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30/01/2023
Evening

Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A force is applied to a steel wire 'A', rigidly clamped at one end. As a result elongation in the wire is 0.2 mm. If same force is applied to another steel wire 'B' of double the length and a diameter 2.4 times that of the wire 'A', the elongation in the wire 'B' will be (wires having uniform circular cross-sections)

(1) 6.9×10^{-2} mm (2) 3.0×10^{-2} mm

(3) 6.06×10^{-2} mm (4) 2.77×10^{-2} mm

Answer (1)

Sol. $\because \quad \Delta\ell = \dfrac{F\ell(4)}{Y\pi d^2}$

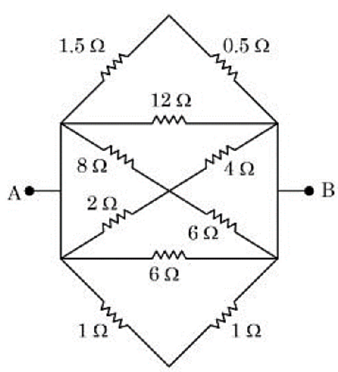
$$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{\ell_1}{d_1^2} \times \frac{d_2^2}{\ell_2}$$

$$\frac{0.2}{\Delta\ell_2} = \frac{1}{2} \times (2.4)^2$$

$$\Delta\ell_2 = \frac{2 \times 0.2}{(2.4)^2}$$

$= 6.9 \times 10^{-2}$ mm

2. The equivalent resistance between *A* and *B* is

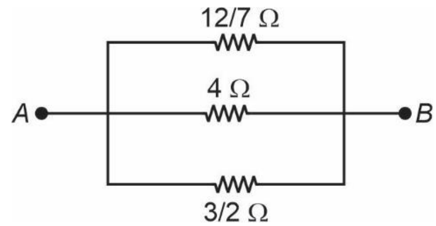


(1) $\dfrac{2}{3}\Omega$ (2) $\dfrac{1}{2}\Omega$

(3) $\dfrac{3}{2}\Omega$ (4) $\dfrac{1}{3}\Omega$

Answer (1)

Sol. Equivalent circuit can be drawn as

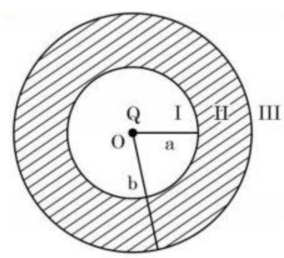


$$\therefore \quad \frac{1}{R_{AB}} = \frac{7}{12} + \frac{1}{4} + \frac{2}{3}$$

$$\boxed{R_{AB} = \frac{2}{3}\Omega}$$

3. As shown in the figure, a point charge Q is placed at the centre of conducting spherical shell of inner radius *a* and outer radius *b*. The electric field due to charge Q in three different regions I, II and III is given by:

(I : $r < a$, II : $a < r < b$, III : $r > a$)



(1) $E_I \neq 0, E_{II} = 0, E_{III} \neq 0$

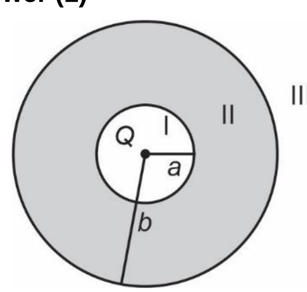
(2) $E_I = 0, E_{II} = 0, E_{III} = 0$

(3) $E_I \neq 0, E_{II} = 0, E_{III} = 0$

(4) $E_I = 0, E_{II} = 0, E_{III} \neq 0$

Answer (1)

Sol.



$E_I \neq 0$ (inside region)

$E_{II} = 0$ (conducting region)

$E_{III} \neq 0$

$= \dfrac{KQ}{r^2} \quad (r > b)$

4. Match List I with List II:

	List I		List II
A.	Torque	I.	kg m^{-1} s^{-2}
B.	Energy density	II.	kg ms^{-1}
C.	Pressure gradient	III.	kg m^{-2} s^{-2}
D.	Impulse	IV.	kg m^2 s^{-2}

Choose the ***correct*** answer from the options given below:

(1) A-IV, B-I, C-II, D-III

(2) A-IV, B-III, C-I, D-II

(3) A-IV, B-I, C-III, D-II

(4) A-I, B-IV, C-III, D-II

Answer (3)

Sol. Torque → kg m^2 s^{-2} (IV)

Energy density → kg m^{-1} s^{-2} (I)

Pressure gradient → kg m^{-2} s^{-2}(III)

Impulse → kg m s^{-1} (II)

5. Match List I with List II:

	List I		List II
A.	Attenuation	I.	Combination of a receiver and transmitter.
B.	Transducer	II.	Process of retrieval of information from the carrier wave at receiver
C.	Demodulation	III.	Converts one form of energy into another
D.	Repeater	IV.	Loss of strength of a signal while propagating through a medium

Choose the ***correct*** answer from the options given below:

(1) A-IV, B-III, C-I, D-II

(2) A-II, B-III, C-IV, D-I

(3) A-IV, B-III, C-II, D-I

(4) A-I, B-II, C-III, D-IV

Answer (3)

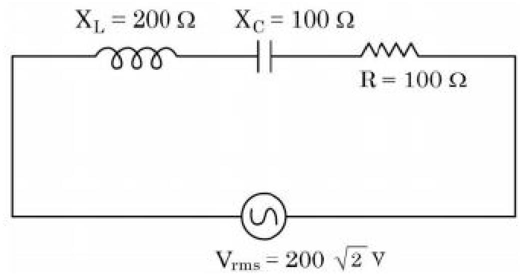
Sol. Theoretical attenuation → (IV)

Transducer → (III)

Demodulation → (II)

Repeater → (I)

6. In the given circuit, rms value of current (I_{rms}) through the resistor R is



(1) 20 A

(2) $2\sqrt{2}$ A

(3) 2 A

(4) $\frac{1}{2}$ A

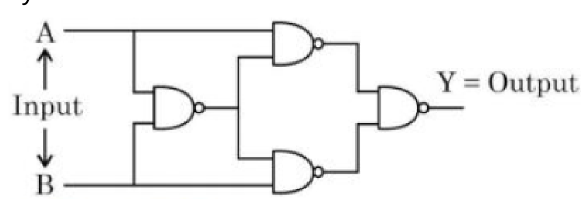
Answer (3)

Sol. $I_{rms} = \frac{V_{rms}}{z} = \frac{200\sqrt{2}}{\sqrt{100^2 + (200-100)^2}}$

$= \frac{200\sqrt{2}}{100\sqrt{2}}$

= 2 A

7. The output Y for the inputs A and B of circuit is given by



Truth table of the shown circuit is

(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

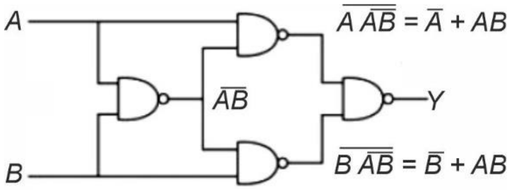
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Answer (2)

Sol.



$$Y = \overline{(\overline{A} + AB)(\overline{B} + AB)} = (A + B)(\overline{A} + \overline{B})$$

$$= A\overline{B} + B\overline{A} \text{ (XOR gate)}$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

8. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Efficiency of a reversible heat engine will be highest at –273°C temperature of cold reservoir.

Reason R: The efficiency of Carnot's engine depends not only on temperature of cold reservoir but it depends on the temperature of hot reservoir too and is given as $\eta = \left(1 - \frac{T_2}{T_1}\right)$.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(3) **A** is false but **R** is true

(4) **A** is true but **R** is false

Answer (1)

Sol. $\eta = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$

T_{cold} = OK $\Rightarrow \eta = 1$ (max)

A is correct.

R is also correct and explains A.

9. A vehicle travels 4 km with speed of 3 km/ h and another 4 km with speed of 5 km/h, then its average speed is

(1) 4.00 km/h (2) 4.25 km/h

(3) 3.50 km/h (4) 3.75 km/h

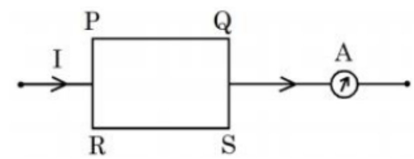
Answer (4)

Sol. Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$$= \frac{8}{\frac{4}{3} + \frac{4}{5}}$$

= 3.75 km/h

10. A current carrying rectangular loop *PQRS* is made of uniform wire. The length *PR* = *QS* = 5 cm and *PQ* = *RS* = 100 cm. If ammeter current reading changes from *I* to 2*I*, the ratio of magnetic forces per unit length on the wire *PQ* due to wire *RS* in the two cases respectively $\left(f_{PQ}^{I} : f_{PQ}^{2I}\right)$ is



(1) 1 : 2 (2) 1 : 4

(3) 1 : 3 (4) 1 : 5

Answer (2)

Sol. Force between two current carrying wire

$$= \frac{\mu_0 I_1 I_2}{2\pi d} \times L$$

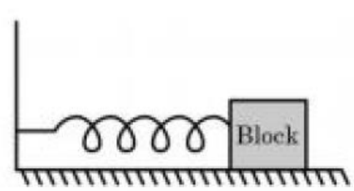
Here I_1 & I_2 are equal

$$F = \frac{\mu_0 I^2}{2\pi d} \times L$$

$$F \propto I^2$$

$$\frac{F_I}{F_{2I}} = \frac{I^2}{4I^2} = \frac{1}{4}$$

11. For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is ω_1. When the mass block is 2 kg, the angular frequency is ω_2. The ratio ω_2/ω_1 is



(1) $\frac{1}{2}$ (2) 2

(3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Answer (4)

Sol. $\omega = \sqrt{\frac{K}{m}} \Rightarrow \omega \propto \frac{1}{\sqrt{m}}$

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{2}}$$

12. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: The nuclear density of nuclides ${}^{10}_{5}B$, ${}^{6}_{3}Li$, ${}^{56}_{26}Fe$, ${}^{20}_{10}Ne$ and ${}^{209}_{83}Bi$ can be arranged as $\rho^N_{Bi} > \rho^N_{Fe} > \rho^N_{Ne} > \rho^N_{B} > \rho^N_{Li}$

Reason R : The radius R of nucleus is related to its mass number A as $R = R_0 A^{1/3}$, where R_0 is a constant.

