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29/01/2023 Evening

Answers & Solutions

Time : 3 hrs.



M.M.: 300

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

- (1) The test is of **3 hours** duration.
- (2) The Test Booklet consists of 90 questions. The maximum marks are 300.
- (3) There are three parts in the question paper consisting of Physics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each part (subject) has two sections.
 - (i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **-1 mark** for wrong answer.
 - Section-B: This section contains 10 questions. In Section-B, attempt any five questions out of 10. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and -1 mark for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which ONLY ONE is correct.

Choose the correct answer:

- 1. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.
 - (1) 6 hours (2) 4 hours
 - (4) 12 hours (3) 3 hours

Answer (3)

Sol. ::
$$T^2 \propto R^3$$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$
$$\frac{24^2}{T_2^2} = \frac{R_1^3}{\left(\frac{R_1}{4}\right)^3}$$
$$\frac{24^2}{T_2^2} = 4^3$$

$$T_2 = \frac{24}{2^3}$$

= 3 hours

2. A force acts for 20 s on a body of mass 20 kg, starting from rest, after which the force ceases and then body describes 50 m in the next 10 s. The value of force will be:

(1)	10 N	(2)	5 N
(3)	20 N	(4)	40 N

Answer (2)

Acceleration
$$=\frac{F}{20}$$
 m/s²

 \therefore v = u + at

$$v = 0 + \left(\frac{F}{20}\right)(20)$$
$$= F \,\mathrm{ms}^{-1}$$

Now for next 10 sec.

$$S = ut$$

$$50 = F(10)$$

$$F = 5$$

3. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same

potential is
$$\frac{1}{\sqrt{m}}$$
, the value of *m* is
(1) 16

(Z) Z

- (3) 8
- (4) 4

Sol.
$$\frac{\lambda_{\alpha}}{\lambda_{\text{proton}}} = \frac{1}{\sqrt{2(4m)(2eV)}} \times \frac{\sqrt{2(m)(e)V}}{1}$$
$$= \frac{1}{\sqrt{8}}$$
$$m = 8$$

At 300 K, the rms speed of oxygen molecule is 4. $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be

(used
$$\pi \quad \frac{22}{7}$$
)
(1) 28
(2) 27
(3) 32
(4) 24
Answer (1)
Sol. 16 $\pi \quad \frac{\pi}{5}$

Sol.
$$v_{\rm rms} = \sqrt{\frac{\alpha+3}{\alpha}}_{\rm avg}$$

 $\sqrt{\frac{3RT}{m}} = \sqrt{\frac{\alpha+5}{5}}\sqrt{\frac{8RT}{\pi m}}$
 $\frac{3 \times \pi}{8} = \frac{\alpha+5}{\alpha}$
 $\frac{33}{28} = \frac{\alpha+5}{\alpha}$
 $\alpha = 28$

5.	Heat energy of 184	kJ is given to ice of mass	(1) C only		
600 g at -12°C. Specific heat of ice is 2222.3 J kg-			(2) B and D only		
	¹ °C ⁻¹ and latent heat of	of ice in 336 kJ/kg ⁻¹	(3) A and C only		
	A. Final temperature	of system will be 0°C.	(4) A only		
	B. Final temperature than 0°C.	of the system will be greater	Answer (3)		
	C. The final system water in the ratio	will have a mixture of ice and of 5 : 1.	Sol. Sensitivity $\propto \frac{1}{k(\text{potential gradient})}$		
	D. The final system	will have a mixture of ice and	\propto length		
	water in the ratio	of 1 : 5.	7. A point charge 2×10^{-2} C is moved from	Ρ	
	E. The final system	vill have water only.	uniform electric field of 30 NC ⁻¹ direc	te	
Choose the correct answer from the options given below:			(1, 2, 0) m and (0, 0, 0) m respectively, done by electric field will be	ctively,	
	(1) A and C only	(2) A and D only	(1) –1200 mJ (2) 600 mJ		
	(3) B and D only	(4) A and E only	(3) -600 mJ (4) 1200 mJ		
Ans	swer (2)		Answer (3)		

Sol. Heat required to raise the temperature of ice to 0°C is

$$=rac{60}{1000}(2222.3)(12)$$

= 16000.5 J

 $\approx 16 \text{ kJ}$

Heat required to melt ice completely

$$=\left(\frac{600}{1000}\right)(336) \text{ kJ}$$

Energy left = (184 - 16) = 168 kJ

... Partial ice will melt

$$\therefore \quad 168 = (m_{\text{ice melted}})336$$

 $0.5 \text{ kg} = (m_{\text{ice melted}})$

- $\therefore m_{\text{ice}}: m_{\text{water}} = 1:5$
- With the help of potentiometer, we can determine 6. the value of emf of a given cell. The sensitivity of the potentiometer is
 - (A) Directly proportional to the length of the potentiometer wire
 - (B) Directly proportional to the potential gradient of the wire
 - (C) inversely proportional to the potential gradient of the wire
 - (D) inversely proportional to the length of the potentiometer wire

Choose the correct option for the above statements:

to S in a ed along d S are the work



$$w = \vec{F} \cdot \vec{d} = (0.6\hat{i}) \cdot (-\hat{i}, -2\hat{j})$$

= -0.6 J

= -600 mJ

8. A square loop of area 25 cm² has a resistance of 10 Ω. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0 sec, will be

(1)
$$2.5 \times 10^{-3} \text{ J}$$

(4) $1.0 \times 10^{-3} \text{ J}$

Answer (4)

Sol. From energy conservation.

Work done to pull the loop out

= Energy lost is resistance

Emf in the loop =
$$\frac{d\phi}{dt} = \frac{B \times A}{t} = \frac{40 \times 25 \times 10^{-4}}{1 \text{ s}}$$

= 0.1 V
Energy lost = $\frac{emf^2}{R} = -\frac{(0.1)^2}{10} = 10^{-3} \text{ J}$

The time taken by an object to slide down 45° rough 9. inclined plane is n times as it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the incline plane is:

(1)
$$1 - \frac{1}{n^2}$$
 (2) $\sqrt{1 - \frac{1}{n^2}}$
(3) $\sqrt{\frac{1}{1 - n^2}}$ (4) $1 + \frac{1}{n^2}$

R

Answer (1)



Smooth case:

$$a = g \sin 45^{\circ} \frac{9}{\sqrt{2}}$$

 $t_1 = \sqrt{\frac{2L}{g}} = \sqrt{\frac{2L}{g/\sqrt{2}}} = \sqrt{\frac{2\sqrt{2}L}{g}} \dots (1)$

Rough case:

 $a = g \sin 45^\circ - \mu g \cos 45^\circ$

$$=\frac{g}{\sqrt{2}}(1-\mu)$$

$$t_2 = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2\sqrt{2}L}{g(1-\mu)}}$$
(2)

From (1) to (2) and
$$t_1 = \frac{t_2}{n}$$
 we have

$$\sqrt{\frac{2\sqrt{2L}}{g}} = \frac{1}{n}\sqrt{\frac{2\sqrt{2L}}{g(1-\mu)}} \Rightarrow \boxed{\mu = 1 - \frac{1}{n^2}}$$

10. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is 500 m². It is in level flight with a speed of 1080 km/h. If the density of air ρ is 1.2 kg m⁻³, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be. $(q = 10 \text{ m/s}^2)$

Answer (3)

Velocity of aircraft = 1080 km/h = 300 m/s

Now, weight of aircraft = ΔPA

$$\Delta P = \frac{5.4 \times 10^5 \quad g}{500} = 10800 \text{ Pa}$$

From Bernoulli's principle

$$\Delta P = \frac{1}{2} \rho \left[V_{upper}^2 - V_{lower}^2 \right]$$

$$10800 = \frac{1}{2} \times 1.2 \times V_{lower}^2 \left[\left(\frac{V_{upper}}{V_{lower}} \right)^2 - 1 \right]$$

$$\left(\frac{V_{upper}}{V_{lower}} \right)^2 = 1 + \frac{10800 \times 2}{1.2 \times (300)^2} = 1.2$$

$$\frac{V_{upper}}{V_{lower}} = 1.096$$

$$\Rightarrow \text{ Fractional increases} = 9.6\%.$$

11. Given below are two statements:

Statement I : Electromagnetic waves are not deflected by electric and magnetic field.

Statement II : The amplitude of electric field and the magnetic field in electromagnetic waves are related

to each other as $E_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}B_0$.

