{2}}{r}$
$v=\sqrt{\frac{2 k \lambda e}{m}}=\sqrt{\frac{2 \times 9 \times 10^{9} \times 2 \times 10^{-8} \times 1.6 \times 10^{-19}}{9 \times 10^{-31}}}$
$=\sqrt{1.6 \times 4 \times 10^{13}}=4 \times 2 \times 10^{6} \mathrm{~m} / \mathrm{s}$
$=8 \times 10^{6} \mathrm{~m} / \mathrm{s}$
54. A square loop of side 2.0 cm is placed inside a long solenoid that has 50 turns per centimetre and carries a sinusoidally varying current of amplitude 2.5 A and angular frequency $700 \mathrm{rad} \mathrm{s}^{-1}$. The central axes of the loop and solenoid coincide. The amplitude of the emf induced in the loop is $x \times 10^{-4} \mathrm{~V}$. The value of $x$ is
$\qquad$ -.
(Take, $\pi=\frac{22}{7}$ )

## Answer (44)

Sol. $\qquad$

$B=\left(\mu_{0} n i\right)=\mu_{0} \times 5000 \times 2.5 \sin (700 t)$
$B=12500 \times 2.5 \sin (700 t)$
$\phi=(2 \times 2) \times 10^{-4} \times 12500 \mu_{0} \sin (700 t)$
$\phi=5 \mu_{0} \sin (700 t)$
$\frac{d \phi}{d t}=\varepsilon_{\text {ind }}=3500 \times \frac{22}{7} \times 4 \times 10^{-7} \cos (700 t)$
$=44 \times 10^{-4} \cos (700 t)$
55. If the maximum load carried by an elevator is 1400 $\mathrm{kg}(600 \mathrm{~kg}-$ Passengers +800 kg - elevator), which is moving up with a uniform speed of $3 \mathrm{~m} \mathrm{~s}^{-1}$ and the frictional force acting on it is 2000 N , then the maximum power used by the motor is $\qquad$ $\mathrm{kW}\left(g=10 \mathrm{~m} / \mathrm{s}^{2}\right)$
Answer (48)
Sol.

$F_{\text {net }}=14000+2000$
$=16000 \mathrm{~N}$
$p=16000 \times 3$
$=48000 \mathrm{~W}$
$=48 \mathrm{~kW}$
56. If $917 \AA$ be the lowest wavelength of Lyman series then the lowest wavelength of Balmer series will be
$\qquad$ Å.

## Answer (3668)

Sol. $\Delta E=\frac{-13.6 z^{2}}{1}$ Lyman series
$\lambda_{1}=\left(\frac{h c}{\Delta E}\right)$
Similarly for Balmer series, $\Delta E=\frac{-13.6 z^{2}}{4}$

$$
\begin{aligned}
& \lambda_{2}=\frac{h c}{\left(\frac{-13.6 z^{2}}{4}\right)}=\left(\frac{h c}{\Delta E}\right) \\
& \therefore \quad \frac{\lambda_{1}}{\lambda_{2}}=\left(\frac{\Delta E_{2}}{\Delta E_{1}}\right)=\frac{1}{4} \\
& \Rightarrow \quad \lambda_{2}=4 \lambda \\
& \Rightarrow \quad \lambda_{2}=4 \times 917 \\
& \quad=3668 \AA
\end{aligned}
$$

57. A rectangular parallelopiped is measured as $1 \mathrm{~cm} \times 1 \mathrm{~cm} \times 100 \mathrm{~cm}$. If its specific resistance is $3 \times 10^{-7} \Omega \mathrm{~m}$, then the resistance between its two opposite rectangular faces will be $\qquad$ $\times 10^{-7} \Omega$.

## Answer (3)

Sol.