In the light of the above statements, choose the correct answer from the options given below

(1) Both **A** and **R** are true but **R** is **Not** the correct explanation of **A**

(2) **A** is true but **R** is false

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) **A** is false but **R** is true

Answer (4)

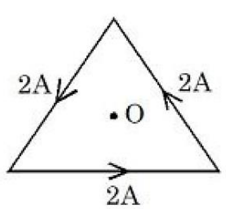
Sol. $R = R_0 A^{\frac{1}{3}}$, using this

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{Am_P}{\frac{4}{3}\pi R_0^3 A} = \frac{m_P}{\frac{4}{3}\pi R_0^3}$$

ρ is independent of mass number.

$\therefore$ A is false

13. As shown in the figure, a current of 2 A flowing in an equilateral triangle of side $4\sqrt{3}$ cm. The magnetic field at the centroid O of the triangle is



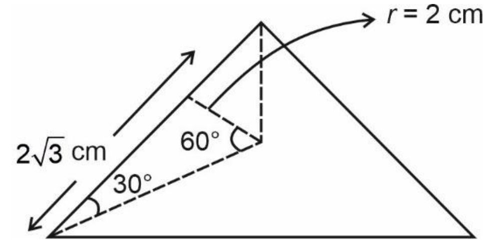
(Neglect the effect of earth's magnetic field)

(1) $3\sqrt{3} \times 10^{-}$ T (2) $\sqrt{3} \times 10^{-}$ T

(3) $4\sqrt{3} \times 10^{-}$ T (4) $4\sqrt{3} \times 10^{-}$ T

Answer (1)

Sol.



$$B_{net} = \frac{\mu_0 i}{4\pi r}(\sin\alpha + \sin\beta) \times 3$$

$$= \frac{\mu_0 \times 2}{4\pi \times (2 \times 10^{-2})} \times \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) \times 3$$

$$= 10^{-7} \times 10^2 (3\sqrt{3})$$

$$= 3\sqrt{3} \times 10^{-5}\,\text{T}$$

14. An object is allowed to fall from a height R above the earth, where R is the radius of earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be

(1) $\sqrt{\frac{gR}{2}}$

(2) $2\sqrt{gR}$

(3) $\sqrt{gR}$

(4) $\sqrt{2gR}$

Answer (3)

Sol. $U_P = -\frac{GMm}{2R}$

$$U_S = -\frac{GMm}{R}$$

$\Rightarrow$ Energy conservation

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{2R}$$

$$v^2 = \frac{GM}{R}$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

15. A flask contains hydrogen and oxygen in the ratio of 2 : 1 by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is:

(1) 1 : 1 (2) 2 : 1

(3) 4 : 1 (4) 1 : 4

Answer (1)

Sol. K.E. per molecule = $\left(\frac{f}{2}KT\right)$

$$\frac{\text{average}(\text{K.E})_{\text{hydrogen}}}{\text{average}(\text{K.E})_{\text{oxygen}}} = \frac{f_{\text{hydrogen}}}{f_{\text{oxygen}}} = 1$$

16. A thin prism P_1 with an angle 6° and made of glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism P_2 is

(1) 6°

(2) 1.3°

(3) 4.5°

(4) 7.8°

Answer (3)

Sol. $(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$

$\Rightarrow (1.54 - 1)6 = (1.72 - 1)A_2$

$$A_2 = \left(\frac{0.54}{0.72} \times 6\right) = \frac{18}{4} = \left(\frac{9}{2}\right) = 4.5°$$

17. A point source of 100 W emits light with 5% efficiency. At a distance of 5 m from the source, the intensity produced by the electric field component is:

(1) $\frac{1}{40\pi}\frac{\text{W}}{\text{m}^2}$

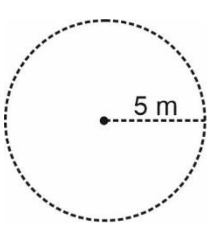
(2) $\frac{1}{2\pi}\frac{\text{W}}{\text{m}^2}$

(3) $\frac{1}{20\pi}\frac{\text{W}}{\text{m}^2}$

(4) $\frac{1}{10\pi}\frac{\text{W}}{\text{m}^2}$

Answer (1)

Sol.



Intensity at 5 m = $\frac{5}{4\pi \times 5^2}\left(\frac{W}{m^2}\right)$

$$= \frac{1}{20\pi}\left(\frac{W}{m^2}\right)$$

Intensity due to electric field $= \frac{1}{40\pi}\left(\frac{W}{m^2}\right)$

$$= \left(\frac{W}{40\pi}\right)$$

18. An electron accelerated through a potential difference V_1 has a de-Broglie wavelength of λ. When the potential is changed to V_2. Its de-Broglie wavelength increases by 50%. The value of $\left(\frac{V_1}{V_2}\right)$ is equal to

(1) $\frac{3}{2}$

(2) $\frac{9}{4}$

(3) 4

(4) 3

Answer (2)

Sol. $P = \sqrt{2\,eVm}$

$\lambda = \left(\frac{h}{P_1}\right)$ …(i)

$\frac{3\lambda}{2} = \frac{h}{P_2}$ …(ii)

Dividing (i) by (ii)

$\Rightarrow \frac{2}{3} = \left(\frac{P_2}{P_1}\right) = \sqrt{\frac{2}{1}}$

$\Rightarrow \frac{4}{9} = \left(\frac{v_2}{v_1}\right)$

$\frac{v_1}{v_2} = \left(\frac{9}{4}\right)$

19. A machine gun of mass 10 kg fires 20 g bullets at the rate of 180 bullets per minute with a speed of 100 m s^{-1} each. The recoil velocity of the gun is

(1) 0.6 m/s

(2) 0.02 m/s

(3) 1.5 m/s

(4) 2.5 m/s

Answer (1)

Sol. Momentum of bullets per unit time

$$= \frac{180 \times \frac{20}{1000} \times 100}{60} \text{ kg m/s}^2$$

= 6 N

$\Rightarrow$ Force on gun = 6 N

We cannot calculate recoil velocity with the given data.

If we consider recoil velocity at t = 1 s, then

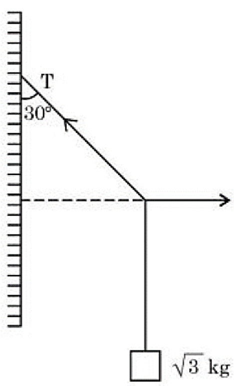
$V_{recoil} = u + at$

$$= 0 + \frac{6}{10} \times 1$$

= 0.6 m/s

20. A block of $\sqrt{3}$ kg is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T is:

(Given g = 10 ms^{-2})



(1) 15 N

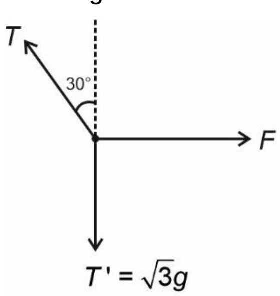
(2) 20 N

(3) 10 N

(4) 25 N

Answer (2)

Sol. Drawing the FBD of the point where F is applied



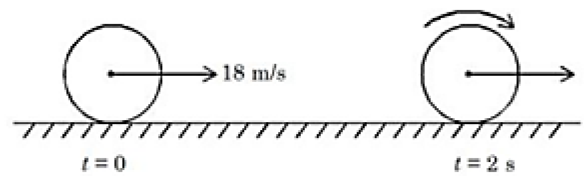
$\Rightarrow T\cos 30° = \sqrt{3}\, g$

$\Rightarrow T = 2g = 20$ N

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A uniform disc of mass 0.5 kg and radius r is projected with velocity 18 m/s at t = 0 s on a rough horizontal surface. It starts off with a purely sliding motion kinetic energy of the disc after 2 s will be_______ J (given, coefficient of friction is 0.3 and g =10 m/s^2).