In the light of the above statements, choose the correct answer from the options given below :

- (1) Both statement I and statement II are false
- (2) Both statement I and statement II are true
- (3) Statement I is true but statement II is false
- (4) Statement I is false but statement II is true

Sol. Statement I is correct as photon do not carry any charge, hence cannot feel force from either fields. Statement II is wrong as $E_0 = cB_0$

$$E_0 = \frac{B_0}{\sqrt{\mu_0 \varepsilon_0}}$$

12. For the given figures, choose the correct options:



- (1) The rms current in circuit (b) can be larger than that in (a)
- (2) At resonance, current in (b) is less than that in(a)
- (3) The rms current in figure (a) is always equal to that in figure (b)
- (4) The rms current in circuit (b) can never be larger than that in (a)

Answer (4)

Sol. For (a),
$$i = \frac{V}{R} = \frac{220}{40} = 5.5 \text{ A}$$

for (b), $X_C = \frac{1}{2 \ fC} = \frac{1}{2\pi 50 \times 0.5 \times 10^{-6}} = \frac{10^6}{50\pi} \Omega$
 $X_I = 2\pi fL = 2\pi \times 50 \times 50 \times 10^{-3} = 50\pi \Omega$

 $X_C > X_L$, hence impedance is greater than 40 Ω .

$$\dot{i}_{\rm rms} = \frac{220}{Z}$$
$$\dot{i}_{\rm rms} \Big|_b < \dot{i}_{\rm rms} \Big|_a$$

- 13. Identify the correct statements from the following:
 - A. Work done by a man in lifting a bucket out of a well by means of rope tied to the bucket is negative.
 - B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.
 - C. Work done by friction on a body sliding down an inclined plane is positive.
 - D. Word done by an applied force on a body moving on a rough horizontal plane with uniform velocity is zero.
 - E. Work done by the air resistance on an oscillating pendulum is negative.

Choose the correct answer from the options given below:

- (1) A and C only (2) B and E only
- (3) B, D and E only (4) B and D only

Answer (2)

- **Sol.** B. Work done by gravitation will be negative if something is lifted upward.
 - D. Work done by air resistance is negative.
- 14. Substance *A* has atomic mass number 16 and half life of 1 day. Another substance *B* has atomic mass

number 32 and half life of $\frac{1}{2}$ day. If both A and B

simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of *A* and *B* combined would be left after 2 days.

- (1) 6.76×10^{24}
- (2) 3.38 × 10²⁴
- (3) 6.76 × 10²³
- (4) 1.69 × 10²⁴

Answer (2)

=

Sol. *n*_{*A*} = 20 moles

$$n_B = 10 \text{ moles}$$

$$N = N_0 e^{-\lambda t}$$

$$N_A = (20 \text{ N}) e^{-\left(\frac{\ln 2}{1} \times 2\right)}$$

$$= \frac{20 \text{ N}}{4} = 5 \text{ N}$$

$$N_A = 10 \text{ N} e^{-4\ln 2}$$

$$= \left(\frac{10 \text{ N}}{16}\right)$$

$$N_A + N_B = 5 \text{ N} + \frac{10 \text{ N}}{16} = \left(\frac{90 \text{ N}}{16}\right) = 3.38 \times 10^{24}$$

- 15. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)
 - (1) Decrease the diameter of the objective lens
 - (2) Decrease the focal length of the eye piece
 - (3) Increase the refractive index of the medium between the object and objective lens
 - (4) Increase the wavelength of the light

Sol. Resolving power of microscope = $\left(\frac{2n\sin\theta}{\lambda}\right)$

 $n\sin\theta =$ Numerical aperture

n is the refractive index of medium.

- The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is
 - (1) 1.4
 - (2) 0.6
 - (3) 0.4
 - (4) 0.2

Answer (3)

Sol. $A_c + A_m = 14 \text{ mV}$

$$A_c - A_m = 6 \text{ mV}$$

$$\Rightarrow 2A_c = 20 \text{ mV}$$

$$A_c = 10 \text{ mV}$$

$$A_m = 4 \text{ mV}$$

Modulation Index =
$$\frac{A_m}{A_c} = \left(\frac{4}{10}\right) = 0.4$$

17. The equation of a circle is given by $x^2 + y^2 = a^2$, where *a* is the radius. If the equation is modified to change the origin other than (0, 0), then find out the correct dimensions of *A* and *B* in a new equation

 $(x - At)^2 + (y - \frac{t}{B})^2 = a^2$. The dimension of *t* is given as $[T^{-1}]$.

- (1) $A = [LT], B = [L^{-1}T^{-1}]$
- (2) $A = [L^{-1}T], B = [LT^{-1}]$
- (3) $A = [L^{-1}T^{-1}], B = [LT]$
- (4) $A = [L^{-1}T^{-1}], B = [LT^{-1}]$

Answer (1)

Sol.
$$[At] = [x] = [L]$$

 $\begin{bmatrix} A \end{bmatrix} = \frac{\begin{bmatrix} x \\ t \end{bmatrix}}{\begin{bmatrix} t \end{bmatrix}} = \begin{bmatrix} LT \end{bmatrix}$
 $\begin{bmatrix} \frac{t}{B} \end{bmatrix} = \begin{bmatrix} y \end{bmatrix} = \begin{bmatrix} L \end{bmatrix}$
 $\Rightarrow \begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} \frac{t}{L} \end{bmatrix} \begin{bmatrix} L^{1}T^{-1} \end{bmatrix}$

- 18. An object moves at a constant speed along a circular path in a horizontal plane with centre at the origin. When the object is at x = +2 m, its velocity is $-4\hat{j}$ m/s. The object's velocity (*v*) and acceleration (*a*) at x = -2 m will be
 - (1) $v = 4\hat{i}$ m/s, $a = 8\hat{j}$ m/s²
 - (2) $v = 4\hat{j} \text{ m/s}, a = 8\hat{i} \text{ m/s}^2$
 - (3) $v = -4\hat{i}$ m/s, $a = -8\hat{j}$ m/s²

(4)
$$v = -4\hat{j}$$
 m/s, $a = 8\hat{i}$ m/s²

Answer (2)

Sol.



$$a = \frac{V^2}{R} = \frac{16}{2} = 8 \text{ m/s}^2$$
$$\vec{a} = 8 (\text{m/s}^2)(\hat{i})$$

19. For the given logic gates combination, the correct truth table will be



Sol. As per the circuit,

$$X = A'B + AB'$$

If: $A = 0, B = 0 \Rightarrow X = 0$
 $A = 0, B = 1 \Rightarrow X = 1$
 $A = 1, B = 0 \Rightarrow X = 1$
 $A = 1, B = 1 \Rightarrow X = 0$

20. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be

(1)	4 T	(2)	16 T
(3)	8 T	(4)	2 T

Answer (4)

Sol. By given information

$$32 = 4 \quad \frac{\mu_0 i}{2r} \qquad ...(i)$$
Also, $r' = 4r \qquad ...(ii)$
and $B' = 1 \times \frac{\mu_0 i}{2r'} \qquad ...(iii)$

$$\Rightarrow B' = \frac{i}{2(4r)} = \frac{\mu_0 i}{8} = \frac{1}{8} \times 16 = 2 \text{ T}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. When two resistances R_1 and R_1 connected in series and introduced into the left gap of a meter bridge and a resistance of 10 Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3 Ω is introduced into the right gap to get null point at 40 cm from left end. The product of R_1R_2 is _____ Ω^2

Answer (30)

Sol. As per given information

$$\frac{1+R_2}{10} = \frac{0.6}{0.4} \qquad \dots (1)$$

$$\& \frac{\frac{R_1 R_2}{1 + R_2}}{3} = \frac{0.4}{0.6} \qquad \dots (2)$$

$$\Rightarrow \begin{array}{c} R_1 + R_2 = 15 \\ \& R_1 R_2 = 30 \end{array} \right] \Rightarrow R_1 R_2 = 30 \ \Omega^2$$

22. For a charged spherical ball, electrostatic potential inside the ball varies with *r* as $V = 2ar^2 + b$.

Here *a* and *b* are constant and *r* is the distance from the center. The volume charge density inside the ball is $-\lambda a \epsilon$. The value of λ is _____.

 ϵ = permittivity of the medium

Answer (12)

Sol.
$$V = 2ar^2 + b$$

$$\Rightarrow E = -\frac{dV}{dr} = -4ar$$
$$\Rightarrow \frac{1}{4\pi} \frac{Q}{r^2} = -ar$$
$$\Rightarrow \frac{Q}{\frac{4}{3}\pi r^3} = 3 \times \varepsilon \times (-4a) = -12 \varepsilon$$

- $\Rightarrow \lambda = 12$
- 23. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km/hr is $t(1 e^{-\pi/2})$ s. The value of *t* is_____.