$I=1 \mathrm{~cm}=0.01 \mathrm{~m}$
$A=100 \times 1=100 \mathrm{~cm}^{2}$

$$
=0.01 \mathrm{~m}^{2}
$$

$R=\frac{\rho l}{A}=\frac{3 \times 10^{-7} \times 0.1}{0.1}=3 \times 10^{-7} \Omega$
58. A straight wire carrying a current of 14 A is bent into a semicircular arc of radius 2.2 cm as shown in the figure. The magnetic field produced by the current at the centre $(\mathrm{O})$ of the arc is $\qquad$ $\times 10^{-4} \mathrm{~T}$


## Answer (2)

Sol.


$$
B=\frac{\mu_{0} i}{4 R}=\frac{4 \pi \times 10^{-7} \times 14}{4 \times 2.2 \times 10^{-2}}
$$

$=\frac{22}{7} \times \frac{14}{22} \times 10^{3} \times 10^{-7}$
$=2 \times 10^{-4} \mathrm{~T}$
59. Figure below shows a liquid being pushed out of the tube by a piston having area of cross section $2.0 \mathrm{~cm}^{2}$. The area of cross section at the outlet is $10 \mathrm{~mm}^{2}$. If the piston is pushed at a speed of $4 \mathrm{~cm} \mathrm{~s}^{-1}$, the speed of outgoing fluid is $\qquad$ $\mathrm{cm} \mathrm{s}^{-1}$


## Answer (80)

Sol. $A_{1} V_{1}=A_{2} V_{2}$
$\Rightarrow \quad V_{2}=\left(\frac{A_{1} V_{1}}{A_{2}}\right)=\frac{2 \times 10^{-4} \times 4}{10 \times 10^{-6}}$
$=80 \mathrm{~cm} / \mathrm{s}$
60. A rectangular block of mass 5 kg attached to a horizontal spiral spring executes simple harmonic motion of amplitude 1 m and time period 3.14 s . The maximum force exerted by spring on block is $\qquad$ N

## Answer (20)

Sol. $F=m a$

$$
\begin{aligned}
& a_{\max }=\omega^{2} A \\
& T=\frac{2 \pi}{\omega}=3.14 \\
& \Rightarrow \quad \omega=2 \mathrm{rad} / \mathrm{sec} \\
& a_{\max }=4 \times 1 \\
& \quad=4 \mathrm{~m} / \mathrm{s}^{2} \\
& \therefore \quad F=5 \times 4=20 \mathrm{~N}
\end{aligned}
$$

## CHEMISTRY

## SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

## Choose the correct answer:

61. Ferric chloride is applied to stop bleeding because
(1) Blood absorbs $\mathrm{FeCl}_{3}$ and forms a complex.
(2) $\mathrm{Cl}^{-}$ions cause coagulation of blood.
(3) $\mathrm{Fe}^{3+}$ ions coagulate blood which is a negatively charged sol.
(4) $\mathrm{FeCl}_{3}$ reacts with the constituents of blood which is a positively charged sol.

## Answer (3)

Sol. Blood is negatively charged sol and $\mathrm{Fe}^{3+}$ lons will coagulate blood.
62. Match List-I with List-II.

| List-I <br> Complex |  | List-II <br> Crystal Field splitting <br> energy ( $\Delta 0$ |  |
| :--- | :--- | :--- | :--- |
| A. | $\left[\mathrm{Ti}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+}$ | I. | -1.2 |
| B. | $\left[\mathrm{V}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{2+}$ | II. | -0.6 |
| C. | $\left[\mathrm{Mn}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+}$ | III. | 0 |
| D. | $\left[\mathrm{Fe}\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}\right]^{3+}$ | IV. | -0.8 |

Choose the correct answer from the options given below:
(1) A-II, B-IV, C-I, D-III
(2) A-IV, B-I, C-III, D-II
(3) A-IV, B-I, C-II, D-III
(4) A-II, B-IV, C-III, D-I