Answer (54)

Sol. $v = v_0 - \mu g t$

$\Rightarrow$ $v = 18 - 0.3 \times 10 \times 2 = 12$ m/s

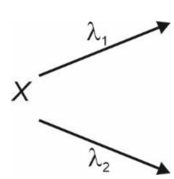
$\Rightarrow$ Kinetic energy = $\frac{1}{2}mv^2 + \frac{1}{2}\frac{mv^2}{2}$

$$= \frac{3}{4}mv^2 = \frac{3}{4} \times 0.5 \times 144 \text{ J} = 54 \text{ J}$$

22. A radioactive nucleus decays by two different process. The half life of the first process is 5 minutes and that of the second process is 30 s. The effective half-life of the nucleus is calculated to be $\frac{a}{11}$ s. The value of a is __________.

Answer (300)

Sol.



$\Rightarrow$ $\lambda_{eff} = \lambda_1 + \lambda_2$

$$\Rightarrow \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(t_{1/2})_1} + \frac{\ln 2}{(t_{1/2})_2}$$

$\Rightarrow \quad t_{1/2} = \dfrac{(t_{1/2})_1 \times (t_{1/2})_2}{(t_{1/2})_1 + (t_{1/2})_2} = \dfrac{300 \times 30}{300 + 30} s = \dfrac{300}{11} s$

$\Rightarrow \quad \alpha = 300$

23. A faulty thermometer reads 5°C in melting ice and 95°C in stream. The correct temperature on absolute scale will be ___________ K when the faulty thermometer reads 41°C.

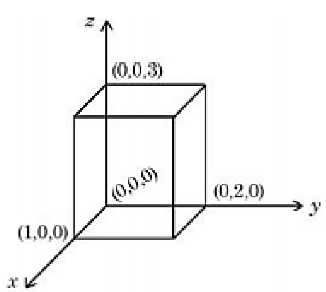
Answer (313)

Sol. let the correct temperature be X°C

$\Rightarrow \quad \dfrac{X-0}{100-0} = \dfrac{41-5}{95-5} \Rightarrow X = 40$

$\Rightarrow$ Temperature is 273 + 40 K = 313 K

24. As shown in figure, a cuboid lies in a region with electric field $E = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}\ \dfrac{N}{C}$. The magnitude of charge within the cuboid is $n\epsilon_0 C$. The value of n is ______ (if dimension of cuboid is 1 × 2 × 3 m^3).



Answer (12)

Sol. Flux through planes parallel to y-z = $2(1)^2$ × Area

$= 2(1)^2 \times 2 \times 3$

$= 12$ Nm^2/C

Flux through planes parallel to x-z = –4(2) × Area

$= -4(2) \times 1 \times 3$

$= -24$ Nm^2/C

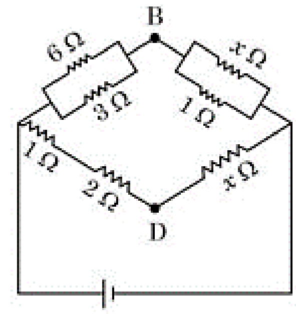
Flux through planes parallel to x-y = 0

$\Rightarrow \quad \phi_{Total} = 12 - 24 = -12$

$\Rightarrow \quad -12 = \dfrac{q_{enc}}{\varepsilon_0} \Rightarrow |q_{enc}| = 12\varepsilon_0$

$\Rightarrow \quad n = 12$

25. If the potential difference between B and D is zero, the value of x is $\dfrac{1}{n}\Omega$. The value of n is _________.



Answer (2)

Sol. The circuit is a wheat stone bridge, so

$$\dfrac{\dfrac{6\times 3}{6+3}}{\dfrac{x\times 1}{x+1}} = \dfrac{1+2}{x}$$

$\Rightarrow \quad \dfrac{2(x+1)}{x} = \dfrac{3}{x}$

$\Rightarrow \quad x = \dfrac{1}{2}$

So $n = 2$

26. The velocity of a particle executing SHM varies with displacement (x) as $4v^2 = 50 - x^2$. The time period of oscillation is $\dfrac{x}{7}$s. The value of x is __________.

$\left(\text{Take } \pi = \dfrac{22}{7}\right)$

Answer (88)

Sol. $4v^2 = 50 - x^2$

or $v = \dfrac{1}{2}\sqrt{50 - x^2}$

Comparing the above equation with $v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow \omega = \dfrac{1}{2}$ & $A = \sqrt{50}$

so $\dfrac{2\pi}{T} = \dfrac{1}{2}$

$\Rightarrow T = 4\pi$ sec

$= 4 \times \dfrac{22}{7}$ sec

$T = \dfrac{88}{7}$ sec

so $x = 88$

27. In an ac generator, a rectangular coil of 100 turns each having area 14×10^{-2} m^2 is rotated at 360 rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be ________ V. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Answer (1584)

Sol. $\phi = B.A$

ϕ = BNA cosωt

So $Emf = \frac{-d\phi}{dt} = NBA\omega \sin\omega t$

So maximum value of emf is

E_{max} = NBAω

$= 100 \times 3 \times 14 \times 10^{-2} \times \frac{360 \times 2\pi}{60}$

= 1584

28. A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4 s is $\frac{1}{3}\alpha^2\sqrt{P}$ m. The value of α will be ________.

Answer (04)

Sol. $P = Fv$

$m\frac{vdv}{dt} = P$

$m\int_0^v vdv = \int_0^t Pdt$

$\frac{mv^2}{2} = Pt$

$v = \sqrt{\frac{2P}{m}}t^{1/2}$

$\int_0^s ds = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$

$s = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2}$

or $s = \frac{2}{3}\sqrt{\frac{2P}{2}} \times 4^{3/2}$

$= \frac{16}{3}\sqrt{P}\, m$

So $\alpha = 4$

29. In a Young's double slit experiment, the intensities at two points, for the path differences $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ [λ being the wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each one of the individual slits, then $\frac{I_1 + I_2}{I_0} =$ _____.

Answer (03)

Sol. $I' = I\cos^2\left(\frac{k\Delta x}{2}\right)$

so $I_1 = 4I_0\cos^2\left(\frac{2\pi}{2\lambda} \times \frac{\lambda}{4}\right)$

$I_1 = 2I_0$

& $I_2 = 4I_0\cos^2\left(\frac{2\pi}{2\lambda} \times \frac{\lambda}{3}\right)$

$I_2 = I_0$

So $\frac{I_1 + I_2}{I_0} = 3$

30. A stone tied to 180 cm long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is $\frac{1936}{x}$ms^{-2}. The value of x ______. $\left(\text{Take } \pi = \frac{22}{7}\right)$

Answer (125)

Sol. Acceleration of stone $a = \frac{v^2}{r} = \omega^2 R$

$a = \left(\frac{28 \times 2}{60} \times \frac{22}{7}\right)^2 \times 1.8$

$= \frac{1936}{125}$

So $x = 125$

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. Match **List I** with **List II**:

	List I (Complexes)		List II (Hybridisation)
A.	$[Ni(CO)_4]$	I.	sp^3
B.	$[Cu(NH_3)_4]^{2+}$	II.	dsp^2
C.	$[Fe(NH_3)_6]^{2+}$	III.	sp^3d^2
D.	$[Fe(H_2O)_6]^{2+}$	IV.	d^2sp^3

(1) A-II, B-I, C-IV, D-III (2) A-I, B-II, C-IV, D-III

(3) A-I, B-II, C-III, D-IV (4) A-II, B-I, C-III, D-IV

Answer (2)

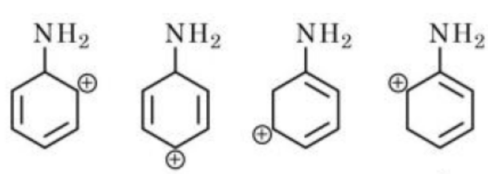
Sol. $Ni(CO)_4 \longrightarrow sp^3$

$\left[Cu(NH_3)_4\right]^{2+} \longrightarrow dsp^2$

$\left[Fe(NH_3)_6\right]^{2+} \longrightarrow d^2sp^3$

$\left[Fe(H_2O)_6\right]^{2+} \longrightarrow sp^3d^2$

32. The most stable carbocation for the following is:

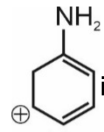


a b c d

(1) a (2) b

(3) d (4) c

Answer (3)

Sol. 
(c)
is the most stable carbocation because its resonance goes up to nitrogen atom.

33. Which of the following reaction is correct?

(1) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 2N_2O_4 + O_2$

(2) $2LiNO_3 \longrightarrow 2Li + 2NO_2 + O_2$

(3) $2LiNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$

(4) $4LiNO_3 \xrightarrow{\Delta} 2Li_2O + 4NO_2 + O_2$

Answer (4)

Sol. $4LiNO_3 \xrightarrow{\text{Heat}} 2Li_2O + 4NO_2 + O_2$

$LiNO_3$ on heating produces Li_2O, NO_2 and O_2

34. The wave function (Ψ) of 2s is given by

$$\Psi_{2s} = \frac{1}{2\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{1/2}\left(2 - \frac{r}{a_0}\right)e^{-r/2a_0}$$

At r = r_0, radial node is formed. Thus, r_0 in terms of a_0

(1) $r_0 = 2a_0$ (2) $r_0 = 4a_0$

(3) $r_0 = a_0$ (4) $r_0 = \frac{a_0}{2}$

Answer (1)

Sol. For radial node $\Psi_{2s} = 0$

$\therefore r = 2a_0$

35. 1 L, 0.02 M solution of $[Co(NH_3)_5SO_4]$ Br is mixed with 1 L, 0.02 M solution of $[Co(NH_3)_5Br]SO_4$. The resulting solution is divided into two equal parts (X) and treated with excess of $AgNO_3$ solution and $BaCl_2$ solution respectively as shown below:

1 L Solution (X) + $AgNO_3$ solution (excess) $\rightarrow$ Y

1 L Solution (X) + $BaCl_2$ solution (excess) $\rightarrow$ Z

The number of moles of Y and Z respectively are

(1) 0.02, 0.02 (2) 0.01, 0.01

(3) 0.02, 0.01 (4) 0.01, 0.02

Answer (2)

Sol. On misxing both $\left[Co(NH_5)_5SO_4\right]Br$

and $\left[Co(NH_3)_5Br\right]SO_4$ becomes 0.01 molar.