Answer (40)

Sol.
$$\frac{dv}{dt} = \frac{v^2}{R} \Rightarrow \frac{v^2}{R} = v \frac{dv}{ds}$$

 $\Rightarrow \frac{dv}{v} = \frac{ds}{R} \Rightarrow \ln v \Big|_{15}^v = \frac{s}{R}$
 $\Rightarrow v = 15e^{\Delta/R} = \frac{ds}{dt} \Rightarrow dt = \frac{1}{15}e^{-\Delta/R}ds$

$$\Delta t = \frac{R}{15} [1 - e^{-\Delta R}]$$
$$= 40 [1 - e^{-\pi/2}] \text{ seconds}$$
$$\Rightarrow t = 40$$

24. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium - 1) and 6.8 (medium -2), respectively . To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be

$$\tan^{-}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$$
 the value of θ is _____.

(Given for dielectric media, $\mu_r = 1$)

Answer (7)

Sol We know that

$$\tan \theta_0 = \frac{\mu_2}{\mu_1}$$
$$\tan \theta_0 = \sqrt{\frac{6.8}{2.8}} = \sqrt{\frac{17}{7}}$$
$$\theta_0 = \tan^- \sqrt{1 + \frac{10}{7}} \Rightarrow \theta = 7$$

11.

25. A metal block of base area 0.20 m² is placed on a table as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is 5.0×10^{-3} Pl, the speed of block is _____ \times 10⁻³ m/s



Answer (25)

Sol. As the block moves with constant speed, the horizontal force is balanced by viscous force thus

$$F = \eta A \frac{\Delta v}{\Delta z}$$

$$0.1 = 5 \times 10^{-1} \times 0.2 \times \frac{v}{.25 \times 10^{-1}}$$

 \Rightarrow v = 25 x 10⁻³ m/s

26. A particle of mass 250 g executes a simple harmonic motion under a periodic force F = (-25 x)N. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is ____ cm.

Answer (40)

Sol. F =
$$-25x$$

 $.250 \frac{d^2x}{dt^2} = -25x$
 $\frac{d^2x}{dt^2} = -100x$
 $\Rightarrow \omega = 10 \text{ rad/sec}$
& $\omega A = v_{max}$
 $10 A = 4$
 $\Rightarrow A = 0.4 \text{ m}$
 $= 40 \text{ cm}$

0E v

27. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on vernier scale is equal to 49 divisions on main scale. The estimated uncertainty in the measurement of refractive index of the slab is

$$\frac{x}{10} \times 10^{-3}$$
, where x is _____.

Answer (41)

Sol.
$$\mu = \frac{\text{real depth } (l_1)}{\text{apparent depth} (l_2)}$$
$$= \frac{5.25}{5} = 1.05$$
$$\frac{d\mu}{\mu} = \frac{dl_1}{l_1} + \frac{dl_2}{l_2}$$

$$d\mu = \left(\frac{dl_1}{l_1} + \frac{dl_2}{l_2}\right)\mu$$
$$= \left(\frac{0.01}{5.25} + \frac{0.01}{5.00}\right) \times 1.05$$
$$= \frac{41}{10} \times 10^{-3}$$

28. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5 Ω . When a resistance of 15 Ω is used for shunting, null point moves to 300 cm. The internal resistance of the cell is _____ Ω .

Answer (05)

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Sol. Let the emf is *E* and internal resistance is *r* of this secondary cell so

$$\frac{RE}{r+R} \propto l$$
so $\frac{R_1E}{r+R_1} \propto l_1$

$$\& \frac{R_2E}{r+R_2} \propto l_2$$

$$\Rightarrow \frac{R_1(r+R_2)}{R_2(r+R_1)} = \frac{l_1}{l_2}$$
OR $\frac{5(r+15)}{15(r+5)} = \frac{200}{300}$

29. An inductor of inductance 2 μ H is connected in series with a resistance, a variable capacitor and an AC source of frequency 7 kHz. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}F$, where the value of x is

(Take
$$\pi = \frac{22}{7}$$
)

 \Rightarrow r = 5 Ω

Answer (3872)

Sol. Current drawn is maximum when circuit is in resonance.

$$\omega = \frac{1}{\sqrt{LC}}$$

$$2\pi(7000) \quad \frac{1}{\sqrt{2 \times 10^{-6}C}}$$

$$\Rightarrow C = \frac{1}{3872}$$

30. A particle of mass 100 g is projected at time t = 0 with a speed 20 ms⁻¹ at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time t = 2 s is found to be \sqrt{K} kg m²/s. The value of K is _____.



Sol. Horizontal displacement $x = v \cos\theta t$

$$=10\sqrt{2}t$$

So torque of weight about point of projection is

$$\tau = mgx \cdot (-\hat{k})$$

$$\frac{d\vec{L}}{dt} = mgx(-\hat{k})$$

$$\int_{0}^{\vec{L}} d\vec{L} = 0.1 \times 10 \times 10\sqrt{2} \int_{0}^{2} t \, dt(-\hat{k})$$

$$\vec{L} = -20\sqrt{2}\hat{k}$$

$$\left|\vec{L}\right| = 20\sqrt{2} = \sqrt{800} \text{ kg m}^{2}/\text{s}$$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. Match List I with List II.

	List I		List II
Α.	Elastomeric	Ι.	Urea
	polymer		formaldehyde
			resin
В.	Fibre polymer	II.	Polystyrene
C.	Thermosetting polymer	III.	Polyester
D.	Thermoplastic polymer	IV.	Neoprene

Choose the correct answer from the options given below?

- (1) A-IV, B-III, C-I, D-II
- (2) A-II, B-III, C-I, D-IV
- (3) A-IV, B-I, C-III, D-II
- (4) A-II, B-I, C-IV, D-III

Answer (1)

Sol.	Α.	Elastomeric fibre	(IV)	Neopr	ene
	В.	Fibre polymer	(III)	Polyes	ster
	C.	Thermosetting	(I)	Urea	Formaldehyde
		polymer		Resin	
	D.	Thermoplastic	(II)	Polyst	yrene
		polymer			
32.	A s	olution of CrO₅ in an	nyl a	alcohol	has a
	colo	our.			
	(1)	Blue	(2)	Yellow	1
	(3)	Green	(4)	Orang	e-Red
Ans	wer	(1)			
Sol.	CrC	D_5 is blue in colour in a	amy	alcoho	ol.

33. The concentration of dissolved oxygen in water for growth of fish should be more than <u>X</u> ppm and Biochemical Oxygen Demand in clean water should be less than <u>Y</u> ppm. X and Y in ppm are, respectively.

(1)	Х	Y	(2)	Х	Y
	4	15		6	12
(3)	Х	Y	(4)	Х	Y
	4	8		6	5

Answer (4)

- **Sol.** For high growth, dissolved oxygen should be less than 6 ppm and clean water has dissolved oxygen less than 5 ppm.
- 34. Correct order of spin only magnetic moment of the following complex ions is

(Given At.no. Fe:26, Co:27)

- $(1) \ [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$
- (2) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$
- $(3) \ [\text{FeF}_6]^{3-} > [\text{CoF}_6]^{3-} > [\text{Co}(\text{C}_2\text{O}_4)_3]^{3-}$
- (4) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Sol.	. $[FeF_6]^{3-} \longrightarrow 5$ unpaired electrons			
	$\left[Co\big(C_2O_4\big)_3\right]^{\!\!3\!-}$	→ 0 u	npa	ired electron
	$\left[\operatorname{CoF}_{6}\right]^{3-} \longrightarrow 6^{3-}$	4 unpaireo	d ele	ectrons
	So, correct answ	ver is : (3)	
35.	According to M	O theory	the	bond orders for O_2^{2-} ,
	CO and NO ⁺ res	spectively	, ar	e
	(1) 1, 2 and 3		(2)	1, 3 and 2
	(3) 2, 3 and 3		(4)	1, 3 and 3
Ans	wer (4)			
Sol.	Species	B.O.		
	O ₂ ⁻²	1		
	СО	3		
	NO⊕	3		

36. Match List-I with List-II.

List-I		List-II
van't Hoff	I.	Cryoscopic constant
factor, i		
k f	II.	Isotonic solutions
Solutions	III.	Normal molar mass
with same		Abnormal molar mass
osmotic		
pressure		
Azeotropes	IV.	Solutions with same
		composition of vapour above it
	List-I van't Hoff factor, i k _f Solutions with same osmotic pressure Azeotropes	List-Ivan't Hofffactor, ikfII.Solutionswith sameosmoticpressureAzeotropesIV.

Choose the correct answer from the options given below.

