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**06/04/2023**
**Morning**

# Answers & Solutions

**Time : 3 hrs.** *for* **M.M. : 300**

# JEE (Main)-2023 (Online) Phase-2

## (Mathematics, Physics and Chemistry)

**IMPORTANT INSTRUCTIONS:**

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Mathematics, Physics** and **Chemistry** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

# MATHEMATICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

1. The straight lines $l_1$ and $l_2$ pass through the origin and trisect the line segment of the line $L : 9x + 5y = 45$ between the axes. If $m_1$ and $m_2$ are the slopes of the lines $l_1$ and $l_2$, then the point of intersection of the line $y = (m_1 + m_2)x$ with $L$ lies on

(1) $y - 2x = 5$

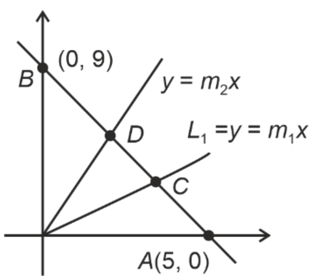
(2) $6x + y = 10$

(3) $y - x = 5$

(4) $6x - y = 15$

**Answer (3)**

**Sol.**




$L : 9x + 5y = 45$

$\Rightarrow \frac{x}{5} + \frac{y}{9} = 1$

$\therefore C \equiv \left(\frac{10}{3}, 3\right)$

$D \equiv \left(\frac{5}{3}, 6\right)$

$\therefore m_1 = \frac{9}{10}, m_2 = \frac{6 \times 3}{5} = \frac{18}{5}$

$\therefore y = \left(\frac{9}{10} + \frac{36}{10}\right)x = \frac{9}{2}x$ …(i)

So, intersection point with $L$

$7y = 45 \Rightarrow y = \frac{45}{7}, x = \frac{10}{7}$

$\therefore$ Option (3) is correct.

2. If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6} : 1$, then the third term from the beginning is:

(1) $30\sqrt{2}$ (2) $30\sqrt{3}$

(3) $60\sqrt{2}$ (4) $60\sqrt{3}$

**Answer (4)**

**Sol.** Given expansion $\left(\sqrt[4]{2} + \frac{1}{4\sqrt{3}}\right)^n$

(5th term from beginning)

$$T_5 = {}^nC_4 \left(\sqrt[4]{2}\right)^{n-4} \cdot \left(\frac{1}{\sqrt[4]{3}}\right)^4$$

(5th term from end)

$$T_5' = {}^nC_4 \left(\frac{1}{\sqrt[4]{3}}\right)^{n-4} \left(\sqrt[4]{2}\right)^4$$

Now, $\frac{T_5}{T_5'} = \frac{\sqrt{6}}{1} \Rightarrow 2^{\frac{n-8}{4}} \cdot 3^{\frac{n-8}{4}} = \sqrt{6}$

$\therefore (\sqrt{6})^{\frac{n-8}{2}} = (\sqrt{6})$

$\therefore n = 10$

$\therefore T_3 = {}^{10}C_2 \left(\sqrt[4]{2}\right)^8 \left(\frac{1}{\sqrt[4]{3}}\right)^2 = \frac{45 \times 4}{\sqrt{3}}$

$= 60\sqrt{3}$

3. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and $\sigma^2$ respectively. If the variance of all the 30 numbers in the two sets is 13, then $\sigma^2$ is equal to

(1) 10

(2) 11

(3) 9

(4) 12

**Answer (1)**

**Sol.** $\sum x_i = 15\times 12$ and $\dfrac{\sum x_i^2}{15} - 12^2 = 14$

And $\sum y_i = 15\times 14$ and $\dfrac{\sum y_i^2}{15} - 14^2 = \sigma^2$

Now, $13 = \dfrac{(14+144)\times 15 + (\sigma^2 + 196)\times 15}{30} - 13^2$

$\therefore \quad \boxed{\sigma^2 = 10}$

4. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\dfrac{k}{3^{11}}$, then $k$ is equal to

(1) 82 (2) 75

(3) 164 (4) 123

**Answer (4)**

**Sol.** $P$(success) = $\dfrac{4}{36} = \dfrac{1}{9}$

$P$(failure) = $\dfrac{8}{9}$

$\therefore$ Required probability = ${}^5C_4\left(\dfrac{1}{9}\right)^4\dfrac{8}{9} + {}^5C_5\left(\dfrac{1}{9}\right)^5$

$= 5\cdot\dfrac{8}{9^5} + \dfrac{1}{9^5} = \dfrac{41}{9^5}$

$= \dfrac{123}{3^{11}}$

$\therefore \quad k = 123$

5. Let the position vectors of the points $A$, $B$, $C$ and $D$ be $5\hat{i} + 5j + 2\lambda\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $-2\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 6\hat{k}$. Let the set $S = \{\lambda \in \mathbb{R}$ : the points $A$, $B$, $C$ and $D$ are coplanar$\}$. The $\sum_{\lambda\in S}(\lambda+2)^2$ is equal to

(1) 25 (2) $\dfrac{37}{2}$

(3) 13 (4) 41

**Answer (4)**

**Sol.** $A(5, 5, 2\lambda)$

$B(1, 2, 3)$

$C(-2, \lambda, 4)$

$D(-1, 5, 6)$

$\overrightarrow{AB}(-4, -3, 3-2\lambda)$

$\overrightarrow{AC}(-7, \lambda-5, 4-2\lambda)$

$\overrightarrow{AD}(-6, 0, 6-2\lambda)$

$\therefore \quad [\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

$$\begin{vmatrix} -4 & -3 & 3-2\lambda \\ -7 & \lambda-5 & 4-2\lambda \\ -6 & 0 & 6-2\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad -4(\lambda - 3)(\lambda - 2) = 0$

$\therefore \quad \lambda = 3, \lambda = 2$

$\sum_{\lambda\in S}(\lambda+2)^2 = 5^2 + 4^2$

$= 41$

6. Let $I(x) = \int \dfrac{x^2\left(x\sec^2 + \tan x\right)}{(x\tan x + 1)^2}dx$. If $I(0) = 0$, then $I\left(\dfrac{\pi}{4}\right)$ is equal to