$\therefore$ moles of y and z formed will also be 0.01 both.

36. Bond dissociation energy of "E-H" bond of the "H_2E" hydrides of group 16 elements (given below), follows order.

A. O B. S

C. Se D. Te

Choose the correct from the options given below:

(1) A > B > C > D (2) B > A > C > D

(3) A > B > D > C (4) D > C > B > A

Answer (1)

Sol. The correct order of bond strength is

$H_2O > H_2S > H_2Se > H_2Te$

37. Chlorides of which metal are soluble in organic solvents?

(1) Be (2) Mg

(3) K (4) Ca

Answer (1)

Sol. $BeCl_2$ is a covalent molecule

So, it is soluble in organic solvents, rest are ionic compounds.

38. Match List I with List II.

	List I (Mixture)		List II (Separation Technique)
A.	$CHCl_3 + C_6H_5NH_2$	I.	Steam distillation
B.	$C_6H_{14} + C_5H_{12}$	II.	Differential extraction
C.	$C_6H_5NH_2 + H_2O$	III.	Distillation
D.	Organic compound in H_2O	IV.	Fractional distillation

(1) A - III, B - I, C - IV, D - II

(2) A - IV, B - I, C - III, D - II

(3) A - III, B - IV, C - I, D - II

(4) A - II, B - I, C - III, D - IV

Answer (3)

Sol. Mixture → Separation technique

(A) $CHCl_3 + C_6H_5NH_2$ → Distillation

(B) $C_6H_4 + C_5H_{12}$ → Fractional distillation

(C) $C_6H_5NH_2 + H_2O$ → Steam distillation

(D) Organic compound in H_2O → Differential extraction

39. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Antihistamines do not affect the secretion of acid in stomach.

Reason R: Antiallergic and antacid drugs work on different receptors.

In the light of the above statements, Choose the correct answer from the options given below.

(1) A is false but R is true

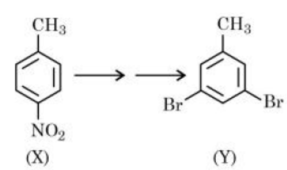
(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is not the correct explanation of A

(4) A is true but R is false

Answer (2)

Sol. Antihistamines not affect the secretion of acid in stomach, the reason is that antiallergic and antacid drugs work on different receptors.

40. 

In the above conversion of compound (X) to product (Y), the sequence of reagents to be used will be

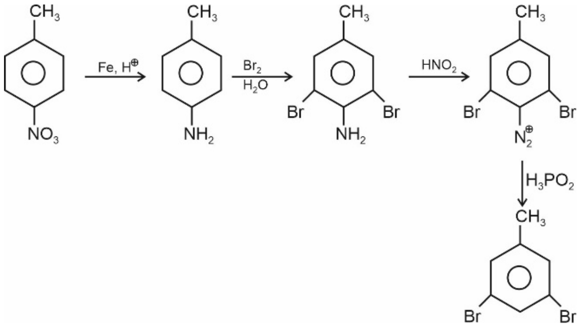
(1) (i) Fe, H^+ (ii) Br_2(aq) (iii) HNO_2 (iv) CuBr

(2) (i) Br_2(aq) (ii) $LiAlH_4$ (iii) H_3O^+

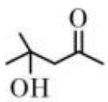
(3) (i) Fe, H^+ (ii) Br_2(aq) (iii) HNO_2 (iv) H_3PO_2

(4) (i) Br_2, Fe (ii) Fe, H^+ (iii) $LiAlH_4$

Answer (3)

Sol. 

41. Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A :  can be easily reduced using Zn-Hg/HCl to .

Reason R : Zn-Hg/HCl is used to reduce carbonyl group to $-CH_2-$ group.

In the light of the above statements, choose the **correct** answer from the options given below :

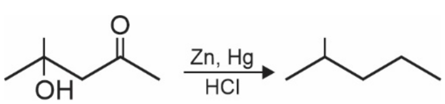
(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) A is false but R is true

(4) Both A and R are true but R is not the correct explanation of A

Answer (3)

Sol. 

Acid sensitive group also react in clemmensen reduction.

42. Boric acid is solid, whereas BF_3 is gas at room temperature because of

(1) Strong hydrogen bond in Boric acid

(2) Strong covalent bond in BF_3

(3) Strong van der Waal's interaction in Boric acid

(4) Strong ionic bond in Boric acid

Answer (1)

Sol. Boric acid is solid because of strong hydrogen bond in it.

43. Maximum number of electrons that can be accommodated in shell with n = 4 are :

(1) 72 (2) 32

(3) 16 (4) 50

Answer (2)

Sol. Maximum electrons accommodated in (n = 4) is $2n^2$ = 32 electrons

44. Given below are two statements :

Statement I : During Electrolytic refining, the pure metal is made to act as anode and its impure metallic form is used as cathode.

Statement II : During the Hall-Heroult electrolysis process, purified Al_2O_3 is mixed with Na_3AlF_6 to lower the melting point of the mixture.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both Statement I and Statement II are incorrect

(2) Both Statement I and Statement II are correct

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Answer (3)

Sol. During electrolytic refining, the pure metal is made to act as cathode and impure metal used as anode.

45. $KMnO_4$ oxidises I^- in acidic and neutral/faintly alkaline solution, respectively, to

(1) I_2 & I_2 (2) I_2 & IO_3^-

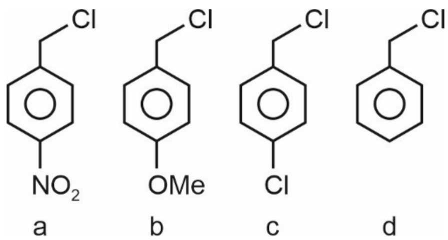
(3) IO_3^- & I_2 (4) IO_3^- & IO_3^-

Answer (2)

Sol. $MnO_4^- + I^- \longrightarrow I_2 + Mn^{2+}$ (acidic medium)

$MnO_4^- + I^- \longrightarrow IO_3^- + MnO_2$ (faintly alkaline)

46. Decreasing order towards S_N1 reaction for the following compounds is

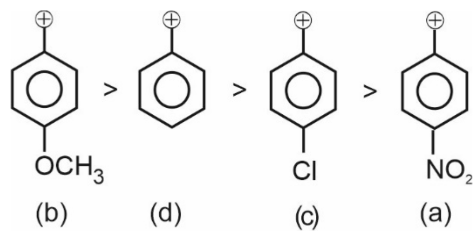


(1) a > c > d > b (2) a > b > c > d

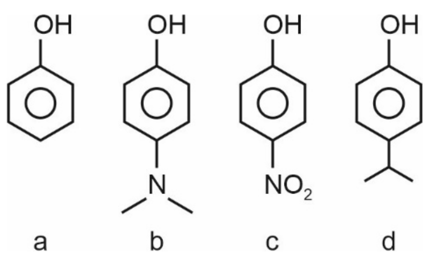
(3) d > b > c > a (4) b > d > c > a

Answer (4)

Sol. Rate of $S_N1 \propto$ stability of carbocation produced



47. The correct order of pK_a values for the following compounds is

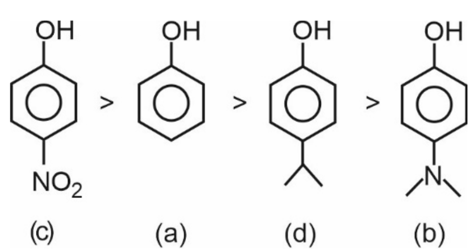


(1) c > a > d > b (2) b > a > d > c

(3) a > b > c > d (4) b > d > a > c

Answer (4)

Sol. The correct acidic order is



$\therefore$ Order of pK_a will be = b > d > a > c

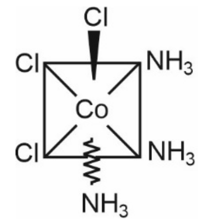
48. The Cl–Co–Cl bond angle values in a fac–$[Co(NH_3)_3Cl_3]$ complex is/are

(1) 180° (2) 90° & 180°

(3) 90° (4) 90° & 120°

Answer (3)

Sol.



fac - $[Co(NH_3)_3Cl_3]$

All the Cl–Co–Cl bond angles are of 90°.

49. The water quality of a pond was analysed and its BOD was found to be 4. The pond has

(1) Slightly polluted water

(2) Very clean water

(3) Highly polluted water

(4) Water has high amount of fluoride compounds

Answer (2)

Sol. A clean water would have BOD value less than 5 ppm.