## Answer (3)

Sol. (A) $\mathrm{Ti}^{2+}: 4 s^{0} 3 d^{2}$

$$
\Delta_{0}=2(-.4 \Delta 0)=-.8 \Delta_{0}
$$

(B) $\mathrm{V}^{2+}: 4 s^{0} 3 d^{\beta}$

$$
\begin{aligned}
\Delta_{0} & =3(-.4 \Delta 0) \\
& =-1.2 \Delta_{0}
\end{aligned}
$$

(C) $\mathrm{Mn}^{3+}=4 s^{0} 3 d^{4}$

$$
\begin{aligned}
\Delta_{0} & =3\left(-.4 \Delta_{0}\right)+1\left(.6 \Delta_{0}\right) \\
& =-1.2 \Delta_{0}+0.6 \Delta_{0} \\
& =-.6 \Delta_{0}
\end{aligned}
$$

(D) $\mathrm{Fe}^{3+}=4 \mathrm{~s}^{0} 3 d^{5}$

$$
\begin{aligned}
\Delta_{0} & =3\left(-.4 \Delta_{0}\right)+2\left(.6 \Delta_{0}\right) \\
& =-1.2 \Delta_{0}+1.2 \Delta_{0}=0
\end{aligned}
$$

63. The major product ' $P$ ' formed in the given reaction is

(2)

(3)

(4)


Answer (1)
Sol.

(P)
$\mathrm{OCH}_{3}$ is ortho/para directing, major product will be obtained from para attack.
64. In the reaction given below


The product ' $X$ ' is :
(1)

(2)

(3)

(4)


Answer (3)
Sol. Amide $\xrightarrow{\mathrm{LAAH}_{4}}$ Amine
Ketone $\xrightarrow{\mathrm{LiAl}_{4}}$ Alcohol
Double Bond $\xrightarrow{\mathrm{LiAl}_{4}}$ No effect
65. Number of water molecules in washing soda and soda ash respectively are :
(1) 1 and 10
(2) 10 and 1
(3) 10 and 0
(4) 1 and 0

Answer (3)
Sol. $\mathrm{Na}_{2} \mathrm{CO}_{3} .10 \mathrm{H}_{2} \mathrm{O}$ is washing soda
$\mathrm{Na}_{2} \mathrm{CO}_{3}$ is soda ash
66. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Physical properties of isotopes of hydrogen are different.

Reason R : Mass difference between isotopes of hydrogen is very large.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both $\mathbf{A}$ and $\mathbf{R}$ are true and $\mathbf{R}$ is the correct explanation of $\mathbf{A}$
(2) $\mathbf{A}$ is false but $\mathbf{R}$ is true
(3) $\mathbf{A}$ is true but $\mathbf{R}$ is false
(4) Both $\mathbf{A}$ and $\mathbf{R}$ are true but $\mathbf{R}$ is NOT the correct explanation of $\mathbf{A}$

## Answer (1)

Sol. Physical properties of isotopes of hydrogen are different.

Mass of D is $100 \%$ more than mass of H and mass of T is $200 \%$ more than mass of H
67. The decreasing order of hydride affinity for following carbocations is:
(A)

(B)

(C)

(D)


Choose the correct answer from the options given below:
(1) C, A, D, B
(2) $C, A, B, D$
(3) A, C, D, B
(4) A, C, B, D

## Answer (2)

Sol. Least stable carbocation will have highest hydride affinity.

Stability of carbocation:
D $>\mathrm{B}>\mathrm{A}>\mathrm{C}$
Hydride affinity order:
C $>\mathrm{A}>\mathrm{B}>\mathrm{D}$
68. The correct order of the number of unpaired electrons in the given complexes is
(A) $\left[\mathrm{Fe}(\mathrm{CN})_{6}\right]^{3-}$
(B) $\left[\mathrm{FeF}_{6}\right]^{3-}$
(C) $\left[\mathrm{CoF}_{6}\right]^{3-}$
(D) $\left[\mathrm{Cr}(\right.$ oxalate $) 3^{3-}$
(E) $\left[\mathrm{Ni}(\mathrm{CO})_{4}\right]$