- (1) A-III, B-I, C-II, D-IV
- (2) A-III, B-I, C-IV, D-II
- (3) A-III, B-II, C-I, D-IV
- (4) A-I, B-III, C-II, D-IV

Answer (1)

Sol. A.	Sol. A. van't Hoff factor I	III.	Normal molar mass
•••••			Abnormal molar mass

- B. k_f I. Cryoscopic constant
- C. Solutions with same II. Isotonic solutions osmotic pressure
- D. Azeotropes IV. Solutions with same composition of vapour above it
- 37. Find out the major products from the following reaction sequence.





Answer (4)



Correct answer is (4)

38. Find out the major product for the following reaction.



(2)







- The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)
 - (1) 1-Bromo-2-methylbutane
 - (2) 2-Bromopentane
 - (3) 2-Bromo-3, 3-dimethylpentane
 - (4) 2-Bromopropane





40. Given below are two statements:

Statement I : The decrease in first ionization enthalpy from B to AI is much larger than that from AI to Ga.

Statement-II : The d orbitals Ga are in completely filled.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (1) Statement I is correct but statement II is incorrect
- (2) Statement I is incorrect but statement II is correct
- (3) Both the statements I and II are correct
- (4) Both the statements I and II are incorrect

Answer (3)

Sol. Statement I : It is correct as decrease in first ionisation enthalpy is much larger from B to Al.

Statement II : It is also correct

- 41. An indicator 'X' is used for studying the effect of variation in concentration of iodide on the rate of reaction of iodide ion with H₂O₂ at room temp. The indicator 'X' forms blue colored complex with compound 'A' present in the solution. The indicator 'X' and compound 'A' respectively are
 - (1) Starch and H₂O₂
 - (2) Starch and iodine
 - (3) Methyl orange and H₂O₂
 - (4) Methyl orange and iodine

Answer (2)

Sol. Starch forms blue coloured complex with iodine.

42. Given below are two statements:

Statement I : Nickel is being used as the catalyst for producing syn gas and edible fats.

Statement II: Silicon forms both electron rich and electron deficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Both the statements I and II are incorrect
- (2) Statement I is incorrect but statement II is correct
- (3) Both the statements I and II are correct
- (4) Statement I is correct but statement II is incorrect

Answer (4)

Sol. Statement I is correct.

Statement II is incorrect as Si forms electron precise hydride.

- 43. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?
 - (1) Hyperacidity
 - (2) Depression and hypertension
 - (3) Stomach ulcers
 - (4) Anxiety and stress

Answer (2)

Sol. Equanil is for depression and hypertension.

- 44. When a hydrocarbon A undergoes combustion in the presence of air, it requires 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A?
 - (1) C_9H_9 (2) C_8H_6
 - (3) C₆H₆ (4) C₉H₆

Answer (2)

Sol.
$$C_x H_y + \left(x + \frac{y}{4}\right) O_2 \rightarrow x C O_2 + \frac{y}{2} H_2 O$$

 $\frac{y}{2} = 3$
 $y = 6$
 $x + \frac{y}{4} = \frac{19}{2}$
 $x = \frac{19}{2} - \frac{3}{2} = 8$

So, formula is C₈H₆.

45. Match List I and List II.

	List I		List II
A.	Osmosis	Ι.	Solvent molecules pass through semi permeable membrane towards solvent side.
В.	Reverse osmosis	Ш.	Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C.	Electro osmosis	III.	Solvent molecules pass through semi permeable membrane towards solution side.
D.	Electrophoresis	IV.	Dispersion medium moves in an electric field.
Choo	ose the correct ans	swer	from the options given

(1) A-III, B-I, C-IV, D-II	(2) A-I, B-III, C-IV, D-II
(3) A-III, B-I, C-II, D-IV	(4) A-I, B-III, C-II, D-IV

Answer (1)

Sol. Correct match is:

Α.	Osmosis	III.	Solvent molecules pass through semi permeable membrane towards solution side.
В.	Reverse osmosis	I.	Solvent molecules pass to solvent side through SPM.
C.	Electro osmosis	IV.	Dispersion medium moves in an electric field.
D.	Electrophoresis	II.	Movementofcolloidalparticletowardsoppositelychargedelectrodes.

46. Which of the following relations are correct?

(A) ∆U = q + p∆V

(B) $\Delta G = \Delta H - T \Delta S$

(C) $\Delta S = \frac{q_{rev}}{T}$ (D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below:

- (1) B and C Only
- (2) C and D Only
- (3) A and B Only
- (4) B and D Only

Answer (1)

Sol. (A) $\Delta U = q - p \Delta V$

(B) $\Delta G = \Delta H - T \Delta S$

(C)
$$\Delta S = \frac{q_{rev}}{T}$$

(D) $\Delta H = \Delta U + (\Delta nRT)$

Hence, (B) and (C) relations are correct.

47. The major component of which of the following ore is sulphide based mineral?

(1) Sphalerite (2) Calamine

(3) Malachite (4) Siderite

Answer (1)

Sol. Sphalerite \rightarrow ZnS Calamine \rightarrow ZnCO₃ Malachite \rightarrow CuCO₃·Cu(H)₂

 $\text{Siderite} \rightarrow \text{FeCO}_3$

- 48. Reaction of propanamide with Br₂/KOH(aq) produces :
 - (1) Propanenitrile (2) Ethylnitrile
 - (3) Ethylamine (4) Propylamine
- Answer (3)
- **Sol.** Propanamide $\xrightarrow{\text{Br}_2/\text{KOH}}$ Ethylamine
- 49. Following tetrapeptide can be represented as



- (F, L, D, Y, I, Q, P are one letter codes for amino acids)
- (1) YQLF(2) FLDY(3) PLDY(4) FIQY

Answer (2)

Sol. The tetrapeptide code is

FLDY

- $F \rightarrow Phenyl alanine$
- $\mathsf{L} \to \mathsf{Leucine}$
- $D \rightarrow Aspartic acid$
- $\mathsf{Y} \to \mathsf{Tyrosine}$
- 50. The set of correct statements is:
 - Manganese exhibits +7 oxidation state in its oxide.
 - (ii) Ruthenium and Osmium exhibit +8 oxidation in their oxides.
 - (iii) Sc shows +4 oxidation state which is oxidizing in nature.
 - (iv) Cr shows oxidising nature in +6 oxidation state.
 - (1) (ii) and (iii) (2) (i), (ii) and (iv)
 - (3) (ii), (iii) and (iv) (4) (i) and (iii)

Answer (2)

- **Sol.** (i) $Mn_2O_7 \rightarrow Mn$ in (+7) oxidation state
 - (ii) It is also correct
 - (iii) Sc only shows +3 oxidation state
 - (iv) Cr⁺⁶ is oxidising in nature

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

51. The denticity of the ligand present in the Fehling's reagent is _____.

Answer (4)

Sol. The denticity is 2 in Fehling' reagent.



However, the overall maximum denticity can be 4 with other metal ion.

52. On heating, LiNO₃ gives how many compounds among the following?

Li₂O, N₂, O₂, LiNO₂, NO₂

Answer (3)

- **Sol.** $\text{LiNO}_3 \xrightarrow{\Delta} \text{Li}_2 \text{O} + \text{NO}_2 + \text{O}_2$
- 53. The equilibrium constant for the reaction $Zn(s) + Sn^{2+} (aq) \rightleftharpoons Zn^{2+} (aq) + Sn(s)$ is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^{0}_{Zn^{2+}/Zn} = -0.76 V$ is

(Nearest integer).

Given :
$$\frac{2.303 \text{ RT}}{\text{F}} = 0.059 \text{ V}$$

Answer (17)

Sol.
$$E_{cell}^{\circ} = \frac{2.303 \text{ RT}}{2 \text{ F}} \log k$$

$$E_{cell}^{\circ} = \frac{0.059}{2} \log(10^{20})$$
$$E_{Zn^{2+}/Zn}^{\circ} + 0.76 = 0.59$$
$$E_{Zn^{2+}/Zn}^{\circ} = 0.59 - 0.76$$
$$E_{Zn/Zn^{2+}}^{\circ} = 0.17 \text{ V}$$

54. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is ______
× 10²¹ (Nearest integer). (Given N_A = 6.02 × 10²³)

Answer (36)

Sol. Number of voids = $0.02 \times 3 \times 6.02 \times 10^{23}$

55. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO₂ was produced. The molar mass of compound is _____ g mol⁻¹ (Nearest integer).

Answer (200)

Sol. Number of moles of
$$CO_2 = \frac{4.4}{44} = 0.1$$

:. Number of moles of C in 1 mole of compound

= 10

 $\therefore 120 = \frac{60}{100} \times (x) \text{ [where x is molar mass of OC]}$

Molar mass = 200 g mol⁻¹

 Total number of acidic oxides among N₂O₃, NO₂, N₂O, Cl₂O₇, SO₂, CO, CaO, Na₂O and

NO is _____.