50. Formulae for Nessler's reagent is

(1) K_2HgI_4 (2) HgI_2

(3) KHg_2I_2 (4) $KHgI_3$

Answer (1)

Sol. Nessler's reagent is $K_2H_gI_4$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

51. An organic compound undergoes first order decomposition. If the time taken for the 60% decomposition is 540 s, then the time required for 90% decomposition will be is ________s. (Nearest integer).

Given : ln 10 = 2.3; log 2 = 0.3

Answer (1350)

Sol. In 60% decomposition $A_0 = 1$, $A_t = 0.4$

$$k = \frac{1}{t}\ln\left(\frac{A_0}{A_t}\right) = \frac{1}{540}\ln\left(\frac{10}{4}\right)$$

$\therefore$ time t0 complete 90% reaction

$$t = \frac{1}{k}\ln(10)$$

$$= \frac{\ln(10)}{\ln\left(\frac{10}{4}\right)} \times 540$$

$$= \frac{2.3}{2.3 \times 0.4} \times 540$$

= 1350 s

52. A short peptide on complete hydrolysis produces 3 moles of glycine (G), two moles of leucine (L) and two moles of valine (V) per mole of peptide. The number of peptide linkages in it are ________.

Answer (06.00)

Sol. The peptide has seven amino acid units therefore it has six peptide bonds.

53. 1 mole of ideal gas is allowed to expand reversibly and adiabatically from a temperature of 27°C. The work done is 3 kJ mol^{-1}. The final temperature of the gas is ________K (Nearest integer). Given C_V = 20 J mol^{-1} K^{-1}

Answer (150.00)

Sol. T_1 = 300 K   w = 3 kJ/mole

$w = nC_v\Delta T$

$3000 = 1 \times 20 \times (300 - T_2)$

$300 - T_2 = 150$

T_2 = 150 K

54. The electrode potential of the following half cell at 298 K

$X \mid X^{2+}$ (0.001 M) $\|$ Y^{2+} (0.01 M) $\mid Y$ is _______ × 10^{-2} V (Nearest integer).

Given : $E^\circ_{X^{2+}|X} = -2.36$ V

$E^\circ_{Y^{2+}|Y} = +0.36$ V

$$\frac{2.303RT}{F} = 0.06 \text{ V}$$

Answer (275.00)

Sol. $E_{cell} = E^\circ_{cell} - \frac{0.06}{n}\log\frac{x^{2+}}{y^{2+}}$

$$E_{cell} = (2.36 + 0.36) - \frac{0.06}{2}\log\left(\frac{1}{10}\right)$$

= 2.36 + 0.36 + 0.03

$\simeq$ 2.75 V

55. The graph of $\log\frac{x}{m}$ vs log p for an adsorption process is a straight line inclined at an angle of 45° with intercept equal to 0.6020. The mass of gas adsorbed per unit mass of adsorbent at the pressure of 0.4 atm is ________ × 10^{-1} (Nearest integer).

Given : log 2 = 0.3010

Answer (16.00)

Sol. $\frac{1}{n} = \tan(45°) \quad \therefore n = 1$

$\log k = 0.602 \quad \therefore k = 4$

$$\frac{x}{m} = k.p^{1/n}$$

= 4 × 0.4

= 1.6 g

56. Iron oxide FeO, crystallises in a cubic lattice with a unit cell edge length of 5.0 Å. If density of the FeO in the crystal is 4.0 g cm^{-3}, then the number of FeO units present per unit cell is ______________. (Nearest integer)

Given : Molar mass of Fe and O is 56 and 16 g mol^{-1} respectively.

$N_A = 6.0 \times 10^{23}$ mol^{-1}

Answer (04.00)

Sol. $d = \frac{z \times m}{a^3}$

$$4 = \frac{z \times 72}{6 \times 10^{23}\left(5 \times 10^{-8}\right)^3}$$

$$4 = \frac{z \times 72}{6 \times 125 \times 10^{-1}}$$

$= z \approx 4$

57. Lead storage battery contains 38% by weight solution of H_2SO_4. The van't Hoff factor is 2.67 at this concentration. The temperature in Kelvin at which the solution in the battery will freeze is _____________ (Nearest integer).

Given K_f = 1.8 K kg mol^{-1}

Answer (243.00)

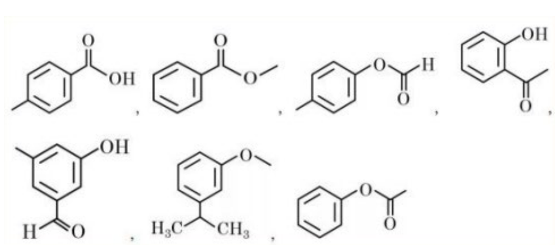
Sol. $\Delta T_f = K_f\, i\, m$

$$= 1.8 \times 2.67 \times \frac{\frac{38}{98}}{0.062}$$

$= 30$

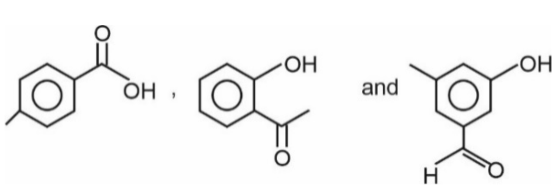
$\therefore$ It freeze at 273 – 30 = 243 K

58. Number of compounds from the following which will not dissolve in cold $NaHCO_3$ and NaOH solutions but will dissolve in hot NaOH solution is _________



Answer (03.00)

Sol.



will dissolve in hot NaOH solution

59. The strength of 50 volume solution of hydrogen peroxide is ___________ g/L (Nearest integer).

Given :

Molar mass of H_2O_2 is 34 g mol^{-1}

Molar volume of gas at STP = 22.7 L.

Answer (150.00)

Sol. $H_2O_2 \longrightarrow H_2O + \frac{1}{2}O_2$

$$\frac{50}{22.7}$$

$\therefore$ Moles of H_2O_2 in solution = $\frac{50}{22.7} \times 2$

$\therefore$ Strength = $\dfrac{\frac{50 \times 2}{22.7} \times 34}{1}$

$= 149.78$

≈ 150

60. Consider the following equation:

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g), \quad \Delta H = -190 \text{ kJ}$$

The number of factors which will increase the yield of SO_3 at equilibrium from the following is ___________

A. Increasing temperature

B. Increasing pressure

C. Adding more SO_2

D. Adding more O_2

E. Addition of catalyst

Answer (03.00)

Sol. $2SO_2 + O_2 \rightleftharpoons 2SO_3 \quad \Delta H = -190$ kJ

It is an exothermic reaction

$\therefore$ factor B, C, D will increase the amount of SO_3.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(1) 76 (2) 72

(3) 64 (4) 81

Answer (2)

Sol. Let a tangent on $y^2 = 16x$ be $y = mx + \frac{4}{m}$

For common to $x^2 + y^2 = 8$

$\frac{4}{m} = 2\sqrt{2}\left(1+m^2\right)$

$\Rightarrow \frac{2}{m^2} = 1+m^2 \Rightarrow m = \pm 1$

Taking one of the tangent $y = x + 4$

Point of tangency with $y^2 = 4x$

$x^2 + 8 + 16 = 4x \Rightarrow x = 4$ & $y = 8$

$\therefore Q(4, 8)$

and for $x^2 + y^2 = 8$

$2x^2 + 8x + 8 = 0$

$x^2 + 4x + 4 = 8 \Rightarrow x = -2, y = 2 \Rightarrow R = (-2, 2)$

$(QR)^2 = 6^2 + 6^2$

$= 72$

62. Let f, g and h be the real valued functions defined of $\mathbb{R}$ as

$$f(x)\begin{cases}\frac{x}{|x|}, & x \neq 0\\ 1, & x = 0\end{cases}, \quad g(x)\begin{cases}\frac{\sin(x+1)}{(x+1)}, & x \neq -1\\ 1, & x = -1\end{cases}$$

and $h(x) = 2[x] - f(x)$, where $[x]$ is the greatest integer $\leq$ x.