Choose the correct answer from the options given below:
(1) E $<$ A $<$ D $<$ C $<$ B
(2) E $<$ A $<$ B $<$ D $<$ C
(3) A $<$ E, C $<$ B $<$ D
(4) $\mathrm{A}<$ E $<$ D $<$ C $<$ B

## Answer (1)

Sol.

| $A$ | $\left[\mathrm{Fe}(\mathrm{CN})_{6}\right]^{3-}$ | 1 |
| :---: | :--- | :---: |
| $B$ | $\left[\mathrm{FeF}_{6}\right]^{3-}$ | 5 |
| $C$ | $\left.[\mathrm{CoF}]_{6}\right]^{3-}$ | 4 |
| $D$ | $\left[\mathrm{Cr}(\mathrm{Ox})_{3}\right]^{3-}$ | 3 |
| $E$ | $\left[\mathrm{Ni}(\mathrm{CO})_{4}\right]$ | 0 |

No. of unpaired electrons order:
E $<$ A $<$ D $<$ C B
69. The reaction used for preparation of soap from fat is :
(1) an addition reaction
(2) reduction reaction
(3) alkaline hydrolysis reaction
(4) an oxidation reaction

Answer (3)
Sol.


Reference: NCERT Class-XII Pg-458
70. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : The energy required to form $\mathrm{Mg}^{2+}$ from Mg is much higher than that required to produce $\mathrm{Mg}^{+}$
Reason $\mathbf{R}$ : $\mathrm{Mg}^{2+}$ is small ion and carry more charge than $\mathrm{Mg}^{+}$
In the light of the above statements, choose the correct answer from the options given below.
(1) Both $\mathbf{A}$ and $\mathbf{R}$ are true but $\mathbf{R}$ is NOT the correct explanation of $\mathbf{A}$
(2) $\mathbf{A}$ is true but $\mathbf{R}$ is false
(3) $\mathbf{A}$ is false but $\mathbf{R}$ is true
(4) Both $\mathbf{A}$ and $\mathbf{R}$ are true and $\mathbf{R}$ is the correct explanation of $\mathbf{A}$
Answer (4)

Sol. $\mathrm{IE}_{2}$ is always greater than $\mathrm{IE}_{1}$ because on removal of $\mathrm{e}^{-}$, size decreases and effective nuclear charge increases.
71. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: 3.1500 g of hydrated oxalic acid dissolved in water to make 250.0 mL solution will result in 0.1 M oxalic acid solution.

Reason R: Molar mass of hydrated oxalic acid is $126 \mathrm{~g} \mathrm{~mol}^{-1}$.

In the light of the above statements, choose the correct answer from the options given below:
(1) Both $\mathbf{A}$ and $\mathbf{R}$ are true but $\mathbf{R}$ is NOT the correct explanation of $\mathbf{A}$
(2) $\mathbf{A}$ is true but $\mathbf{R}$ is false
(3) Both $\mathbf{A}$ and $\mathbf{R}$ are true and $\mathbf{R}$ is the correct explanation of $\mathbf{A}$
(4) $\mathbf{A}$ is false but $\mathbf{R}$ is true

## Answer (3)

Sol. $\quad \mathrm{H}_{2} \mathrm{C}_{2} \mathrm{O}_{4} \cdot 2 \mathrm{H}_{2} \mathrm{O} \equiv 126 \mathrm{gm} \mathrm{mole}^{-1}$
Molarity $(M)=\frac{\left(\frac{3.15}{126}\right)}{.250}=0.10$
72. Buna-S can be represented as:
(1) $\left[\mathrm{CH}=\mathrm{CH}-\mathrm{CH}=\mathrm{CH}-\stackrel{\mathrm{C}_{6} \mathrm{H}_{5}}{\mathrm{CH}}-\mathrm{CH}_{2}\right]_{\mathrm{n}}$
(2)