Answer (4)

Sol. N₂O₃, NO₂, Cl₂O₇, SO₂ are acidic in nature

57. For conversion of compounds A → B, the rate constant of the reaction was found to be 4.6 × 10– 5 L mol⁻¹ s⁻¹. The order of the reaction is _____.

Answer (2)

- **Sol.** As unit is L mol⁻¹ s⁻¹, order of the reaction is 2.
- Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6 Å. The radius of the third Bohr orbit of He⁺ is _____ picometer. (Nearest Integer)

Answer (270)

Sol. Radius of 3rd Bohr orbit of He⁺³

=
$$0.6 \times \frac{(3)^2}{2}$$

= 0.3×9
= 2.7 Å
= $270 \times 10^{-12} \text{ pm}$

59. At 298 K

$$\begin{split} N_{2}(g) + 3H_{2}(g) &= 2NH3(g), K_{1} = 4 \times 10^{5} \\ N_{2}(g) + O_{2}(g) &= 2NO(g), K_{2} = 1.6 \times 10^{12} \\ H_{2}(g) + \frac{1}{2}O_{2}(g) &= H_{2}O(g), K_{3} = 1.0 \times 10^{-13} \end{split}$$

Based on above equilibria, the equilibrium constant of the reaction,

$$2NH_{3}(g) + \frac{5}{2}O_{2}(g) \Longrightarrow 2NO(g) + 3H_{2}O(g)$$

is______ × 10⁻³³ (Nearest integer).

Answer (4)

Sol.
$$2NH_3(g)\frac{5}{2}O_2(g) \Longrightarrow 2NO(g) + 3H_2O(g)$$

Clearly,
$$K_{eq} = \frac{1}{k_1} \times k_2 \times k_3^3$$

= $\frac{1.6 \times 10^{12} \times 10^{-39}}{4 \times 10^5}$
= 0.4 × 10⁻³²
= 4 × 10⁻³³

60. The volume of HCl, containing 73 g L⁻¹, required to completely neutralise NaOH obtained by reacting 0.69 g of metallic sodium with water, is ______ mL. (Nearest Integer) (Given: molar Masses of Na, Cl, O, H, are 23, 35.5, 16 and 1 g mol⁻¹ respectively)

Answer (15)

Sol. $Na + H_2O \rightarrow NaOH + \frac{1}{2}H_2$ $_{0.69g}O \rightarrow NaOH + \frac{1}{2}H_2$ = 0.03 moles $\therefore 0.03 = 2 \times V$ $V = \frac{0.03}{2}L$ = 15 mL

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The area of the region

$$A = \left\{ \begin{pmatrix} x, y \end{pmatrix} : |\cos x - \sin x| \le y \le \sin x, \ 0 \le x \le \frac{\pi}{2} \right\}$$
is
(1) $\sqrt{5} - 2\sqrt{2} + 1$
(2) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$
(3) $\sqrt{5} + 2\sqrt{2} - 4.5$
(4) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$
Answer (1)



62. Let *K* be the sum of the coefficients of the odd powers of *x* in the expansion of $(1 + x)^{99}$. Let *a* be the middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$. If

 $\frac{{}^{200}C_{99}K}{a} = \frac{2^{l}m}{n}, \text{ where } m \text{ and } n \text{ are odd numbers,}$

then the ordered pair (I, n) is equal to

(1) (51, 101)(2) (51, 99)(3) (50, 101)(4) (50, 51)

Sol.
$$K = 2^{98}$$

 $a = {}^{200} {}_{100} 2^{50}$
 $\therefore \frac{2^{00}C_{99} \cdot {}^{98}}{2^{00}C_{100} \cdot {}^{50}} = \frac{2^{l}m}{n}$
 $\Rightarrow \frac{100}{101} \cdot {}^{48} = \frac{2^{l}m}{n}$
 $\Rightarrow \frac{25}{101} \cdot {}^{5} = \frac{2^{l}m}{n}$
 $\therefore l = 50, m = 25, n = 101$
63. If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersect at the point *P*, then the distance of the point *P* from the plane $z = a$ is
(1) 28 (2) 16
(3) 10 (4) 22
Answer (1)
Sol. $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1} = \lambda$ (say)
 $\& \frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1} = \mu$ (say)
 $\therefore \lambda + 1 = 2\mu + a$...(i)
 $2\lambda + 2 = 3\mu + 2$...(ii)
 $\lambda - 3 = \mu + 3$...(iii)
By (i) & (ii)
 $\Rightarrow 3\mu - 2 = 4\mu + 2a + 2$

 $\Rightarrow \mu = -2(1 + a) \& \lambda = 5 - 3a$ Put $\lambda \& \mu$ in (iii) we get a = -9 $\mu = 16$ $\lambda = 22$

- $\therefore \text{ Point of intersection} = (23, 46, 19)$ Distance from z = -9 is 28
- 64. If the tangent at a point *P* on the parabola $y^2 = 3x$ is parallel to the line x + 2y = 1 and the tangents at the

points *Q* and *R* on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line x - y = 2, then the area of the triangle *PQR* is

(1) $5\sqrt{3}$ (2) $3\sqrt{5}$ (3) $\frac{9}{\sqrt{5}}$ (4) $\frac{3}{2}\sqrt{5}$

Answer (2)

Sol.
$$P = \left\lfloor \frac{A}{m^2}, \frac{2A}{m} \right\rfloor$$
 where $\left(A = \frac{3}{4}, m = \frac{-1}{2} \right)$
& $Q, R = \left(\mp \frac{a^2 m_1}{a^2 m_1^2 + b^2}, \frac{\mp \cdot b^2}{\sqrt{a^2 m_1^2 + b^2}} \right)$
Where $a^2 = 4, b^2 = 1$ and $m_1 = 1$
 $\therefore P = (3, -3)$
 $Q = \left(\frac{-4}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right)$ & $R\left(\frac{4}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right)$
Area $= \frac{1}{2} \begin{vmatrix} 3 & -3 & 1 \\ \frac{-4}{\sqrt{5}} & \frac{-1}{\sqrt{5}} & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \end{vmatrix} = \frac{1}{10} \begin{vmatrix} 3 & -3 & 1 \\ -4 & -1 & \sqrt{5} \\ 0 & 0 & 2\sqrt{5} \end{vmatrix}$
 $= \frac{2\sqrt{5}}{10} (-15) = 3\sqrt{5}$

- 65. The statement $B \Rightarrow ((\sim A) \lor B)$ is equivalent to
- (1) $A \Rightarrow (A \Leftrightarrow B)$ (2) $B \Rightarrow ((\sim A) \Rightarrow B)$ (3) $B \Rightarrow (A \Rightarrow B)$ (4) $A \Rightarrow ((\sim A) \Rightarrow B)$ Answer (2, 3, 4)

Sol.
$$B \Rightarrow ((\neg A) \lor B) \equiv \neg B \lor (\neg A \lor B)$$

 $\equiv (\neg B \lor B) \lor (\neg A) \equiv T$
Option (2) $B \Rightarrow ((\neg A) \Rightarrow B) \equiv (\neg B) \lor (\neg A \Rightarrow B)$
 $\equiv (\neg B) \lor (A \lor B) \equiv T$
Option (3) $B \Rightarrow (A \Rightarrow B) \equiv B \Rightarrow ((\neg A) \lor B)$
(same as given)
Option (4) $A \Rightarrow ((\neg A) \Rightarrow B) \equiv (\neg A) \lor (A \lor B) \equiv T$
66. The set of all value of $t \in \mathbb{R}$, for which the matrix
 $\begin{vmatrix} e^t & e^{-t} (\sin t - 2\cos t) & e^{-1} (-2\sin t - \cos t) \\ e^t & e^{-t} (2\sin t + \cos t) & e^{-1} (\sin t - 2\cos t) \end{vmatrix}$ is

 $e^{-1}\cos t$ $e^{-1}\sin t$

invertible, is

 e^t

(1)
$$\left\{ (2k+1)\frac{\pi}{2}, k \in Z \right\}$$
 (2) R
(3) $\left\{ k\pi + \frac{\pi}{4}, k \in Z \right\}$ (4) $\left\{ k\pi, k \in Z \right\}$

Answer (2)

Sol.
$$\begin{vmatrix} 0 & e^{-t} (-\sin t - 3\cos t) & e^{-t} (-3\sin t + \cos t) \\ 8 & e^{-t} (2\sin t) & e^{-t} (-2\cos t) \\ e^{t} & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix} = 0$$