Then the value of $\lim\limits_{x\to 1} g(h(x-1))$ is

(1) 1 (2) –1

(3) sin(1) (4) 0

Answer (1)

Sol. $f(x) = \text{sgn}(x)$

$h(x) = 2[x] - \text{sgn}(x)$

If $x \to 1^+$ then $h(x - 1) = 2[x] - 2 - \text{sgn}(x - 1)$

$= 0 - 1 = -1$

& if $x \to 1^-$ then $h(x - 1) = 2[x] - 2 - \text{sgn}(x - 1)$

$= -2 + 1 = -1$

$$\therefore \lim_{x\to 1^+} g(h(x-1)) = \lim_{x\to 1^+} \frac{\sin(h(x+1)+1)}{h(x-1)+1} = 1$$

$$\lim_{x\to 1^-} g(h(x-1)) = \lim_{x\to 1^-} \frac{\sin(h(x-1))+1)}{h(x-1)+1} = 1$$

63. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots\ldots, a_{100}$ is 25. Then S is

(1) {9} (2) {99}

(3) ϕ (4) $\mathbb{N}$

Answer (4)

Sol. Let $a_1 = a \Rightarrow a_2 = a + 1, \ldots.. a_{100} = a + 90$

$$\mu = \frac{\frac{100a + (99\times 100)}{2}}{100} = a + \frac{99}{2}$$

$$\text{M.D} = \frac{\sum |a_1 - \mu|}{100} = \frac{\left(\frac{99}{2} + \frac{97}{2} + \ldots.. \frac{1}{2}\right)^2}{100} = \frac{2500}{100} = 25$$

$\therefore a \to z$

64. If a plane passes through the points $(-1, k, 0)$, $(2, k, -1)$, $(1, 1, 2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is

(1) $\frac{13}{6}$ (2) $\frac{17}{5}$

(3) $\frac{5}{17}$ (4) $\frac{6}{13}$

Answer (1)

Sol. Let $P \equiv (-1, k, 0)$, $Q \equiv (2, k, -1)$ & $R(1, 1, 2)$

$\vec{PR} = 2\hat{i} + (1-k)\hat{j} + 2\hat{k}$

& $\vec{QR} = -\hat{i} + (1-k)\hat{j} + 3\hat{k}$

$\therefore$ Normal to plane will be

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & (1-k) & 2 \\ -1 & (1-k) & 3 \end{vmatrix} = \hat{i}(1-k) - \hat{j}(8) + 3\hat{k}(1-k)$$

If line is parallel to this we have

$1(1 - k) + 1(-8) + (-3)(1 - k) = 0$

$\Rightarrow 2(1 - k) = -8$

$\Rightarrow 1 - k = -4 \Rightarrow k = 5$

$\therefore \dfrac{k^2+1}{(k-1)(k-2)} = \dfrac{26}{4.3} = \dfrac{13}{6}$

65. $\lim\limits_{n\to\infty} \dfrac{3}{n}\left\{4 + \left(2+\dfrac{1}{n}\right)^2 + \left(2+\dfrac{2}{n}\right)^2 + ... + \left(3-\dfrac{1}{n}\right)^2\right\}$ is equal to

(1) $\dfrac{19}{3}$

(2) 12

(3) 19

(4) 0

Answer (3)

Sol. $\lim\limits_{n\to\infty} \sum\limits_{r=0}^{n-1} \dfrac{3}{n}\left(2+\dfrac{r}{n}\right)^2$

$= \int_0^1 3(2+x)^2\,dx$

$= 3 \cdot \dfrac{(2+x)^3}{3}\Bigg|_0^1$

$= 3^3 - 2^3 = 19$

66. Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $\vec{a}.\vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of $\vec{b}.\vec{c}$ is

(1) –84

(2) –48

(3) –24

(4) –60

Answer (2)

Sol. $\vec{b} \cdot \vec{c} = \vec{b} \cdot (2\vec{a} \times \vec{b}) - 3\vec{b} \cdot \vec{b}$

$= 0 - 3|\vec{b}|^2$

$= -48$

67. Let $\lambda \in R$, $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$

If $\left((\vec{a}+\vec{b}) \times (\vec{a} \times \vec{b})\right) \times (\vec{a} - \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

then $\left|\lambda(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})\right|^2$ is equal to

(1) 132

(2) 136

(3) 140

(4) 144

Answer (3)

Sol. $\left(\vec{a} \times (\vec{a} \times \vec{b}) + \vec{b} \times (\vec{a} \times \vec{b})\right) \times (\vec{a} - \vec{b})$

$= \left(\vec{a}(\vec{a}\cdot\vec{b}) - \vec{b}(\vec{a}\cdot\vec{a}) + \vec{a}(\vec{b}\cdot\vec{b}) - \vec{b}(\vec{a}\cdot\vec{b})\right) \times (\vec{a} - \vec{b})$

$= (\vec{a}\cdot\vec{b})(\vec{a}\times\vec{a} - \vec{a}\times\vec{b}) - (\vec{a}\cdot\vec{a})(\vec{b}\times\vec{a} - \vec{b}\times\vec{b})$

$+ (\vec{b}\cdot\vec{b})(\vec{a}\times\vec{a} - \vec{a}\times\vec{b}) - (\vec{a}\cdot\vec{b})(\vec{b}\times\vec{a} - \vec{b}\times\vec{b})$

$= (\vec{a}\cdot\vec{b})(\vec{b}\times\vec{a}) - (\vec{a}\cdot\vec{a})(\vec{b}\times\vec{a})$

$+ (\vec{b}\cdot\vec{b})(\vec{b}\times\vec{a}) - (\vec{a}\cdot\vec{b})(\vec{b}\times\vec{a})$

$= (\vec{b}\times\vec{a})(\vec{b}\cdot\vec{b} - \vec{a}\cdot\vec{a})$

$= (5\vec{b}\times\vec{a})(5 + \lambda^2 - 13 - \lambda^2)$

$= 8(\vec{a}\times\vec{b})$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix}$$

$= \hat{i}(4 - 3\lambda) - \hat{j}(2\lambda + 3) + \hat{k}(-\lambda^2 - 2)$

$\Rightarrow \boxed{\lambda = 1}$

$\left|\vec{a}\times(\vec{a}-\vec{b}) + \vec{b}\times(\vec{a}-\vec{b})\right|^2$

$= \left|2(\vec{a}\times\vec{b})\right|^2 = 4.35 = 140$

68. If the functions $f(x)=\frac{x^3}{3}+2bx+\frac{ax^2}{2}$ and $g(x)=\frac{x^3}{3}+ax+bx^2, a\neq 2b$ have a common extreme point, then $a+2b+7$ is equal to

(1) 3
(2) 4
(3) 6
(4) $\frac{3}{2}$

Answer (3)

Sol. $f'(x)=x^2+2b+ax$

$g'(x)=x^2+a+2bx$

$\Rightarrow$ x = 1 is common root

$a + 2b + 1 = 0$

69. The solution of the differential equation $\frac{dy}{dx}=-\left(\frac{x^2+3y^2}{3x^2+y^2}\right), y(1)=0$

(1) $\log_e|x+y|-\frac{xy}{(x+y)^2}=0$

(2) $\log_e|x+y|+\frac{2xy}{(x+y)^2}=0$

(3) $\log_e|x+y|+\frac{xy}{(x+y)^2}=0$

(4) $\log_e|x+y|-\frac{2xy}{(x+y)^2}=0$

Answer (2)

Sol. $y = vx$

$$v+x\frac{dv}{dx}=-\left(\frac{1+3v^2}{3+v^2}\right)$$

$$x\frac{dv}{dx}=-\left(\frac{1+3v^2}{3+v^2}+v\right)$$

$$\frac{dv}{dx}=-\left(\frac{(1+v)^3}{3+v^2}\right)$$

$$\Rightarrow \frac{3+v^2}{(1+v)^3}=\frac{-dx}{x}$$

$$\Rightarrow \ln|v+1|+\frac{2v}{(v+1)^2}=C-\ln|x|$$

$$x=1,\ v=0\ \Rightarrow C=0$$

$$\Rightarrow \ln|x+y|-\ln|x|+\frac{2xy}{(x+y)^2}=-\ln|x|$$

70. The parabolas : $ax^2+2bx+cy=0$ and $dx^2+2ex+fy=0$ intersect on the line $y=1$.

If *a, b, c, d, e, f* are positive real numbers and *a, b, c* are in G.P., then

(1) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P.
(2) *d, e, f* are in G.P.
(3) *d, e, f* are in A.P.
(4) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

Answer (1)

Sol. Let point of intersection be (α, 1)

$\alpha x^2+2b\alpha+c=0$ …(i)

and $d\alpha^2+2e\alpha+f=0$ …(ii)

$\Rightarrow a\alpha^2+2\sqrt{ac}\alpha+c=0$ $(\because b^2 = ac)$

$$\left(\sqrt{a}\alpha+\sqrt{c}\right)^2=0$$

$$\alpha=-\sqrt{\frac{c}{a}}$$

Put the value α in (ii),

$$d\frac{c}{a}-2e\sqrt{\frac{c}{a}}+f=0$$

$$\frac{d}{a}-\frac{2e}{\sqrt{ac}}+\frac{f}{c}=0$$

$$\frac{d}{a}+\frac{f}{c}=2\frac{e}{b}$$

$\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

71. Consider the following statements:

P : I have fever

Q : I will not take medicine

R : I will take rest

The statement "If I have fever, then I will take medicine and I will take rest"

(1) $(P\vee\sim Q)\wedge(P\vee\sim R)$

(2) $((\sim P)\vee\sim Q)\wedge((\sim P)\vee\sim R)$

(3) $((\sim P)\vee\sim Q)\wedge((\sim P)\vee R)$

(4) $(P\vee Q)\wedge((\sim P)\vee R)$

Answer (3)

Sol. The given expression is

$P\rightarrow\sim Q\wedge R$

$\equiv(\sim P)\vee(\sim Q\wedge R)$

$\equiv(\sim P\vee\sim Q)\wedge(\sim P\vee R)$

72. Let $a_1 = 1, a_2, a_3, a_4, \ldots$ be consecutive natural numbers. Then

$$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{1}{1+a_2a_3}\right)+\ldots + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$$ is equal to