(1) $\log_e\dfrac{(\pi+4)^2}{16} + \dfrac{\pi^2}{4(\pi+4)}$

(2) $\log_e\dfrac{(\pi+4)^2}{16} - \dfrac{\pi^2}{4(\pi+4)}$

(3) $\log_e\dfrac{(\pi+4)^2}{32} - \dfrac{\pi^2}{4(\pi+4)}$

(4) $\log_e\dfrac{(\pi+4)^2}{32} + \dfrac{\pi^2}{4(\pi+4)}$

**Answer (3)**

**Sol.** $\int x^2\left(\dfrac{x\sec^2 x + \tan x}{(x\tan x + 1)^2}\right)dx$

$= \dfrac{x^2}{(x\tan x + 1)} + \int\dfrac{2x}{x\tan x + 1}dx$

$I = 2\int\dfrac{x}{x\tan x + 1}dx$

$= 2\int\dfrac{x\cos x}{x\sin x + \cos x}dx$

Let $x\sin x + \cos x = t$

$(x\cos x + \sin x - \sin x)\,dx = dt$

$= 2\int\dfrac{dt}{t} = 2\log t + c$

$= 2\log|x\sin x + \cos x| + c$

$\therefore \quad \int \frac{x^2(x\sec^2 x + \tan x)}{(x\tan + 1)^2} dx$

$= \frac{-x^2}{x\tan x + 1}$ + 2 log |$x$sin$x$ + cos$x$| + $c$

$I(0) = 0$

$\Rightarrow \quad c = 0$

$I\left(\frac{\pi}{4}\right) = -\frac{\left(\frac{\pi}{4}\right)^2}{\frac{\pi}{4}\times 1 + 1} + 2\log\left|\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\right) + 1\right|$

$= \log_e \frac{(\pi+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$

7. Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a}\cdot\vec{d} = 18$, then $\left|\vec{a}\times\vec{d}\right|^2$ is equal to

(1) 640 (2) 680

(3) 720 (4) 760

**Answer (3)**

**Sol.** $\vec{b}\times\vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$

$\therefore \quad \vec{d} = \lambda\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$

$\vec{a}\cdot\vec{d} = 18$

$\Rightarrow \quad \lambda = 2$

$\therefore \quad \left|\vec{a}\times\vec{d}\right|^2 = |\vec{a}|^2 \cdot |\vec{d}|^2 - \left(\vec{a}\cdot\vec{d}\right)^2$

= 720

8. Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0.$ Then $18\int_1^2 f(x)dx$ is equal to

(1) 5 $\log_e$ 2 + 3 (2) 10 $\log_e$ 2 + 6

(3) 10 $\log_e$ 2 – 6 (4) 5 $\log_e$ 2 – 3

**Answer (3)**

**Sol.** $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$ …(i)

Replace $x \to \frac{1}{x}$

$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3$ …(ii)

Solving (i) and (ii) we get

$9f(x) = \frac{5}{x} - 4x + 3$

$\int_1^2 f(x)dx = \frac{1}{9}\int_1^2 \left(\frac{5}{x} - 4x + 3\right)dx$

$= \frac{1}{9}\left[5\log x - 2x^2 + 3x\right]_1^2$

$= \frac{1}{9}\left[5\log x - 3\right]$

$\therefore \quad 18\int_1^2 f(x)dx = 10\log_e 2 - 6$

9. Statement $(P \Rightarrow Q)\wedge(R \Rightarrow Q)$ is logically equivalent to

(1) $(P \Rightarrow R)\vee(Q \Rightarrow R)$

(2) $(P \vee R) \Rightarrow Q$

(3) $(P \Rightarrow R)\wedge(Q \Rightarrow R)$

(4) $(P \wedge R) \Rightarrow Q$

**Answer (4)**

**Sol.** $(P \to Q)\wedge(R \to Q)$

$\Rightarrow \quad (P' \vee Q)\wedge(R' \vee Q)$

$\Rightarrow \quad Q \vee (P' \wedge R')$

$\Rightarrow \quad Q \vee (P \wedge R)'$

$\Rightarrow \quad (P \wedge R)' \vee Q$

$\Rightarrow \quad (P \wedge R) \to Q$

10. If the system of equations

$x + y + az = b$

$2x + 5y + 2z = 6$

$x + 2y + 3z = 3$

has infinitely many solutions, then $2a + 3b$ is equal to

(1) 25 (2) 20

(3) 23 (4) 28

**Answer (3)**

**Sol.** $x + y + az = b$ …(i)

$2x + 5y + 2z = 6$ …(ii)

$x + 2y + 3z = 3$ …(iii)

3 (iii) – (ii)

$x + y + 7z = 3$

$x + y + az = b$

$\therefore \quad a = 7, b = 3$ ($\because$ solutions are infinite)

$\therefore \quad 2a + 3b$

= 14 + 9 = 23

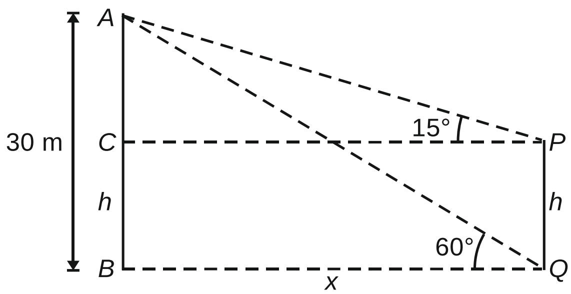
11. From the top $A$ of a vertical wall $AB$ of height 30 m, the angles of depression of the top $P$ and bottom $Q$ of a vertical tower $PQ$ are 15° and 60° respectively, $B$ and $Q$ are on the same horizontal level. If $C$ is a point on $AB$ such that $CB = PQ$, then the area (in $m^2$) of the quadrilateral $BCPQ$ is equal to

(1) $300(\sqrt{3}-1)$ (2) $300(\sqrt{3}+1)$

(3) $600(\sqrt{3}-1)$ (4) $200(\sqrt{3}-1)$

**Answer (3)**

**Sol.**




$\frac{30}{x} = \sqrt{3}$;

$\frac{30-h}{x} = 2-\sqrt{3}$

$\Rightarrow \quad 30-h = 20\sqrt{3}-30$

$\Rightarrow \quad x = 10\sqrt{3}$

$\Rightarrow h = 60-20\sqrt{3}$

$\therefore$ area = $hx$

$= (60-20\sqrt{3})10\sqrt{3}$

$= 200\sqrt{3}(3-\sqrt{3})$

$= 600(\sqrt{3}-1)$

$\therefore$ (3) is correct.

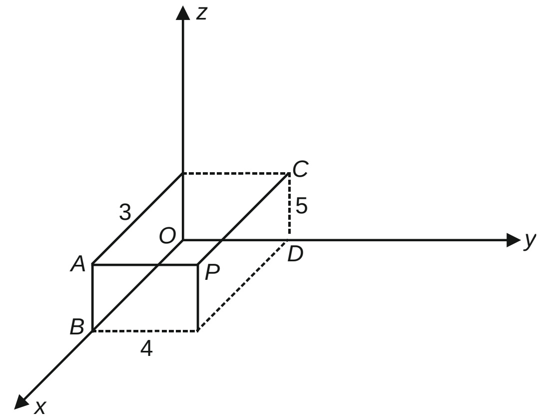
12. One vertex of a rectangular parallelopiped is at the origin $O$ and the lengths of its edges along $x$, $y$ and $z$ axes are 3, 4 and 5 units respectively. Let $P$ be the vertex (3, 4, 5). Then the shortest distance between the diagonal $OP$ and an edge parallel to $z$ axis, not passing through $O$ or $P$ is

(1) $\frac{12}{\sqrt{5}}$ (2) $12\sqrt{5}$

(3) $\frac{12}{5\sqrt{5}}$ (4) $\frac{12}{5}$

**Answer (4)**

**Sol.**




Line $OP$ : $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$

Line $AB$ : $\frac{x-3}{0} = \frac{y}{0} = \frac{z}{1}$

$$\overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 0 & 0 & 1 \end{vmatrix}$$

$= \hat{i}(4) - \hat{j}(3) + \hat{k}(0)$

$= 4\hat{i} - 3\hat{j}$

$$\text{Distance} = \frac{(\overrightarrow{a_2}-\overrightarrow{a_1}) \cdot (\overrightarrow{n_1} \times \overrightarrow{n_2})}{|\overrightarrow{n_1} \times \overrightarrow{n_2}|}$$

$= \frac{(3\hat{i}) \cdot (4\hat{i} - 3\hat{j})}{5}$

$= \frac{12}{5}$

Option (4) is correct.