(3) $-\left[\mathrm{CH}_{2}-\mathrm{CH}=\stackrel{\mathrm{C}_{6} \mathrm{H}_{5}}{\mathrm{C}}-\mathrm{CH}=\mathrm{CH}-\mathrm{CH}_{2}\right]_{\mathrm{n}}$
(4)


Answer (2)

Sol. BUNA-S is polymer of
(i) Buta-1,3-diene
(ii) Styrene

It's correct structure is represented in option-2.
73. Incorrect method of preparation for alcohols from the following is:
(1) Reaction of Ketone with RMgBr followed by hydrolysis.
(2) Reaction of alkyl halide with aqueous NaOH .
(3) Hydroboration-oxidation of alkene.
(4) Ozonolysis of alkene

## Answer (4)

Sol. Reductive ozonolysis of alkenes will lead to formation of aldehyde or ketones, oxidative ozonolysis of alkenes will lead to formation of carboxylic acids or ketones.

So, alcohol is not formed by ozonolysis of alkenes.
74. The correct order for acidity of the following hydroxyl compound is
(A) $\mathrm{CH}_{3} \mathrm{OH}$
(B) $\left(\mathrm{CH}_{3}\right)_{3} \mathrm{COH}$
(C)

(D)

(E)


Choose the correct answer from the options given below:
(1) C $>$ E $>$ D $>$ B $>$ A
(2) E $>$ D $>$ C $>$ B $>$ A
(3) D $>$ E $>$ C $>$ A $>$ B
(4) E $>$ C $>$ D $>$ A $>$ B

## Answer (4)

Sol. $\mathrm{NO}_{2}$ group will increase the acidity, $\mathrm{OCH}_{3}$ group at para position will decrease the acidity, phenol is more acidic than methanol.

So, Acidic strength $\mathrm{E}>\mathrm{C}>\mathrm{D}>\mathrm{A}>\mathrm{B}$
75. Match List-I with List-II.

|  | List-I |  | List-II |
| :--- | :--- | :--- | :--- |
| A. | 16 g of $\mathrm{CH}_{4}(\mathrm{~g})$ | I | Weighs 28 g |
| B. | 1 g of $\mathrm{H}_{2}(\mathrm{~g})$ | II | $60.2 \times 10^{23}$ electrons |
| C. | 1 mole of $\mathrm{N}_{2}(\mathrm{~g})$ | III | Weighs 32 g |
| D. | 0.5 mol of <br> $\mathrm{SO}_{2}(\mathrm{~g})$ | IV | Occupies 11.4 L <br> volume at STP |

Choose the correct answer from the options given below:
(1) A-II, B-III, C-IV, D-I
(2) A-II, B-IV, C-I, D-III
(3) A-I, B-III, C-II, D-IV
(4) A-II, B-IV, C-III, D-I

## Answer (2)

Sol. (A) Moles of $\mathrm{CH}_{4}=\frac{16}{16}=1$
Molecules $=6.02 \times 10^{23}$
Electrons $=10 \times 6.02 \times 10^{23}$

$$
=60.2 \times 10^{23}
$$

(B) Moles of $\mathrm{H}_{2}=\frac{1}{2}=0.5$ mole

Volume at STP $\approx \frac{1}{2} \times 22.8=11.4$ lit.
(C) $1 \mathrm{~mole}_{2}=28 \mathrm{gm}$
(D) $0.5 \mathrm{~mole} \mathrm{SO}_{2}=\frac{1}{2} \times 64=32 \mathrm{gm}$
76. Gibbs energy vs T plot for the formation of oxides is given below.


For the given diagram, the correct statement is-
(1) At $600^{\circ} \mathrm{C}, \mathrm{CO}$ cannot reduce FeO
(2) At $600^{\circ} \mathrm{C}, \mathrm{CO}$ can reduce ZnO
(3) At $600^{\circ} \mathrm{C}, \mathrm{C}$ can reduce ZnO
(4) At $600^{\circ} \mathrm{C}, \mathrm{C}$ can reduce FeO

## Answer (4)

Sol. Element below in Ellingham diagram can reduce oxide of element above it.