(1) $\tan^{-1}(2022)-\frac{\pi}{4}$ (2) $\cot^{-1}(2022)-\frac{\pi}{4}$

(3) $\frac{\pi}{4}-\cot^{-1}(2022)$ (4) $\frac{\pi}{4}-\tan^{-1}(2022)$

Answer (1)(3)*

Sol. $\tan^{-1}(a_2)-\tan^{-1}(a_1)+\tan^{-1}(a_3)-\tan^{-1}(a_2)+\ldots+\tan^{-1}a_{2022}-\tan^{-1}a_{2021}$

$= \tan^{-1}(a_{2022})-\tan^{-1}(a_1)$

$= \tan^{-1}(2022)-\frac{\pi}{4}$

73. Let $x=\left(8\sqrt{3}+13\right)^{13}$ and $y=\left(7\sqrt{2}+9\right)^{9}$. If $[t]$ denotes the greatest integer $\le t$, then

(1) $[x]$ is odd but $[y]$ is even

(2) $[x] + [y]$ is even

(3) $[x]$ and $[y]$ are both odd

(4) $[x]$ is even but $[y]$ is odd

Answer (2)

Sol. If $I_1+f=(8\sqrt{3}+13)^{13}$, $f'=\left(8\sqrt{3}-13\right)^{13}$

$I_1+f-f' = \text{Even}$

I_1 = Even

$I_2+f-f'=\left(7\sqrt{2}+9\right)^{9}+\left(7\sqrt{2}-9\right)^{9}$

= Even

I_2 = Even

74. For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

Has infinitely many solutions. Then α and β are the roots of

(1) $x^2 + 14x + 24 = 0$ (2) $x^2 - 18x + 56 = 0$

(3) $x^2 - 10x + 16 = 0$ (4) $x^2 + 18x + 56 = 0$

Answer (2)

Sol. $\Delta = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$

$\Rightarrow \alpha = 4$

$\Delta_3 = 0$

$= \begin{vmatrix} 1 & -1 & 5 \\ 2 & 2 & 8 \\ 3 & -1 & \beta \end{vmatrix} = 0$

$\Rightarrow \beta = 14$

$\therefore x^2 - 18x + 56 = 0$

75. The number of ways of selecting two numbers a and b, $a \in \{2, 4, 6, \ldots, 100\}$ and $b \in (1, 3, 5, \ldots, 99)$ such that 2 is the remainder when $a + b$ is divided by 23 is

(1) 108

(2) 186

(3) 54

(4) 268

Answer (1)

Sol. $a + b = 23\lambda + 2$

$\lambda = 0, 1, 2, \ldots$, but λ cannot be even as $a + b$ is odd

$\therefore \lambda = 1$ $(a, b) \to$ 12 pairs

$\lambda = 3$ $(a, b) \to$ 35 pairs

$\lambda = 5$ $(a, b) \to$ 42 pairs

$\lambda = 7$ $(a, b) \to$ 19 pairs

$\lambda = 9$ $(a, b) \to$ 0 pairs

$\vdots$

Total = 12 + 35 + 42 + 19 = 108

76. If P is a 3×3 real matrix such that $P^T = aP + (a - 1)I$, where a > 1, then

(1) $|\text{Adj}P| = \frac{1}{2}$

(2) $|\text{Adj}P| > 1$

(3) P is a singular matrix

(4) $|\text{Adj}P| = 1$

Answer (4)

Sol. Let $P = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

Given : $P^T = aP + (a-1)I$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} aa_1 + a - 1 & ab_1 & ac_1 \\ aa_2 & ab_2 + a - 1 & ac_2 \\ aa_3 & ab_3 & ac_3 + a - 1 \end{bmatrix}$$

$\Rightarrow \ a_1 = aa_1 + a - 1 \Rightarrow a_1(1-a) = a - 1 \Rightarrow a_1 = -1$

Similarly, $a_1 = b_2 = c_3 = -1$

Now, $\left.\begin{matrix} a_2 = ab_1 \\ b_1 = aa_2 \end{matrix}\right] \rightarrow a_2 = a^2 a_2 \Rightarrow a_2 = 0 \Rightarrow b_1 = 0$

$c_1 = aa_3$

Similarly all other elements will also be 0

$a_2 = a_3 = b_1 = b_3 = c_1 = c_2 = 0$

$\therefore \quad P = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$|P| = -1$

$|\text{Adj}(P)|_{n\times n} = |A|^{(n-1)}$

$\Rightarrow \quad |\text{Adj}(P)| = (-1)^2 = 1$

77. A vector $\vec{v}$ in the first octant is inclined to the x- axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2}, -1, 1)$ and (a, b, c) is normal to $\vec{v}$, then

(1) $\sqrt{2a} + b + c = 1$ (2) $a + \sqrt{2b} + c = 1$

(3) $\sqrt{2a} - b + c = 1$ (4) $a + b + \sqrt{2c} = 1$

Answer (2)

Sol. $l = \frac{1}{2}, m = \frac{1}{\sqrt{2}}, n = \cos\theta$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \ \frac{1}{4} + \frac{1}{2} + n^2 = 1 \Rightarrow n^2 = \frac{1}{4} \Rightarrow n = \pm\frac{1}{2}$

θ is acute $\therefore \ n = \frac{1}{2}$

$\therefore \quad \vec{v} = k\left(\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}\right), k \in R$

$\vec{v} \cdot (\vec{a} - \vec{b}) = 0$

$(\sqrt{2} - a)\frac{1}{2} + (-1-b)\frac{1}{\sqrt{2}} + (1-c)\frac{1}{2} = 0$

$\Rightarrow \ \frac{a}{2} + \frac{b}{\sqrt{2}} + \frac{c}{2} = \frac{1}{2}$

$\Rightarrow \ a + \sqrt{2}b + c = 1$

$\therefore$ Option (2) is correct.

78. Let $a, b, c > 1$ a^3, b^3 and c^3 be in A.P., and $\log_a b$, $\log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is –444, then abc is equal to:

(1) $\frac{343}{8}$

(2) $\frac{125}{8}$

(3) 343

(4) 216

Answer (4)

Sol. $2b^3 = a^3 + c^3$

$\left(\frac{\log a}{\log c}\right)^2 = \left(\frac{\log b}{\log a}\right)\left(\frac{\log c}{\log b}\right)$

$\Rightarrow \ (\log a)^3 = (\log c)^3$

$\Rightarrow \ \log a = \log c$

$\Rightarrow \ a = c$

$\Rightarrow \ a = b = c$

$T_1 = 2a, \ d = -\frac{3a}{5}$

$S_{20} = -444$

$\Rightarrow \ \frac{20}{2}\left(2(2a) + (19)\left(-\frac{3a}{5}\right)\right) = -444$

$\Rightarrow \ 10\frac{(20a - 57a)}{5} = -444$

$\Rightarrow \ 37a = 222$

$\Rightarrow \ a = 6$

$\Rightarrow \ abc = (6)^3 = 216$

79. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is:

(1) $\left[\sqrt{5}, \sqrt{10}\right]$ (2) $\left[\sqrt{2}, \sqrt{7}\right]$

(3) $\left[\sqrt{5}, \sqrt{13}\right]$ (4) $\left[2\sqrt{2}, \sqrt{11}\right]$

Answer (1)

Sol. $f(x) = \sqrt{3-x} + \sqrt{x+2}$

$$y' = \frac{-1}{2\sqrt{3-x}} + \frac{1}{2\sqrt{x+2}} = 0$$

$\Rightarrow \sqrt{x} + 2 = \sqrt{3} - x$

$\Rightarrow x = \frac{1}{2}$

$$y\left(\frac{1}{2}\right) = \sqrt{\frac{5}{2}} + \sqrt{\frac{5}{2}} = \sqrt{10}$$

y_{min} at $x = -2$ or $x = 3$, i.e., $\sqrt{5}$

$\therefore f(x) \in \left[\sqrt{5}, \sqrt{10}\right]$

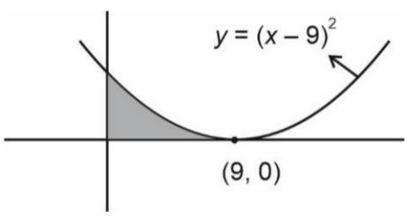
80. Let q be the maximum integral value of p in [0, 10] for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $\left\{(x, y) : 0 \le y \le (x-q)^2, 0 \le x \le q\right\}$ is

(1) $\frac{125}{3}$ (2) 164

(3) 243 (4) 25

Answer (3)

Sol. Given equation : $4x^2 - 4px + 5p = 0$



for rational roots, D must be perfect square

$D = 16p^2 - 4 \times 4 \times 5p = 16p(p-5)$

So, max. Integral value of $p = 9$ for making D is perfect square

$\therefore q = 9$

Area of shared region

$$= \int_0^9 (x-9)^2\, dx$$

$$= \left.\frac{(x-9)^3}{3}\right|_0^9 = \frac{9^3}{3} = 243 \text{ sq. units}$$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. If $\int \sqrt{\sec 2x - 1}\, dx = \alpha \log_e \left|\cos 2x + \beta + \sqrt{\cos 2x\left(1 + \cos\frac{1}{\beta}x\right)}\right| +$ constant, then $\beta - \alpha$ is equal to ________.