13. If ${}^{2n}C_3 : {}^{n}C_3 = 10 : 1$, then the ratio $(n^2 + 3n) : (n^2 - 3n + 4)$ is

(1) 35 : 16 (2) 27 : 11

(3) 65 : 37 (4) 2 : 1

**Answer (4)**

**Sol.** ${}^{2n}C_3 : {}^{n}C_3 = 10 : 1$

$\frac{(2n)!}{3!\,(2n-3)!} \times \frac{3!\,(n-3)!}{n!} = 10$

$\Rightarrow \quad 4(2n - 1) = 10n - 20$

$\Rightarrow \quad n = 8$

Now $\frac{(n^2+3n)}{(n^2-3n+4)} = \frac{64+24}{64-24+4} = \frac{88}{44} = 2$

14. If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$, then $a + b + c$ is equal to

(1) 12 (2) 14

(3) 15 (4) 13

**Answer (2)**

**Sol.** $P_1 = 2x - y + z = 3$

$P_2 = 4x - 3y + 5z + 9 = 0$

$P_1 = \lambda P_2 = 0$

$(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$

$P_3 = (2 + 4\lambda)x - (1 + 3\lambda)y + (1 + 5\lambda)z - (3 - 9\lambda) = 0$

$P_3$ is parallel to $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$

$-2(2 + 4\lambda) - 4(1 + 3\lambda) + 5(1 + 5\lambda) = 0$

$-3 + 5\lambda = 0$

$\Rightarrow \lambda = \frac{3}{5}$

$P_3 : \frac{22x}{5} - \frac{14y}{5} + \frac{20z}{5} + \frac{12}{5} = 0$

$P_3 = 11x - 7y + 10z + 6 = 0$

$\therefore$ $a = 11$

$b = -7$

$c = 10$

$\therefore$ $a + b + c = 11 - 7 + 10 = 14$

15. The sum of the first 20 terms of the series 5 + 11 + 19 + 29 + 41 + … is

(1) 3520 (2) 3450

(3) 3250 (4) 3420

**Answer (1)**

**Sol.**

$$S = 5 + 11 + 19 + 29 + 41 + \dots\dots\dots\dots T_n$$
$$S = 5 + 11 + 19 + 29 + \dots\dots\dots\dots + T_{n-1} + T_n$$
$$T_n = 5 + 6 + 8 + \dots\dots\dots + (T_n - T_{n-1})$$

$$T_n = n^2 + 3n + 1$$

$$S_n = \sum T_n = \sum n^2 + 3\sum n + \sum 1$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$$

$n = 20$

$$S_{20} = \frac{20 \times 21 \times 41}{6} + \frac{3 \times 20 \times 21}{2} + 20$$

$= 3520$

16. Let $a_1, a_2, a_3, \dots, a_n$ be $n$ positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference, then

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left( \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right)$$ is

(1) $\frac{1}{\sqrt{d}}$ (2) $\sqrt{d}$

(3) 1 (4) 0

**Answer (3)**

**Sol.** $$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left( \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} \right)$$

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} \left( \frac{\sqrt{a_2} - \sqrt{a_1}}{a_2 - a_1} + \frac{\sqrt{a_3} - \sqrt{a_2}}{a_3 - a_2} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}} \right)$$

$$\lim_{n\to\infty} \sqrt{\frac{d}{n}} - \frac{1}{d}\left(\sqrt{a_n} - \sqrt{a_1}\right)$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{d}} \left( \frac{\sqrt{a_1 + (n-1)d} - \sqrt{a_1}}{\sqrt{n}} \right)$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{d}} \left( \sqrt{\frac{a_1}{n} + d - \frac{d}{n}} - \sqrt{\frac{a_1}{n}} \right)$$

$$\frac{1}{\sqrt{d}} \times \sqrt{d} = 1$$

17. Let $A = [a_{ij}]_{2\times 2}$, where $a_{ij} \neq 0$ for all $i, j$ and $A^2 = I$. Let $a$ be the sum of all diagonal elements of $A$ and $b = |A|$. Then $3a^2 + 4b^2$ is equal to

(1) 4 (2) 14

(3) 7 (4) 3

**Answer (1)**

**Sol.** Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$$A^2 = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} p^2 + qr & pq + qs \\ rp + rs & qr + s^2 \end{bmatrix}$$

$A^2 = I$

$\Rightarrow$ $p^2 + qr = 1$ $\quad q(p + s) = 0$

$r(p + s) = 0$ $\quad qr + s^2 = 1$

$q \neq 0 \Rightarrow p + s = 0 \Rightarrow a = 0$

$b = |A| = ps - qr = -p^2 - qr = -1$ ($\because s = -p$)

$\therefore$ $3a^2 + 4b^2 = 4$

18. The sum of all the roots of the equation

$\left|x^2 - 8x + 15\right| - 2x + 7 = 0$ is:

(1) $9-\sqrt{3}$ (2) $9+\sqrt{3}$

(3) $11-\sqrt{3}$ (4) $11+\sqrt{3}$

**Answer (2)**

**Sol.** **Case:I** $x^2 - 8x + 15 \geq 0 \Rightarrow (x-3)(x-5) \geq 0$

$\Rightarrow x \leq 3$ OR $x \geq 5$

$x^2 - 8x + 15 - 2x + 7 = 0$

$x^2 - 10x + 22 = 0 \Rightarrow x = 5+\sqrt{3}, 5-\sqrt{3}$ (rejected)

$\alpha = 5+\sqrt{3}$

**Case: II** $x^2 - 8x + 15 < 0 \Rightarrow 3 < x < 5$

$x^2 - 8x + 15 + 2x - 7 = 0$

$x^2 - 6x + 8 = 0 \Rightarrow x = 4, 2$ (rejected)