C can reduce FeO at $600^{\circ} \mathrm{C}$.
77. The correct relationships between unit cell edge length ' $a$ ' and radius of sphere ' $r$ ' for face-centred and body-centred cubic structures respectively are:
(1) $2 \sqrt{2} r=a$ and $\sqrt{3} r=4 a$
(2) $r=2 \sqrt{2} a$ and $4 r=\sqrt{3} a$
(3) $r=2 \sqrt{2} a$ and $\sqrt{3} r=4 a$
(4) $2 \sqrt{2} r=a$ and $4 r=\sqrt{3} a$

## Answer (4)

Sol. For FCC: $a \sqrt{2}=4 r$

$$
a=2 \sqrt{2} r
$$

For BCC : $a \sqrt{3}=4 r$
78. The correct order of metallic character is
(1) $\mathrm{K}>\mathrm{Be}>\mathrm{Ca}$
(2) $\mathrm{Be}>\mathrm{Ca}>\mathrm{K}$
(3) $\mathrm{Ca}>\mathrm{K}>\mathrm{Be}$
(4) $\mathrm{K}>\mathrm{Ca}>\mathrm{Be}$

Answer (4)
Sol. K is an alkali metal.
For alkaline earth metals, metallic character increases down the group.
79. The delicate balance of $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$ is NOT disturbed by
(1) Respiration
(2) Burning of coal
(3) Deforestation
(4) Burning of petroleum

## Answer (1)

Sol. Burning of coal; deforestation and burning of petroleum will lead to disturbance of $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$.
80. In Carius tube, an organic compound ' $X$ ' is treated with sodium peroxide to form a mineral acid ' Y '.

The solution of $\mathrm{BaCl}_{2}$ is added to ' Y ' to form a precipitate ' $Z$ '. ' $Z$ ' is used for the quantitative estimation of an extra element. ' $X$ ' could be
(1) Cytosine
(2) A nucleotide
(3) Methionine
(4) Chloroxylenol

Answer (3)
Sol. Z is $\mathrm{BaSO}_{4}$
Y is $\mathrm{H}_{2} \mathrm{SO}_{4}$
It is clear that " $X$ " must contain sulphur. Methionine is $\mathrm{C}_{5} \mathrm{H}_{11} \mathrm{NO}_{2} \mathrm{~S}$.

## SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10 . The answer to each question is a NUMERICAL VALUE. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, $-00.33,-00.30,30.27,-27.30$ ) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
81. The difference in the oxidation state of Xe between the oxidised product of Xe formed on complete hydrolysis of $\mathrm{XeF}_{4}$ and $\mathrm{XeF}_{4}$ is $\qquad$ -.

Answer (2)
Sol. Hydrolysis of $\mathrm{XeF}_{4}$ will give $\mathrm{XeO}_{3}$
$\mathrm{XeO}_{3}: \mathrm{Xe}(+6)$
$\mathrm{XeF}_{4}: \mathrm{Xe}(+4)$
difference $=2$
82.


The electron in the $\mathrm{n}^{\text {th }}$ orbit of $\mathrm{Li}^{2+}$ is excited to $(n+1)$ orbit using the radiation of energy $1.47 \times 10^{-17} \mathrm{~J}$ (as shown in the diagram). The value of $n$ is $\qquad$ .

Given: $R_{H}=2.18 \times 10^{-18} \mathrm{~J}$
Answer (1)

Sol. $1.47 \times 10^{-17}=2.18 \times 10^{-18} \times(9)\left(\frac{1}{(n)^{2}}-\frac{1}{(n+1)^{2}}\right)$
$0.749=\frac{1}{\mathrm{n}^{2}}-\frac{1}{(\mathrm{n}+1)^{2}}$
$n=1 \quad$ If we take $0.749 \simeq 0.75=\frac{3}{4}$
83. For a metal ion, the calculated magnetic moment is 4.90 BM. This metal ion has $\qquad$ number of unpaired electrons.