Answer (01)

Sol. $I = \int \sqrt{\sec 2x - 1}\, dx \int \sqrt{\frac{1-\cos 2x}{\cos 2x}}\, dx$

$$= \int \sqrt{\frac{2\sin^2 x}{2\cos^2 x - 1}}\, dx$$

Let $\sqrt{2}\cos x = t$

$-\sqrt{2}\sin x\, dx = dt$

$$I = \int -\frac{dt}{\sqrt{t^2 - 1}} = -\ln\left|t + \sqrt{t^2 - a^2}\right| + c$$

$$= -\ln\left|\sqrt{2}\cos x + \sqrt{2\cos^2 x - 1}\right| + c$$

$$= -\frac{1}{2}\ln\left|2\cos^2 x + \cos 2x + 2\sqrt{2}\sqrt{\cos 2x.\cos^2 x}\right| + c$$

$$= -\frac{1}{2}\ln\left|2\cos 2x + 1 + 2\sqrt{\cos 2x(1 + \cos 2x)}\right| + c$$

$$= -\frac{1}{2}\ln\left|\cos 2x + \frac{1}{2} + \sqrt{\cos 2x(1 + \cos 2x)}\right| + c$$

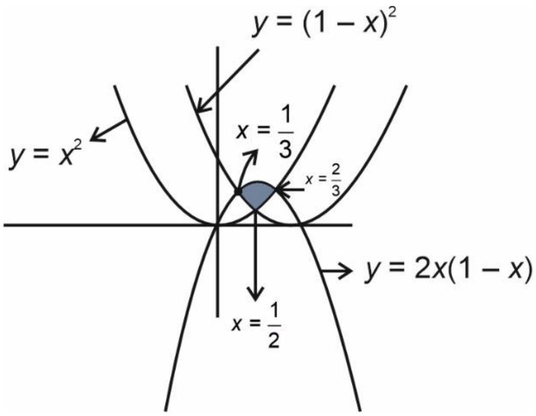
$\therefore \alpha = \frac{-1}{2}, \beta = \frac{1}{2}$

$\therefore \beta - \alpha = 1$

82. Let A be the area of the region $\left\{(x, y) : y \ge x^2, y \ge (1-x)^2, y \le 2x(1-x)\right\}$. Then 540 A is equal to ________.

Answer (25)

Sol.



$y = x^2$ and $y = 2x(1 - x)$

$\Rightarrow$ $x^2 = 2x - 2x^2$

$\Rightarrow$ $x = 0, x = \frac{2}{3}$

Now,

$y = (1 - x)^2$ and $y = 2x(1 - x)$

$\Rightarrow$ $1 + x^2 - 2x = 2x - 2x^2$

$\Rightarrow$ $3x^2 - 4x + 1 = 0$

$x = 1, x = \frac{1}{3}$

$$\therefore \quad A = \int_{\frac{1}{3}}^{\frac{2}{3}} (2x - 2x^2)\,dx - \left\{ \int_{\frac{1}{3}}^{\frac{1}{2}} (1-x)^2 dx + \int_{\frac{1}{2}}^{\frac{2}{3}} x^2 dx \right\}$$

$$= \left(x^2 - \frac{2x^3}{3} \right)_{\frac{1}{3}}^{\frac{2}{3}} - \left\{ \frac{(x-1)^3}{3} \Bigg|_{\frac{1}{3}}^{\frac{1}{2}} + \frac{x^3}{3} \Bigg|_{\frac{1}{2}}^{\frac{2}{3}} \right\}$$

$$= \frac{5}{108}$$

$\therefore$ 540A = 25

83. 50th root of a number x is 12 and 50th root of another number y is 18. Then the remainder obtained on dividing $(x + y)$ by 25 is _______.

Answer (23)

Sol. Given $x^{\frac{1}{50}} = 12 \Rightarrow x = 12^{50}$

$y^{\frac{1}{50}} = 18 \Rightarrow y = 18^{50}$

$12 \equiv 13$ (Mod 25)

$12^2 \equiv 19$ (Mod 25)

$12^3 \equiv -3$ (Mod 25)

$12^9 \equiv -2$ (Mod 25)

$12^{10} \equiv -1$ (Mod 25)

$12^{50} \equiv -1$ (Mod 25) …(i)

Now

$18 \equiv 7$ (Mod 25)

$18^2 \equiv -1$ (Mod 25)

$18^{50} \equiv -1$ (Mod 25) …(ii)

$\therefore$ $12^{50} + 18^{50} \equiv -2$ (Mod 25)

$\equiv 23$ (Mod 25)

$\therefore$ Answer = 23

84. The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is _______.

Answer (240)

Sol. $\ldots\ldots 1 \rightarrow \frac{6!}{2!3!} = 60$

$\ldots\ldots 3 \rightarrow \frac{6!}{3!} = 120$

$\ldots\ldots 5 \rightarrow \frac{6!}{3!2!} = 60$

Total = 240

85. Let $A = \{1, 2, 3, 5, 8, 9\}$. Then the number of possible functions $f : A \rightarrow A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to _______.

Answer (432)

Sol. $f(1.n) = f(1).f(n) \Rightarrow f(1) = 1.$

$f(3.3) = (f(3))^2$

Hence the possibilities for $(t(3), t(9))$ are (1, 1) and (3, 9).

Other three *i.e.* $f(2), f(5), f(8)$

Can be chosen in 6^3 ways.

Hence total number of functions

$= 6^3 \times 2$

= 432

86. Let a line L pass through the point $P(2, 3, 1)$ and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$. If the distance of L from the point (5, 3, 8) is α, then $3\alpha^2$ is equal to _________.

Answer (158)

Sol. $L : \frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1} = \lambda$

Any point on L can be taken as

$B(\lambda + 2, -\lambda + 3, -\lambda + 1)$

Let $A(5, 3, 8)$

So, $AB \cdot \left(\hat{i} - \hat{j} - \hat{k}\right) = 0$

$[(\lambda - 3)\,\hat{i} - \lambda\hat{j} - (\lambda + 7)\hat{k}] \cdot [\hat{i} - \hat{j} - \hat{k}] = 0$

$\lambda - 3 + \lambda + \lambda + 7 = 0$

$\therefore \quad \lambda = \frac{-4}{3}$

$\overrightarrow{AB} = \frac{13}{3}\hat{i} + \frac{4}{3}\hat{i} - \frac{17}{3}\hat{k}$

$|\overrightarrow{AB}| = \sqrt{\frac{169}{9} + \frac{16}{9} + \frac{289}{9}}$

$= \frac{\sqrt{474}}{3} = \alpha$

$3\alpha^2 = \frac{474}{9} \times 3 = 158$

87. If the value of real number $\alpha > 0$ for which $x^2 - 5\alpha x + 1 = 0$ and $x^2 - \alpha x - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to ________.

Answer (13)

Sol. $x^2 - 5\alpha x + 1 = 0$ …(1)

$x^2 - \alpha x - 5 = 0$ …(2)

have a common root.

Subtracting (1) with (2) we'll get $x = \frac{6}{4\alpha}$.

Substituting in (1)

$\frac{36}{16\alpha^2} - \frac{30}{4} + 1 = 0$

$\Rightarrow \quad \alpha^2 = \frac{9}{26}$

$\alpha = \frac{3}{\sqrt{2 \times 13}}$

$\therefore \quad \beta = 13$

88. The 8th common term of the series

$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots,$

$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots,$

is ________.

Answer (151)

Sol. First common term is 11

Common difference of series of common terms is LCM (4, 5) = 20

$a_8 = a + 7d$

$= 11 + 7 \times 20 = 151$

89. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to ________.

Answer (14)

Sol. $p = \frac{6}{36} = \frac{1}{6}$

$q = \frac{{}^6C_1 \times {}^5C_1 \times \frac{4!}{3!}}{6^4} = \frac{120}{1296} = \frac{5}{54}$

$\frac{p}{q} = \frac{\frac{1}{6}}{\frac{5}{54}} = \frac{54}{6 \times 5} = \frac{9}{5} = \frac{m}{n}$

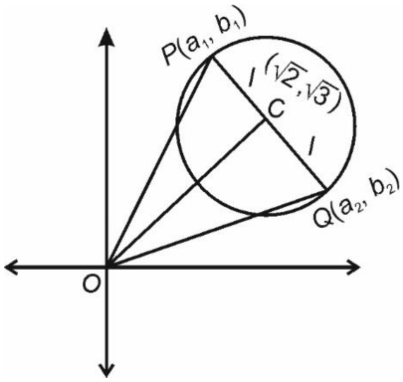
$m + n = 14$

90. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C\left(\sqrt{2}, \sqrt{3}\right)$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ________.

Answer (24)

Sol. $OC \perp CP$ and $OC \perp CQ$

$\Rightarrow$ *PCQ* is a straight line



$OC = \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{3}\right)^2} = \sqrt{5}$

Let $CP = CQ = l$

$[OCP] = \frac{1}{2} \times OC \times l = \frac{\sqrt{35}}{2}$

$l = \sqrt{7}$

$OP = OQ = \sqrt{(OC)^2 + l^2} = \sqrt{5+7} = \sqrt{12}$

$a_1^2 + a_2^2 + b_1^2 + b_2^2 = \left(a_1^2 + b_2^2\right) + \left(a_2^2 + b_2^2\right)$

$OP^2 + OQ^2 = 12 + 12 = 24$

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