$\gamma = 4$

$\alpha + \gamma = 9+\sqrt{3}$

19. If $2x^y + 3y^x = 20$, then $\frac{dy}{dx}$ at (2, 2) is equal to:

(1) $-\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$ (2) $-\left(\frac{3+\log_e 16}{4+\log_e 8}\right)$

(3) $-\left(\frac{3+\log_e 8}{2+\log_e 4}\right)$ (4) $-\left(\frac{3+\log_e 4}{2+\log_e 8}\right)$

**Answer (1)**

**Sol.** $2x^y + 3y^x = 20$

Differentiating both sides w.r.t. $x$,

$$2x^y\left(y'\ln x + \frac{y}{x}\right) + 3y^x\left(\ln y + \frac{x}{y}\cdot y'\right) = 0$$

Putting $x = 2 = y$,

$8(\ln 2 \cdot y' + 1) + 12(\ln y + y') = 0$

$(8\ln 2 + 12)y' = -(12\ln 2 + 8)$

$$y' = -\left(\frac{3\ln 2 + 2}{2\ln 2 + 3}\right) = -\left(\frac{2+\ln 8}{3+\ln 4}\right)$$

20. Let $A = \{x \in \mathbb{R} : [x+3] + [x+4] \leq 3\}$,

$B = \left\{x \in \mathbb{R} : 3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$, where $[t]$ denotes greatest integer function. Then,

(1) $B \subset C, A \neq B$ (2) $A \cap B = \phi$

(3) $A \subset B, A \neq B$ (4) $A = B$

**Answer (4)**

**Sol.** $\because$ $[x+3] + [x+4] \leq 3 \Rightarrow [x] \leq -2$

$\Rightarrow x < -1 \Rightarrow A = (-\infty, -1)$

$\because$ $3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}$

$\Rightarrow 3^x\left(\frac{3\cdot\frac{1}{10}}{1-\frac{1}{10}}\right)^{x-3} < 3^{-3x}$

$\Rightarrow 3^x\left(\frac{1}{3}\right)^{x-3} < 3^{-3x}$

$\Rightarrow 3^{x-x+3+3x} < 1$

$\Rightarrow 3^{3(x+1)} < 1$

$\Rightarrow x < -1 \Rightarrow B = (-\infty, -1)$

$\Rightarrow A = B$

## SECTION - B

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The coefficient of $x^{18}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is ________

**Answer (5005)**

**Sol.** General term $(T_{r+1}) = {}^{15}C_r\left(x^4\right)^{15-r}\left(-\frac{1}{x^3}\right)^r$

$T_{r+1} = {}^{15}C_r\ x^{60-7r}(-1)^r$

For coefficient of $x^{18}$, put $60 - 7r = 18$

$\Rightarrow r = 6$

Coefficient of $x^{18} = (-1)^6 . {}^{15}C_6 = 5005$

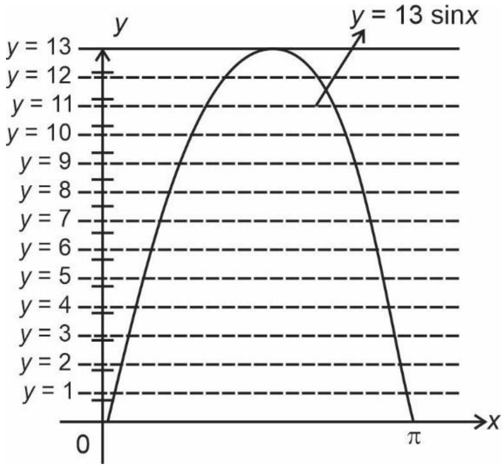
22. Let $a \in \mathbb{Z}$ and $[t]$ be the greatest integer $\leq t$. Then the number of points, where the function $f(x) = [a + 13\sin x]$, $x \in (0, \pi)$ is not differentiable, is ________

**Answer (25)**

**Sol.** $f(x) = [a + 13\sin x]$, $x \in (0, \pi)$, $a \in Z$

$f(x) = [13\sin x]$

Let $g(x) = 13\sin x$




$f(x) = [13\sin x]$ is not differentiable at intersection points of $y = K$, $K \in [1, 13]$, $K \in Z$

$\Rightarrow$ Points of non-differentiability = 25

23. The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is _______

**Answer (171*)**

**Sol.** Number of ways

$= \text{coefficient of } x^{20} \text{ in } (x + x^2 + \ldots + x^{18})^3$

$= \text{coefficient of } x^{17} \text{ in } (1 - x)^{-3}$

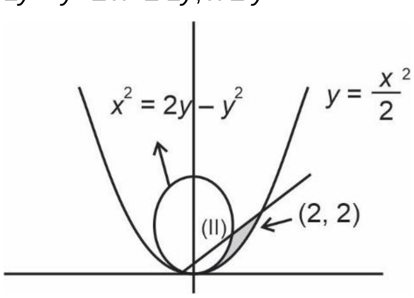
$= {}^{19}C_2 = 171$

**Note:** Here 3 children are considered identical but children showed be considered distinct and number of ways should be number of ways $= 3^{20} - {}^3C_1 2^{20} + {}^3C_2 1^{20}$

24. If the area of the region $S = \{(x, y) : 2y - y^2 \le x^2 \le 2y, x \ge y\}$ is equal to $\frac{n+2}{n+1} - \frac{\pi}{n-1}$, then the natural number $n$ is equal to _______

**Answer (05.00)**

**Sol.** $2y - y^2 \le x^2 \le 2y$, $x \ge y$




$$\text{Area} = \int_1^2 \left(\sqrt{2y} - y\right) dy + \int_0^1 \left(\sqrt{2y} - \sqrt{2y - y^2}\right) dy$$

$$= \frac{7 - 2^{\frac{5}{2}}}{6} + \frac{2^{\frac{7}{2}} - 3\pi}{12}$$

$$= \frac{14 - 2^{\frac{7}{2}} + 2^{\frac{7}{2}} - 3\pi}{12} = \frac{7}{6} - \frac{\pi}{4}$$

$\boxed{n = 5}$

25. A circle passing through the point $P(\alpha, \beta)$ in the first quadrant touches the two coordinate axes at the points $A$ and $B$. The point $P$ is above the line $AB$. The point $Q$ on the line segment $AB$ is the foot of perpendicular from $P$ on $AB$. If $PQ$ is equal to 11 units, then the value of $\alpha\beta$ is _______

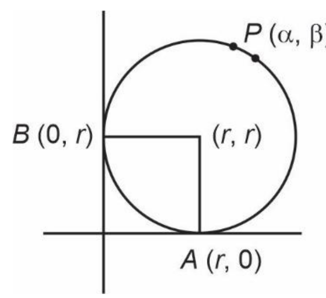
**Answer (121.00)**

**Sol.** $(x - r)^2 + (y - r)^2 = r^2$ passes through $(\alpha, \beta)$

$AB : x + y = r$

$Q(h, k)$

$$\frac{h - \alpha}{1} = \frac{k - \beta}{1} = \frac{-(\alpha + \beta - r)}{2}$$




$$(h, k) = \left(\alpha - \frac{\alpha}{2} - \frac{\beta}{2} + \frac{r}{2}, \beta - \frac{\alpha}{2} - \frac{\beta}{2} + \frac{r}{2}\right)$$

$$= \left(\frac{\alpha - \beta + r}{2}, \frac{-\alpha + \beta + r}{2}\right)$$

$$(PQ)^2 = \left(\frac{-\alpha - \beta + r}{2}\right)^2 + \left(\frac{-\alpha - \beta + r}{2}\right)^2 = 121$$