(If matrix is non invertible)

$$-2\cos t(-\sin t - 3\cos t) - 2\sin t(\cos t - 3\sin t) = 0$$

$$\Rightarrow \quad 6\cos^2 t + 6\sin^2 t = 0$$

$$t \in 0$$

67. Let $S = \{w_1, w_2, \dots\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}, n \ge 2$. Let $A = \{2k+3l : k, l \in N\}$ and $B = \{w_n : n \in A\}$. Then P(B) is equal to

(1)
$$\frac{1}{16}$$
 (2) $\frac{1}{32}$

(3)
$$\frac{3}{64}$$
 (4) $\frac{3}{32}$

Sol.
$$P(w_1) + \frac{P(w_1)}{2} + \frac{P(w)}{2^2} + \dots = 1$$

 $\therefore P(w_1) = \frac{1}{2}$

Hence, $P(w_n) = \frac{1}{2^n}$ Every number except 1, 2, 3, 4, 6 is representable in the form 2k + 3l where $k, l \in N$. $\therefore P(B) = 1 - P(w_1) - P(w_2)$ $-P(w_3) - P(w_4) - P(w_6)$ $= \frac{3}{64}$

68. The value of the integral $\int_{1}^{2} \left(\frac{t^{4}+1}{t^{6}+1}\right) dt$ is

(1) $\tan^{1}2 - \frac{1}{3}\tan^{-}8 + \frac{\pi}{3}$ (2) $\tan^{1}2 + \frac{1}{3}\tan^{-}8 - \frac{\pi}{3}$ (3) $\tan^{-}\frac{1}{2} + \frac{1}{3}\tan^{-1}8 - \frac{\pi}{3}$ (4) $\tan^{-}\frac{1}{2} - \frac{1}{3}\tan^{-1}8 + \frac{\pi}{3}$

Answer (2)

Sol.
$$\int_{1}^{2} \frac{t^{4} + 1}{t^{6} + 1} dt$$
$$= \int_{1}^{2} \frac{\left(t^{2} + 1\right)^{2}}{+1} dt - 2\int_{1}^{2} \frac{t^{2}}{t^{6} + 1} dt$$
$$= \int_{1}^{2} \frac{t^{2} + 1}{t^{4} - t^{2} + 1} dt - 2\int_{1}^{2} \frac{t^{2}}{\left(t^{3}\right)^{2} + 1} dt$$
$$= \tan^{-1}\left(2 + \sqrt{3}\right) + \tan^{-1}\left(2t - \sqrt{3}\right)\Big|_{1}^{2}$$
$$- \frac{2}{3}\tan^{-1}\left(t^{3}\right)\Big|_{1}^{2}$$
$$= \tan^{-1}\left(4 + \sqrt{3}\right) + \tan^{-1}\left(4 - \sqrt{3}\right) - \tan^{-1}\left(2 + \sqrt{3}\right)$$
$$-\tan^{-1}\left(2 + \sqrt{3}\right)$$
$$-\tan^{-1}\left(2 + \sqrt{3}\right)$$
$$-\tan^{-1}\left(2 + \sqrt{3}\right)$$
$$= \tan^{-1}\left(2 + \sqrt{3}\right) = \tan^{-1}\left(2 + \sqrt{3}\right)$$

69. Consider a function f: N → R, satisfying
f(1)+2f(2)+3f(3)+....+xf(x) = x(x+1)f(x); x ≥ 2
with f(1) = 1. Then
$$\frac{1}{(2022)} + \frac{1}{f(2028)}$$
 is equal to
(1) 8200 (2) 8400
(3) 8100 (4) 8000
Answer (3)
Sol. f(1)+2f(2)+3f(3)+...+nf(n) = n(n+1)+(n)...(i)
n → n + 1
f(1)+2f(2)+...+(n+1)f(n+1) = (n+1)(n+2)
f(n+1)...(ii)
(i) and (ii) gives
3f(3)-2f(2) = 0
4f(4)-3f(3) = 0
:
(n+1)f(n+1)-nf(n) = 0
⇒ f(n+1) $\frac{2f(2)}{n+1}$
f(n) = $\frac{1}{2n}$
 $\frac{1}{f(2022)} + \frac{1}{f(2028)} = 8100$
70. Let $y = y(x)$ be the solution of the differential
equation $x\log_{e} x \frac{dy}{dx} + y = x^{2}\log_{e} x, (x > 1)$. If $y(2) = 2$,
then y(e) is equal to
(1) $\frac{1+e^{2}}{2}$ (2) $\frac{1+e^{2}}{4}$
(3) $\frac{4+e^{2}}{4}$ (4) $\frac{2+e^{2}}{2}$
Answer (3)
Sol. $x\ln x \frac{dy}{dx} + y = x^{2}\ln x$
 $\frac{dy}{dx} + \frac{1}{x\ln x} \cdot y =$
If $e^{\int \frac{1}{x\ln x} dx} = \int_{1}^{1} t^{dt}$, where $t = \ln x$
 $= e^{\ln t} = t = \ln x$

$$y \cdot \ln x = x \ln x = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x}$$

$$y \ln x = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C \qquad \dots(i)$$

$$y(2) = 2 \Rightarrow C = 1$$
Putting $x = e$ in (i),

$$y = \frac{e^2}{4} + 1 = \frac{4 + e^2}{4}$$
71. The shortest distance between the lines
$$\frac{x - 1}{2} = \frac{y + 8}{-} = \frac{z - 4}{5} \text{ and } \frac{x - 1}{2} = \frac{y - 2}{1} = \frac{z - 6}{-} \text{ is}$$
(1) $4\sqrt{3}$ (2) $3\sqrt{3}$
(3) $5\sqrt{3}$ (4) $2\sqrt{3}$
Answer (1)
Sol. $\vec{r}_1 = \hat{i} - 8\hat{j} + 4\hat{k}$
 $\vec{r}_2 = \hat{i} + 2\hat{j} + 6\hat{k}$
 $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$
 $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$
S.D. $= \frac{|\vec{r}_1 - \vec{r} - \vec{a} - \vec{b}|}{|\vec{a} \times \vec{b}|}$
 $\left[\vec{r}_1 - \vec{r}_2 - \vec{a} - \vec{b} - \frac{|0|}{|2|} - 75 - 5|$
 $\frac{1}{2} - 7 - 5$
 $\frac{1}{2} - 7 - 5|$
 $\frac{1}{2} - 7 - 5$
 $\frac{1}{2} - 7 - 5|$
 $\frac{1}{2} - 7 - 5|$
 $\vec{a} \times \vec{b} = 16\sqrt{3}$
S.D. $= \frac{16\hat{j} - 16\hat{j} - 16\hat{k}}{\vec{a} \times \vec{b} = 16\sqrt{3}$
S.D. $= \frac{192}{16\sqrt{3}} = 4\sqrt{3}$
72. The letters of the word OUGHT are written in all

72. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

Answer (3)	
(3) 89	(4) 79
(1) 84	(2) 86



- **Sol.** f''(x) = g''(x) + 6x $\Rightarrow f'(x) = g'(x) + 3x^2 + C$ f'(1) = g'(1) + 3 + C \Rightarrow $g = 3 + 3 + C \Rightarrow C = 3$ $\Rightarrow f'(x) = q'(x) + 3x^2 + 3$ $\Rightarrow f(x) = g(x) + x^2 + 3x + C'$ x = 2f(2) = g(2) + 14 + C'12 = 4 + 14 + C' $\Rightarrow C' = -6$ $\Rightarrow f(x) = g(2) + x^3 + 3x - 6$ f(-2) = g(-2) - 8 - 6 - 6g(-2) - f(-2) = 20 $f'(x) - q'(x) = 3x^2 + 3$ $x \in (-1, 1)$ $3x^2 + 3 \in (0, 6)$ $\Rightarrow f'(x) - g'(x) \in (0, 6)$ $f(x) - g(x) = x^3 + 3x - 6$ At x = -1|f(-1) - g(-1)| = 10... Option (4) is false. 75. If $\vec{a} = \hat{i} + 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$. Then $\vec{r} \cdot \vec{c}$ is equal to (1) 32 (2) 30 (3) 34 (4) 36 Answer (3) **Sol.** $(\vec{r} - \vec{c}) \times \vec{b} = 0$ $\vec{r} = \lambda \vec{b} + \vec{c}$ $\vec{r} \cdot \vec{a} = 0$ $\Rightarrow \lambda \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a} = 0$ $\Rightarrow \lambda(3) + (7+8) = 0$ $\Rightarrow \lambda = -5$ $\vec{r} = 5\vec{b} + \vec{c}$ $= -5\hat{i} - 5\hat{j} - 5\hat{k} + (7\hat{i} + 3\hat{j} + 4\hat{k})$ $= 2\hat{i} - 8\hat{j} - \hat{k}$ $\vec{r} \cdot \vec{c} = 17 + 24 - 4 = 34$
- 76. The plane 2x y + z = 4 intersects the line segment joining the points A(a, -2, 4) and B(2, b, -3) at the point *C* in the ratio 2 : 1 and the distance of the point *C* from the origin is $\sqrt{5}$. If ab < 0 and *P* is the point (a - b, b, 2b - a) then CP^2 is equal to