## Answer (4)

Sol. $\mu=4.90$
$4.90=\sqrt{(\mathrm{n})(\mathrm{n}+2)}$
$24=(n)(n+2)$
$\mathrm{n}=4$
84. In alkaline medium, the reduction of permanganate anion involves a gain of $\qquad$ electrons.

## Answer (3)

Sol. $\mathrm{MnO}_{+7}^{-} \xrightarrow[\text { medium }]{\text { alkaline }} \mathrm{MnO}_{+4}$
$\underset{+7}{\mathrm{MnO}_{4}^{-}} \xrightarrow[\text { medium }]{\text { strongly akaline }} \underset{+6}{\mathrm{MnO}_{4}^{2-}}$
85. $\quad \mathrm{A}(\mathrm{g}) \rightleftharpoons 2 \mathrm{~B}(\mathrm{~g})+\mathrm{C}(\mathrm{g})$

For the given reaction, if the initial pressure is 450 mm Hg and the pressure at time t is 720 mm Hg at a constant temperature T and constant volume V . The fraction of $\mathrm{A}(\mathrm{g})$ decomposed under these conditions is $x \times 10^{-1}$. The value of $x$ is (nearest integer)

## Answer (3)

Sol. $A(g) \rightleftharpoons 2 B(g)+C(g)$

| 450 | - | - |
| :--- | :--- | :--- |
| $\downarrow$ |  | $\downarrow$ |
| $(450-x)$ | $(2 x)$ | $(x)$ |
| $(450-x)+(2 x)+(x)=720$ |  |  |
|  |  |  |
| $450+2 x=720$ |  |  |

$2 x=270$
$x=135$
Fraction of $\mathrm{A}(\mathrm{g})$ decomposed $=\frac{135}{450}=0.3$
$x=3$
86. The number of endothermic process/es from the following is $\qquad$ .
A. $\quad \mathrm{I}_{2}(\mathrm{~g}) \rightarrow 2 \mathrm{I}(\mathrm{g})$
B. $\mathrm{HCl}(\mathrm{g}) \rightarrow \mathrm{H}(\mathrm{g})+\mathrm{Cl}(\mathrm{g})$
C. $\mathrm{H}_{2} \mathrm{O}(\mathrm{I}) \rightarrow \mathrm{H}_{2} \mathrm{O}(\mathrm{g})$
D. $\mathrm{C}(\mathrm{s})+\mathrm{O}_{2}(\mathrm{~g}) \rightarrow \mathrm{CO}_{2}(\mathrm{~g})$
E. Dissolution of ammonium chloride in water

## Answer (4)

Sol. (D) $\mathrm{C}(\mathrm{s})+\mathrm{O}_{2}(\mathrm{~g}) \longrightarrow \mathrm{CO}_{2}(\mathrm{~g})$
Above reaction is combustion of Carbon and hence exothermic
(E) Dissolution of Ammonium chloride in water is endothermic process.
87. The specific conductance of 0.0025 M acetic acid is $5 \times 10^{-5} \mathrm{~S} \mathrm{~cm}^{-1}$ at a certain temperature. The dissociation constant of acetic acid is $\qquad$ $\times 10^{-7}$. (Nearest integer)

Consider limiting molar conductivity of $\mathrm{CH}_{3} \mathrm{COOH}$ as $400 \mathrm{~S} \mathrm{~cm}^{2} \mathrm{~mol}^{-1}$