$\Rightarrow (r - (\alpha + \beta))^2 = 242$

or $r^2 + (\alpha + \beta)^2 - 2r(\alpha + \beta) = 242$

or $\alpha^2 + \beta^2 + 2\alpha\beta + r^2 - 2r\alpha - 2r\beta = 242$

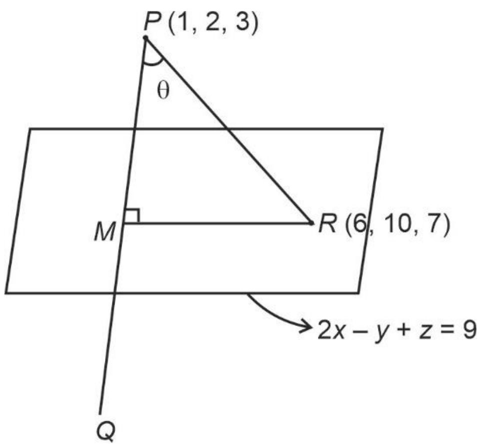
or $2\alpha\beta = 242$

$\Rightarrow \alpha\beta = 121$

26. Let the image of the point $P(1, 2, 3)$ in the plane $2x - y + z = 9$ be $Q$. If the coordinates of the point $R$ are (6, 10, 7), then the square of the area of the triangle $PQR$ is ________.

**Answer (594.00)**

**Sol.**




$R$ lies on plane

$$PR = \sqrt{5^2 + 8^2 + 4^2} = \sqrt{105}$$

$$\cos\theta = \frac{(5\hat{i} + 8\hat{j} + 4\hat{k})(2\hat{i} - \hat{j} + \hat{k})}{\sqrt{105}\sqrt{6}}$$

$$= \frac{6}{\sqrt{630}}$$

Area ($\Delta PQR$) = 2area($\Delta PMR$)

$$= 2 \cdot \frac{1}{2}(PR)^2 \sin\theta\cos\theta$$

$$= 105 \cdot \frac{6}{\sqrt{630}} \cdot \frac{\sqrt{594}}{\sqrt{630}}$$

$$= \sqrt{594}$$

27. Let $y = y(x)$ be a solution of the differential equation $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0$, $0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to ________.

**Answer (02.00)**

**Sol.** $\because$ $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0$

$$\therefore \quad (x\cos x)\frac{dy}{dx} + y(x\sin x + y\cos x) = 1$$

$$\Rightarrow \quad \frac{dy}{dx} + y\left(\frac{x\sin x + \cos x}{x\cos x}\right) = \frac{1}{x\cos x}$$

$\therefore$ Integrating factor = $e^{\int\left(\tan x + \frac{1}{x}\right)dx}$

$= x\sec x$

$$\therefore \quad y \cdot x\sec x = \int \frac{x\sec x}{x\cos x}dx$$

$\therefore$ $xy\sec x = \tan x + C$

$$\because \quad \frac{\pi}{3}y\left(\frac{\pi}{3}\right)\sec\frac{\pi}{3} = \tan\frac{\pi}{3} + C.$$

$$\therefore \quad C = \sqrt{3}$$

$$\therefore \quad xy\sec x = \tan x + \sqrt{3}$$

$$\therefore \quad y(x) = \frac{2\sin\left(x + \frac{\pi}{3}\right)}{x}$$

$$\therefore \quad xy''(x) + 2y'(x) = -2\sin\left(x + \frac{\pi}{3}\right)$$

Thus $\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2\ y'\left(\frac{\pi}{6}\right) = -2$

Hence $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2\ y'\left(\frac{\pi}{6}\right)\right| = 2$

28. Let $A = \{1, 2, 3, 4, ........, 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \left\{(a,b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\right\}$ is ________.

**Answer (18)**

**Sol.** $A = \{1, 2, 3, ..., 10\}$ $\quad B = \{0, 1, 2, 4\}$

$(a, b) \in A \times A$ such that

$2(a - b)^2 + 3(a - b) - k = 0$

where $k \in \{0, 1, 2, 3, 4\}$

We should have

$9 - 4 \times 2(-k)$ a perfect square for any possible $(a, b)$

i.e, $9 + 8k$ is perfect square

$\Rightarrow$ $k = 0$ or $k = 2$

for $k = 0$, $2(a - b)^2 + 3(a - b) = 0$

$\Rightarrow$ $a - b = 0 \Rightarrow (a, b) \in \{(1, 1), (2, 2) ..... (10, 10)\}$.

$\Rightarrow$ Total 10 elements belonging to $R$.

$a - b = -\frac{3}{2}$ is not possible

for $k = 0$ $2(a - b) + 3(a - b) - 2 = 0$

$\Rightarrow$ $a - b = -2$ or $a - b = \frac{1}{2}$(not possible)

$\Rightarrow$ $(a, b) \in \{(1, 3), (2, 4), ..... (8, 10)\}$

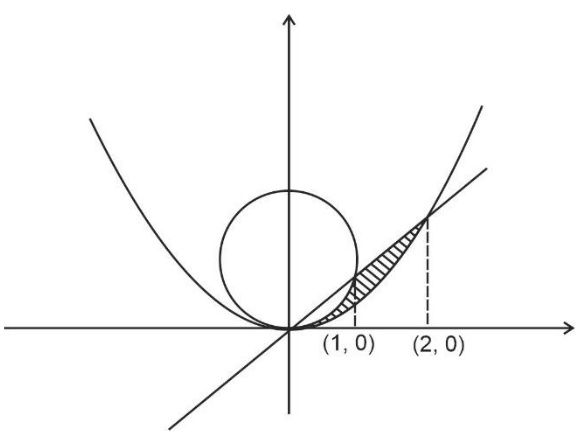
$\Rightarrow$ 8 element belonging to $R$

Total = 18

29. Let the tangent to the curve $x^2 + 2x - 4y + 9 = 0$ at the point $P(1, 3)$ on it meet the $y$-axis at $A$. Let the line passing through $P$ and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at $B$. If $B$ lies on the line $2x - 3y = 8$, then $(AB)^2$ is equal to ________.