(1)
$$\frac{73}{3}$$
 (2) $\frac{16}{3}$

(3)
$$\frac{97}{3}$$
 (4) $\frac{17}{3}$

Answer (4)

Sol.
A
C
C
(2, b, -3)

$$c: \left(\frac{a+4}{3}, \frac{-2+2b}{3}, \frac{4-6}{3}\right)$$

$$= \left(\frac{a+4}{3}, \frac{-2+2b}{3}, \frac{-2}{3}\right)$$
c lies on plane $2x - y + z = y$
 $2\left(\frac{a+4}{3}\right) - \left(\frac{2b-2}{3}\right) - \frac{2}{3} = 4$
 $\Rightarrow 2a + 8 - 2b + 2 - 2 = 12$
 $\Rightarrow a - b = 2$...(i)
Now, $OP = \sqrt{5}$
 $\left(\frac{a+4}{3}\right)^2 + \left(\frac{2b-2}{3}\right)^2 + \frac{4}{9} = 5$ and using (i)
 $a = \frac{11}{5}, 1$
 $\Rightarrow b = \frac{1}{5}, -1$
as also $\Rightarrow a = 1, b = -1$
 $\therefore P(2, -1, -3), c\left(\frac{5}{3}, \frac{-4}{3}, \frac{-2}{3}\right)$
 $CP^2 = \frac{1}{9} + \frac{1}{9} + \frac{49}{9}$
 $= \frac{17}{3}$
 \therefore Option (4) is correct.

JEE (Main)-2023 : Phase-1 (29-01-2023)-Evening				
77.	The number of 3 digit numbers, that are divisible by			
	either 3 or 4 but not divisible by 48, is			
	(1) 432	(2)	507	
	(3) 400	(4)	472	
Ansv	wer (1)			
Sol.	Number divisible by $3 = 3$	800		
	Number divisible by 4 = 225 Number divisible by 12 = 75 Number divisible by 48 = 18 Total required number = 300 + 225 – 75 – 18			
	= 432			
	Option (1) is correct.			
78.	The set of all values of λ for which the equation $\cos^2 2x - 2 \sin^4 x - 2\cos^2 x = \lambda$ has a real solution <i>x</i> ,			
	is			
	(1) $\left\lfloor -1, -\frac{1}{2} \right\rfloor$	(2)	$\left\lfloor -\frac{3}{2}, -1 \right\rfloor$	
	$(3) \left\lfloor -2, -\frac{3}{2} \right\rfloor$	(4)	[-2, -1]	
Answer (2)				
Sol.	$\cos^2 2x - 2\sin^4 x - 2\cos^2 x$	² x	$=\lambda$	

$$\cos^{2} 2x - \frac{1}{2}(2\sin^{2} x)^{2} - (1 + \cos 2x) = \lambda$$

$$\Rightarrow \cos 2x - \frac{1}{2}(1 - \cos 2x)^{2} - (1 + \cos 2x)$$

$$\Rightarrow \cos 2x - \frac{1}{2}(1 + \cos^{2} 2x - 2\cos 2x) - 1 - \cos 2x$$

$$\Rightarrow \frac{\cos^{2} 2x}{2} - \frac{3}{2}$$

$$\cos^{2} 2x \in [0, 1]$$

$$\frac{\cos^{2} 2x}{2} \in \left[0, \frac{1}{2}\right]$$

$$\frac{\cos^{2} 2x}{2} - \frac{3}{2} \in \left[-\frac{3}{2}, -1\right]$$

$$\therefore \lambda \quad \left[-\frac{\pi}{2}, -1\right]$$
Option (2) is correct

79. Let $\vec{a} = 4i + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$. If \vec{c} is a vector such that $\vec{c} \cdot (\vec{a} \times \vec{b}) + 25 = 0$, $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ and projection of \vec{c} on \vec{a} is 1, then the projection of \vec{c} on \vec{b} equals

(1)
$$\frac{1}{5}$$
 (2) $\frac{5}{\sqrt{2}}$

(3)
$$\frac{1}{\sqrt{2}}$$
 (4) $\frac{3}{\sqrt{2}}$

Answer (2)

Sol. $[\vec{c} \ \vec{a} \ \vec{b}] = -25$ Let $\vec{c} = l\hat{i} + n\hat{j} + n\hat{k}$ $\begin{vmatrix} I & m & n \\ 4 & 3 & 0 \\ 3 & -4 & 5 \end{vmatrix} = -25$ $\Rightarrow 3I-4m-5n=-5 \dots (i)$ $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ \Rightarrow *l* + *m* + *n* = 4 ...(ii) $\frac{\vec{c}\cdot\vec{a}}{|\vec{a}|}=1 \Rightarrow \vec{c}\cdot\vec{a}=5$ $\Rightarrow 4l + 3m = 5$...(iii) Using (i), (ii) and (iii) l = 2, m = -1, n = 3Now, $\frac{\vec{c} \cdot \vec{b}}{|\vec{b}|} = \frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$.:. Option (2) is correct. 80. Let R be a relation defined on N as a R b if 2a + 3bis a multiple of 5, $a, b \in N$. Then R is (1) an equivalence relation (2) not reflexive (3) symmetric but not transitive (4) transitive but not symmetric Answer (1) **Sol.** a R b if 2a + 3b = 5m, $m \in I$ (1) $(a, a) \in R$ as $2a + 3a = 5a, a \in N$

Hence R is reflexive

(2) If
$$(a, b) \in R$$
 then $2a + 3b = 5m$

Now, 5(a + b) = 5n

$$3a + 2b + 2a + 3b = 5n$$

$$\therefore 3a + 2b = 5(n - m)$$

$$\therefore (b, a) \in R$$

$$\therefore R \text{ is symmetric}$$

(3) If $(a, b) \in R$ and $(b, c) \in R$ then

$$2a + 3b = 5m, 2b + 3c = 5n$$

$$\Rightarrow 2a + 5b + 3c = 5(m + n)$$

$$\Rightarrow 2a + 3c = 5(m = n - b)$$

$$\therefore (a, c) \in R$$

$$\therefore R \text{ is transitive}$$

Hence R is equivalence relation.
option (1) is correct.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Let $\{a_k\}$ and $\{b_k\}$, $k \in \mathbb{N}$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

Answer (09)

Sol. $\{a_k\}$ be a G.P. with $a_1 = 4$, $r = r_1$ And $\{b_k\}$ be G.P. with $b_1 = 4$, $r = r_2$ $(r_1 < r_2)$ Now $C_k = a_k + b_k$ $c_1 = 4 + 4 = 8$ and $c_2 = 5$ $a_2 + b_2 = 5$ $\therefore \quad _1 + r_2 = \frac{5}{4}$ and $c_3 = \frac{13}{4} \Rightarrow r_4^2 + r_2^2 = \frac{13}{16}$

$$\therefore \quad \frac{25}{16} - 2r_1r_2 = \frac{13}{16} \Longrightarrow 2r_1r_2 = \frac{3}{4}$$
$$\therefore \quad r_2 - r_1 = \sqrt{\frac{25}{16} - \frac{3}{2}} = \frac{1}{4}$$
$$\therefore \quad 2 = \frac{3}{4}, r_1 = \frac{1}{2}$$
$$\therefore \quad a_6 = 4 \times \frac{1}{2^5} = \frac{1}{8}, \quad 4 = 4 \times \frac{27}{64} = \frac{27}{16}$$
and
$$\sum_{K=1}^{\infty} C_K = 4 \left| \frac{1}{1 - \frac{1}{2}} + \frac{1}{1 - \frac{3}{4}} \right| = 24$$
$$\therefore \quad \sum_{K=1}^{\infty} C_K - (12a_6 + 8b_4) = 0$$

82. the total number of 4-digit numbers whose greatest common divisor with 54 is 2, is _____.

Answer (3000)

4 digit no.

=

And
$$54 = 3 \times 3 \times 3 \times 2$$

So, a = all even no. of 4 digits

- Even multiple of 3 (4 digits)

= 3000

83. If the equation of the normal to the curve

$$y = \frac{x-a}{(x+b)(x-2)}$$
 at the point (1, -3) is $x - 4y = 13$,

then the value of *a* + *b* is equal to ______.