## Answer (66)

Sol. $\lambda_{m}=\frac{k \times 1000}{M}=\frac{\left(5 \times 10^{-5}\right) \times\left(10^{3}\right)}{\left(2.5 \times 10^{-3}\right)}=20$

$$
\alpha=\frac{20}{400}=\frac{1}{20}
$$

$$
\mathrm{k}_{\mathrm{a}}=\frac{\mathrm{C} \alpha^{2}}{(1-\alpha)}=\frac{\left(2.5 \times 10^{-3}\right)\left(\frac{1}{20} \times \frac{1}{20}\right)}{\left(\frac{19}{20}\right)}
$$

$$
=65.789 \times 10^{-7}
$$

$$
\approx 66 \times 10^{-7}
$$

88. The number of incorrect statement/s from the following is $\qquad$ -
A. The successive half lives of zero order reactions decreases with time.
B. A substance appearing as reactant in the chemical equation may not affect the rate of reaction
C. Order and molecularity of a chemical reaction can be a fractional number
D. The rate constant units of zero and second order reaction are $\mathrm{mol} \mathrm{L}^{-1} \mathrm{~s}^{-1}$ and $\mathrm{mol}^{-1} \mathrm{~L} \mathrm{~s}^{-1}$ respectively

## Answer (1)

Sol. A. $\mathrm{C}_{0} \xrightarrow{\mathrm{t}=\frac{\mathrm{C}_{0}}{2 \mathrm{k}}} \frac{\mathrm{C}_{0}}{2} \xrightarrow{\mathrm{t}=\frac{\mathrm{C}_{0}}{4 \mathrm{k}}} \frac{\mathrm{C}_{0}}{4} \xrightarrow{\mathrm{t}=\frac{\mathrm{C}_{0}}{8 k}} \frac{\mathrm{C}_{0}}{8}$
Successive half lives of zero order reactions decreases with time

Note: successive half lives of first order reactions remains same
$\Rightarrow A$ is correct
(B) $B$ is correct
(C) Molecularity cannot be fractional

Order can be fractional
$\Rightarrow \mathrm{C}$ is incorrect
(D) Unit of k :

Zero order $=\mathrm{mol} \mathrm{lit}^{-1} \mathrm{sec}^{-1}$
Second order $=\mathrm{mol}^{-1}$ lit sec${ }^{-1}$
D is correct
89. The number of molecules from the following which contain only two lone pair of electrons is $\qquad$ $\mathrm{H}_{2} \mathrm{O}, \mathrm{N}_{2}, \mathrm{CO}, \mathrm{XeF}_{4}, \mathrm{NH}_{3}, \mathrm{NO}, \mathrm{CO}_{2}, \mathrm{~F}_{2}$

Answer (3)

Sol.

$\mathrm{N}_{2}$ ( $\mathrm{N} \equiv \mathrm{N} \odot \rightarrow 2 \mathrm{lps}$




NO $\ddot{\mathrm{N}}=\ddot{\mathrm{O}} \quad \rightarrow 3 \mathrm{lp}$
Reference for structure NCERT- XI pg 105



The number of molecules having only 2 lone pair of electrons $=3$

Which are $\mathrm{H}_{2} \mathrm{O} ; \mathrm{N}_{2}$ and CO
Note:-
$\mathrm{XeF}_{4}$ have 2 lps on central atom, but we are asked about lone pair in molecule
Official answer is given as 4
90. An aqueous solution of volume $300 \mathrm{~cm}^{3}$ contains 0.63 g of protein. The osmotic pressure of the solution at 300 K is 1.29 mbar. The molar mass of the protein is $\qquad$ $\mathrm{g} \mathrm{mol}^{-1}$.
Given : $\mathrm{R}=0.083 \mathrm{~L}^{\mathrm{b}} \mathrm{bar} \mathrm{K}^{-1} \mathrm{~mol}^{-1}$
Answer (40535)
Sol. $\pi=$ CRT

$$
\begin{aligned}
& \left(1.29 \times 10^{-3}\right)=\frac{0.63}{(\mathrm{MW})(0.3)} \times(0.083) \times 300 \\
& M W=\frac{0.63 \times 0.083 \times 300}{(0.3) \times\left(1.29 \times 10^{-3}\right)} \\
& \quad=40534.88 \\
& \quad \approx 40535 \text { (Nearest integer) }
\end{aligned}
$$