**Answer (5)**

**Sol.**




Given region are : $2y - y^2 \le x^2$

$x^2 \le 2y$

$x \ge y$

The required area is as shown

Required area

$$= \int_0^1 \left(1 + \sqrt{1 - x^2} - \frac{x^2}{2}\right) + \int_1^2 \left(x - \frac{x^2}{2}\right) dx$$

$$= \left(x - \frac{x}{2}\sqrt{1 - x^2} - \frac{1}{2} \cdot \sin^{-1} x - \frac{x^3}{6}\right)\Bigg|_0^1 + \left(\frac{x^2}{2} - \frac{x^3}{6}\right)_1^2$$

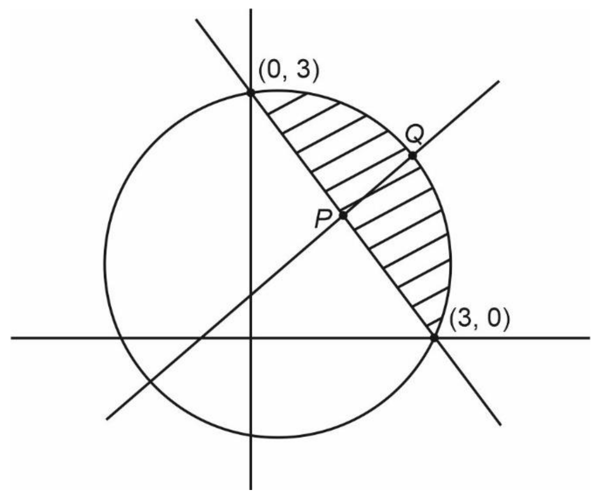
$$= \frac{7}{6} - \frac{\pi}{4} = \frac{n+2}{n+1} - \frac{\pi}{x-1}$$

$\Rightarrow$ $n = 5$

30. Let the point $(p, p + 1)$ lie inside the region $E = \left\{(x, y) : 3 - x \le y \le \sqrt{9 - x^2}, 0 \le x \le 3\right\}$. If the set of all values of $p$ is the interval $(a, b)$, then $b^2 + b - a^2$ is equal to ________.

**Answer (3)**

**Sol.**




$(p, p + 1)$ lies on $y - x = 1$

$\therefore$ Solving $y - x = 1$ and $x + y = 3$

$P(1, 2)$

solving $y - x = 1$ and $x^2 + y^2 = 9$

$x^2 + (1 + x)^2 = 9 \Rightarrow 2x^2 + 2x - 8 = 0$

$\Rightarrow$ $x^2 + x - 4 = 0$ ...(i)

$$x = \frac{-1 \pm \sqrt{1 + 4 \cdot 4}}{2} = \frac{\sqrt{17} - 1}{2}$$

$\therefore$ $p \in \left(1, \frac{\sqrt{17} - 1}{2}\right)$

$\therefore$ $a = 1$, $b^2 + b = 4$ [using (i)]

$\therefore$ $b^2 + b - a^2 = 4 - 1 = 3$

# PHYSICS

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

31. A particle is moving with constant speed in a circular path. When the particle turns by an angle 90°, the ratio of instantaneous velocity to its average velocity is $\pi : x\sqrt{2}$. The value of $x$ will be

(1) 2 (2) 5

(3) 1 (4) 7

**Answer (1)**

**Sol.** Instantaneous velocity = $\omega R$

Time taken $= \frac{\pi}{2\omega}$

Displacement $= R\sqrt{2}$

Average velocity $= \frac{R\sqrt{2} \times 2\omega}{\pi} = \frac{2\sqrt{2}}{\pi}\omega R$

$\Rightarrow \frac{v_{\text{ins}}}{v_{\text{avg}}} = \frac{\omega R \pi}{2\sqrt{2}\omega R}$

$\Rightarrow x = 2$

32. A source supplies heat to a system at the rate of 1000 W. If the system performs work at a rate of 200 W. The rate at which internal energy of the system increases is

(1) 600 W (2) 800 W

(3) 500 W (4) 1200 W

**Answer (2)**

**Sol.** $\frac{dQ}{dt} = 1000 \text{ W}$

$\frac{dW}{dt} = 200 \text{ W}$

$\frac{dU}{dt} = 1000 \text{ W} - 200 \text{ W} = 800 \text{ W}$

33. The number of air molecules per cm$^3$ increased from $3 \times 10^{19}$ to $12 \times 10^{19}$. The ratio of collision frequency of air molecules before and after the increase in number respectively is :

(1) 0.75 (2) 1.25

(3) 0.50 (4) 0.25

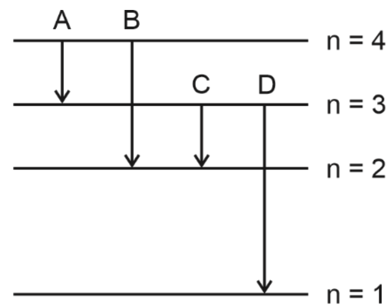
**Answer (4)**

**Sol.** $f \propto \frac{1}{\tau}$

$\Rightarrow f \propto n$

$\frac{f_{\text{before}}}{f_{\text{after}}} = \frac{3}{12} = 0.25$

34. The energy levels of an hydrogen atom are shown below. The transition corresponding to emission of shortest wavelength is




(1) D (2) A

(3) B (4) C

**Answer (1)**

**Sol.** Shortest wavelength will correspond to highest energy difference de-excitation, which in the given figure will correspond to transition from $n$ = 3 to $n$ = 1.

35. The induced emf can be produced in a coil by

A. moving the coil with uniform speed inside uniform magnetic field

B. moving the coil with non uniform speed inside uniform magnetic field

C. rotating the coil inside the uniform magnetic field

D. changing the area of the coil inside the uniform magnetic field

Choose the correct answer from the options given below:

(1) B and C only

(2) A and C only

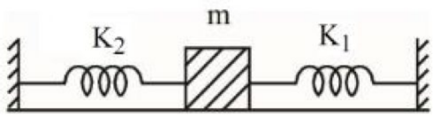
(3) C and D only

(4) B and D only

**Answer (3)**

**Sol.** If flux can change with respect to time then emf can be produced in the coil.

36. A mass m is attached to two strings as shown in figure. The spring constants of two springs are $K_1$ and $K_2$. For the frictionless surface, the time period of oscillation of mass m is




(1) $2\pi\sqrt{\frac{m}{K_1+K_2}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{K_1-K_2}{m}}$

(3) $2\pi\sqrt{\frac{m}{K_1-K_2}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{K_1+K_2}{m}}$

**Answer (1)**

**Sol.** Two springs are effectively in parallel

$\Rightarrow$ $K = K_1 + K_2$

$\Rightarrow$ $T = 2\pi\sqrt{\frac{m}{K_1+K_2}}$

37. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A:** When a body is projected at an angle 45°, it's range is maximum.