Answer (04)

Sol. Given curve :
$$y = \frac{x-a}{(x+b)(x-2)}$$
 at (1,-3)

$$\therefore -3 = \frac{1-a}{(1+b)(-1)} \Rightarrow 3+3b = 1-a$$

$$\beta \Rightarrow a+3b+2 = 0$$

$$y = \frac{x-a}{(x+b)(x-2)}$$

$$\frac{dy}{dx} = \frac{(x+b)(x-2) - (x-a)[(x+b) + (x-2)]}{[(x+b)(x-2)]^2}$$
at (1,-3) $m_T = \frac{-(1+b) - (1-a)(b)}{(1+b)^2} = -4$

$$\therefore 1+b+b-ab = 4(1+b)^2$$

$$\Rightarrow 1+2b+b(3b+2) = 4b^2+4+8b$$

$$\Rightarrow b^2+4b+3 = 0$$

$$(b+1)(b+3) = 0$$

$$b = -1, a = 1$$

$$but 1+b \neq 0$$

$$b = -3, a = 7$$

$$\therefore b \neq -1$$

$$\therefore a+b = 04$$

84. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line x + y + 2 = 0. If *r* is the radius of its circumcircle, then r^2 is equal to _____.

Answer (10)

Sol. Tangent for $y^2 = 2x$ at (2, 2) is

$$L_1: 2y = x + 2$$

Tangent for $x^2 + y^2 = 4x$ at (2, 2) is

$$L_2: y = 2$$



Radius of circumcircle =
$$\frac{abc}{4\Delta}$$

$$=\frac{\left(\sqrt{20}\right)(6)\left(\sqrt{8}\right)}{4\times\frac{1}{2}\times6\times2}$$
$$R=\sqrt{10}$$

85. Let $X = \{11, 12, 13, \dots, 40, 41\}$ and $Y = \{61, 62, 63, \dots, 90, 91\}$ be the two sets of observations. If \overline{x} and \overline{y} are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $|\overline{x} + \overline{y} - \sigma^2|$ is equal to _____.

Answer (603)

 $R^2 = 10$

Sol.
$$x = \{11, 12, 13 ..., 40, 41\}$$

 $y = \{61, 62, 63, ..., 90, 91\}$
 $\overline{x} = \frac{\frac{31}{2}(11+41)}{31} = \frac{1}{2} \times 52 = 26$

$$\overline{y} = \frac{\frac{31}{2}(61+91)}{31} = \frac{1}{2} \times 152 = 76$$

$$\sigma^{2} = \frac{\sum x_{i}^{2} + \sum y_{i}^{2}}{62} - \left(\frac{\sum x + \sum y}{62}\right)^{2}$$

$$= 705$$
Now
$$\left|\overline{x} + \overline{y} - \sigma^{2}\right|$$

$$= |26 + 76 - 705|$$

$$= 603$$
86. Let $\alpha = 8 - 14t$, $A = \left\{z \in \mathbb{C} : \frac{\alpha z - \overline{\alpha}\overline{z}}{z^{2} - (\overline{z})^{2} - 112i}\right\}$ and
$$B = \left\{z \in \mathbb{C} : |z + 3i| = 4\right\}.$$
 Then $\sum_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z)$ is
equal to
$$-----$$
Answer (07)
Sol. Let $z = x + iy$
and $\alpha = 8 - 14i$

$$\frac{\alpha z - \overline{\alpha} \,\overline{z}}{z^{2} - \overline{z}^{2} - 112i} = 1$$

$$(16y - 28x)i = 1$$

$$(16y - 28x)i = 1$$

$$(16y - 28x + 112)i = 4xy$$

$$\therefore z = -7i \text{ or } 4$$
Now, $z = -7i$ satisfy B

$$B : x^{2} + (y + 3)^{2} = 16$$

$$A \cap B = (0, -7)$$

$$Re z - Im z = 7$$
87. Let A be a symmetric matrix such that
$$|A| = 2 \text{ and} \begin{bmatrix} 2 & 1\\ 3 & \frac{3}{2} \end{bmatrix} |A - \begin{bmatrix} 1 & 2\\ \alpha & \beta \end{bmatrix}.$$
 If the sum of the
diagonal elements of A is s, then $\frac{\beta s}{\alpha^{2}}$ is equal to
Answer (05)
Sol. $A = \begin{pmatrix} a & c\\ c & b \end{pmatrix}$

D1.
$$A = \begin{pmatrix} c & b \end{pmatrix}$$
$$|A| = ab - c^{2} = 2 \qquad \dots(1)$$
$$\begin{pmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{pmatrix} \begin{pmatrix} a & c \\ c & b \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ \alpha & \beta \end{pmatrix}$$

2a + c = 1	(2)
2c + b = 2	(3)
$3a+\frac{3}{2}c=\alpha$	(4)
$3c+\frac{3}{2}b=\beta$	(5)
From (1), (2) and (3)	
$a = \frac{3}{4}$ $b = 3$ $c = -\frac{1}{2}$	
\Rightarrow Now, $\alpha = \frac{6}{4}$	
$\beta = 3$	
$s=\frac{15}{4}$	
$\frac{\beta s}{\alpha^2} = \frac{3 \times \frac{15}{4}}{\left(\frac{6}{4}\right)^2} = \frac{\frac{45}{4}}{\frac{9}{4}} =$	= 5

88. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall n \ge 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^8 \frac{n}{2^{n-1}}$, then

2⁷(2S – T) is equal to _____. Answer (461)

Sol. ::
$$a_n = a_{n-1} + (n-1)$$
 and $a_1 = b_1 = 1$

$$b_n = b_{n-1} + a_{n-1}$$

$\therefore b_{n+1} = 2b_n - b_{n-1} + n - 1$							
	n	b _n	b _n -n				
	1	1	0				
	2	2	0				
	3	4	1				
	4	8	4				
	5	15	10				
	6	26	20				
	7	42	35				
	8	64	56				
	9	93	84				
	10	130	120				

$$\therefore \quad 2S - T = \left(\sum_{n=1}^{8} \frac{b_n - n}{2^{n-1}}\right) + \frac{b_9}{2^8} + \frac{b_{10}}{2^9}$$
$$= \frac{461}{128}$$
$$\therefore \quad 2^7 (2S - T) = 461$$

89. A circle with centre (2, 3) and radius 4 intersects the line x + y = 3 at the points *P* and *Q*. If the tangents at *P* and *Q* intersect at the point *S*(α , β), then $4\alpha - 7\beta$ is equal to _____.

Answer (11)

- Sol. The line x + y = 3 ...(i) is polar of $S(\alpha, \beta)$ w.r.t. circle $(x-2)^2 + (y-3)^2 = 16$ $\Rightarrow x^2 + y^2 - 4x - 6y - 3 = 0$ Equation of polar is $\alpha x + \beta y - 2 (x + \alpha) - 3(4 + \beta) - 3 = 0$ $(\alpha - 2) x + (\beta - 3)y - (2\alpha + 3\beta + 3) = 0$...(ii) (i) and (ii) represent the same. $\therefore \frac{\alpha - 2}{1} = \frac{\beta - 3}{1} = \frac{2\alpha + 3\beta + 3}{3}$ $\alpha - \beta + 1 = 0$ $\alpha - 3\beta - 9 = 0$ $\Rightarrow \alpha = -6, \beta = -5$ $4\alpha - 7\beta = 11$
- 90. Let $\alpha_1, \alpha_2, ..., \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \ge |\alpha_2| \ge ... \ge |\alpha_7|$. Then $\alpha_1 \alpha_2 - \alpha_3 \alpha_4 + \alpha_5 \alpha_6$ is equal to _____.

Answer (09) Sol. $x^7 + 3x^5 - 13x^3 - 15x = 0$ $x(x^6 + 3x^4 - 13x^2 - 15) = 0$ $x = 0 = \alpha_7$ Let $x^2 = t$ $t^3 + 3t^2 - 13t - 15 = 0$ (t + 1) (t + 5) (t - 3) = 0 $t = x^2 = -1, -5, 3$ $x = \pm i, \pm \sqrt{5}i, \pm \sqrt{3}$ $\alpha_1, \alpha_2 = \pm \sqrt{5}i, \alpha_3, \alpha_4 \pm = \sqrt{3}, \alpha_5, \alpha_6 = \pm i$ $\alpha_1 \alpha_2 - \alpha_3 \alpha_4 + \alpha_5 \alpha_6 = 5 + 3 + 1 = 9$