**Reason R:** For maximum range, the value of sin 2θ should be equal to one.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R are correct and R is the correct explanation of A

(4) Both A and R are correct but R is NOT the correct explanation of A

**Answer (3)**

**Sol.** On a horizontal ground projectile $R = \frac{u^2 \sin 2\theta}{g}$

for $R_{max}$ sin(2θ) = 1 $\Rightarrow$ θ = 45°

38. A small block of mass 100 g is tied to a spring of spring constant 7.5 N/m and length 20 cm. The other end of spring is fixed at a particular point A. If the block moves in a circular path on a smooth horizontal surface with constant angular velocity 5 rad/s about point A, then tension in the spring is

(1) 0.75 N (2) 0.25 N

(3) 0.50 N (4) 1.5 N

**Answer (1)**

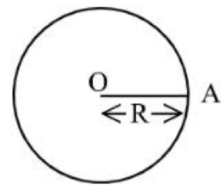
**Sol.** $kx = T = m\omega^2(r + x)$

$7.5x = 2.5(0.2 + x)$

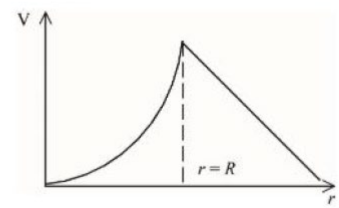
$x = \frac{0.5}{5} = 0.1$

$\Rightarrow$ $kx = 0.75$ N

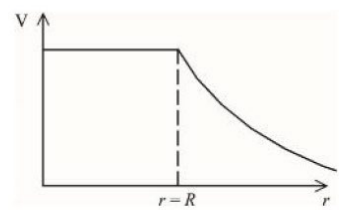
39. For a uniformly charged thin spherical shell, the electric potential (V) radially away from the centre (O) of shell can be graphically represented as



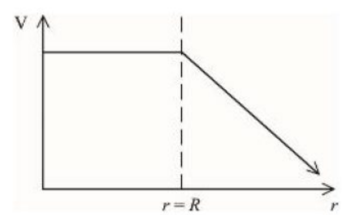

(1)




(2)




(3)

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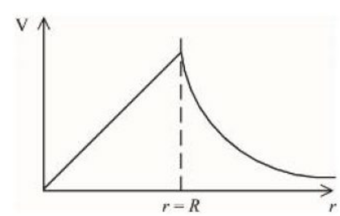
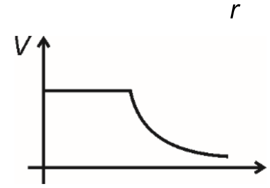



**Answer (2)**

**Sol.** Electric potential inside shell will remain constant.

Outside the shell $V \propto \frac{1}{r}$

$\Rightarrow$



40. Two resistance are given as $R_1 = (10 \pm 0.5)\ \Omega$ and $R_2 = (15 \pm 0.5)\ \Omega$. The percentage error in the measurement of equivalent resistance when they are connected in parallel is

(1) 6.33 (2) 2.33
(3) 5.33 (4) 4.33

**Answer (4)**

**Sol.** $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$\Rightarrow \frac{dR}{R^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}$

$\frac{1}{R} = \frac{1}{10} + \frac{1}{15} \Rightarrow R = \frac{150}{25} = 6\ \Omega$

$\Rightarrow dR = 36\left(\frac{0.5}{100} + \frac{0.5}{15 \times 15}\right) = 0.26$

$\Rightarrow R = 6 \pm 0.26$

$\frac{dR}{R}$ in percentage = 4.33%

41. A planet has double the mass of the earth. Its average density is equal to that of the earth. An object weighing W on earth will weigh on that planet:

(1) W (2) $2^{1/3}$ W
(3) 2 W (4) $2^{2/3}$ W

**Answer (2)**

**Sol.** $g = \frac{GM}{R^2}$

$g \propto M^{\frac{1}{3}} \rho^{\frac{2}{3}}$

$\frac{W'}{W} = \left(\frac{M'}{M}\right)^{\frac{1}{3}} = (2)^{\frac{1}{3}}$

42. A monochromatic light wave with wavelength $\lambda_1$ and frequency $\nu_1$ in air enters another medium. If the angle of incidence and angle of refraction at the interface are 45° and 30° respectively, then the wavelength $\lambda_2$ and frequency $\nu_2$ of the refracted wave are:

(1) $\lambda_2 = \sqrt{2}\lambda_1,\ \nu_2 = \nu_1$

(2) $\lambda_2 = \lambda_1,\ \nu_2 = \frac{1}{\sqrt{2}}\nu_1$

(3) $\lambda_2 = \lambda_1,\ \nu_2 = \sqrt{2}\nu_1$

(4) $\lambda_2 = \frac{1}{\sqrt{2}}\lambda_1,\ \nu_2 = \nu_1$

**Answer (4)**

**Sol.** $\mu = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}$

$\Rightarrow \frac{C_{air}}{C_{med}} = \sqrt{2} = \frac{\lambda_{air}}{\lambda_{med}}$

$\Rightarrow \lambda_1 = \sqrt{2}\lambda_2$ and $\nu_1 = \nu_2$

43. A small ball of mass M and density $\rho$ is dropped in a viscous liquid of density $\rho_0$. After some time, the ball falls with a constant velocity. What is the viscous force on the ball?

(1) $F = Mg\left(1 + \frac{\rho_0}{\rho}\right)$ (2) $F = Mg\left(1 + \frac{\rho}{\rho_0}\right)$

(3) $F = Mg\left(1 - \frac{\rho_0}{\rho}\right)$ (4) $F = Mg(1 \pm \rho\rho_0)$

**Answer (3)**

**Sol.** At terminal velocity

$B + F_v = Mg$

$\Rightarrow F_v = Mg\left(1 - \frac{\rho_0}{\rho}\right)$

44. By what percentage will the transmission range of a TV tower be affected when the height of the tower is increased by 21%?

(1) 15% (2) 12%
(3) 10% (4) 14%

**Answer (3)**

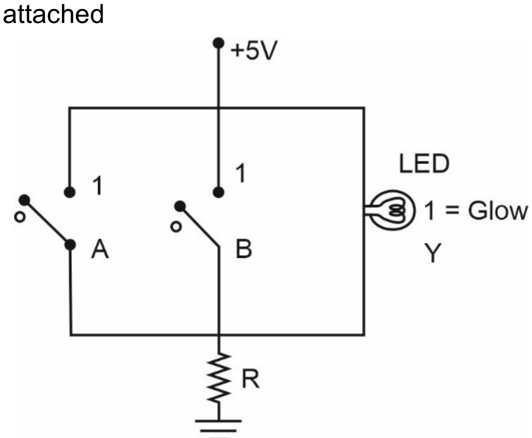
**Sol.** $R_{ange} = \sqrt{2hR_e} = R$ (say)

Now $h' = 1.21h$

$\Rightarrow R' = 1.1R$

$\Rightarrow$ Range increases by 10%

45. Name the logic gate equivalent to the diagram attached




(1) NAND (2) AND
(3) NOR (4) OR

**Answer (3)**

**Sol.** Truth table corresponding to given situation

| A | B | Out |
|---|---|---|
| 1 | 1 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 1 |

$\Rightarrow$ The truth table corresponds to NOR gate.

46. For the plane electromagnetic wave given by $E = E_0 \sin(\omega t - kx)$ and $B = B_0 \sin(\omega t - kx)$, the ratio of average electric energy density to average magnetic energy density is

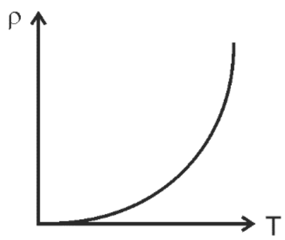
(1) 1/2 (2) 2

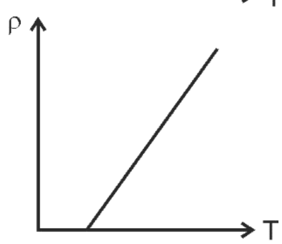
(3) 4 (4) 1

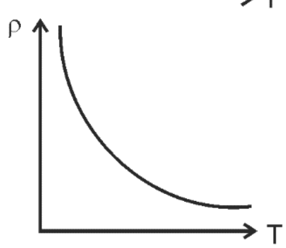
**Answer (4)**

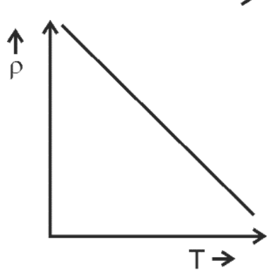
**Sol.** Average electric energy density = Average magnetic energy density

47. The resistivity ($\rho$) of semiconductor varies with temperature. Which of the following curve represents the correct behaviour?

(1)


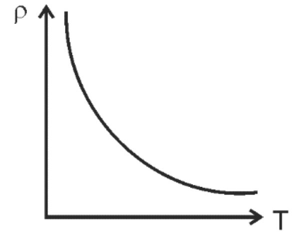

(2)



(3)



(4)



**Answer (3)**

**Sol.** In semiconductors when small amount of energy is supplied then electrons easily move to conduction band becoming free to move within lattice.




48. A long straight wire of circular cross-section (radius a) is carrying steady current I. The current I is uniformly distributed across this cross-section. The magnetic field is

(1) inversely proportional to $r$ in the region $r < a$ and uniform throughout in the region $r > a$

(2) directly proportional to $r$ in the region $r < a$ and inversely proportional to $r$ in the region $r > a$

(3) zero in the region $r < a$ and inversely proportional to $r$ in the region $r > a$

(4) uniform in the region $r < a$ and inversely proportional to distance $r$ from the axis, in the region $r > a$

**Answer (2)**

**Sol.** $B_{in} = \dfrac{\mu_0 Jr}{2}$

$B_{out} = \dfrac{\mu_0 JR^2}{2r}$

49. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A:** Earth has atmosphere whereas moon doesn't have any atmosphere.

**Reason R:** The escape velocity on moon is very small as compared to that on earth.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is false but R is true

(3) Both A and R are correct and R is the correct explanation of A

(4) A is true but R is false

**Answer (3)**

**Sol.** Because of lower escape velocity on surface of moon the average velocity of gas molecules remains higher than the escape velocity making moon unable to hold atmosphere.

50. The kinetic energy of an electron, $\alpha$-particle and a proton are given as 4 K, 2 K and K respectively. The de-Broglie wavelength associated with electron ($\lambda e$), $\alpha$-particle ($\lambda\alpha$) and the proton ($\lambda p$) are as follows:

(1) $\lambda\alpha = \lambda p > \lambda e$ (2) $\lambda\alpha < \lambda p < \lambda e$

(3) $\lambda\alpha = \lambda p < \lambda e$ (4) $\lambda\alpha > \lambda p > \lambda e$

**Answer (2)**

**Sol.** $\lambda = \frac{h}{\sqrt{2mKE}}$

$\Rightarrow \lambda e > \lambda p > \lambda\alpha$

**SECTION - B**

**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. The radius of fifth orbit of $Li^{++}$ is ________ $\times 10^{-12}$ m. Take: radius of hydrogen atom = 0.51 Å

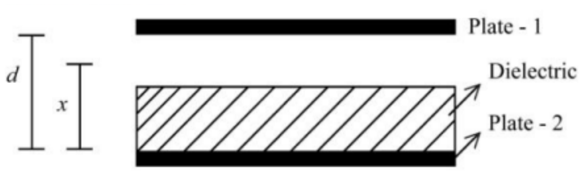
**Answer (425)**

**Sol.** $R_{\text{Li}} = R_{\text{H}} \times \frac{n^2}{Z}$

$= \frac{0.51 \times 25}{3} \times 10^{-10}$

$= 425 \times 10^{-12}$ m

52. A parallel plate capacitor with plate area $A$ and plate separation $d$ is filled with a dielectric material of dielectric constant $K = 4$. The thickness of the dielectric material is $x$, where $x < d$.




Let $C_1$ and $C_2$ be the capacitance of the system for $x = \frac{1}{3}d$ and $x = \frac{2d}{3}$, respectively. If $C_1 = 2$ μF the value of $C_2$ is ___________ μF.

**Answer (3)**

**Sol.** $C_1 = \frac{\varepsilon_0 A}{\frac{2d}{3} + \frac{d/3}{4}} = \frac{4\varepsilon_0 A}{3d}$

$C_2 = \frac{\varepsilon_0 A}{\frac{d}{3} + \frac{2d/3}{4}} = \frac{4\varepsilon_0 A}{2d} = \frac{2\varepsilon_0 A}{d}$

$= 3$ μF

53. A particle of mass 10 g moves in a straight line with retardation $2x$, where $x$ is the displacement in SI units. Its loss of kinetic energy for above displacement is $\left(\frac{10}{x}\right)^{-n}$ J. The value of $n$ will be ___________.

**Answer (2)**

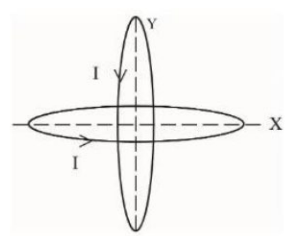
**Sol.** $vdv = -adx$

$\left|\frac{1}{2}m\left(v_f^2 - v_i^2\right)\right| = m2\left(\frac{x^2}{2}\right) = mx^2$

$\Rightarrow |\Delta KE| = 0.01x^2$

$= \left(\frac{10}{x}\right)^{-2}$

54. Two identical circular wires of radius 20 cm and carrying current $\sqrt{2}$ A are placed in perpendicular planes as shown in figure. The net magnetic field at the centre of the circular wires is ________ $\times 10^{-8}$ T.




(Take $\pi = 3.14$)

**Answer (628)**

**Sol.** $B_{\text{net}} = \frac{\sqrt{2} \times 4\pi \times 10^{-7} \times \sqrt{2}}{2 \times 0.2}$

$= 2\pi \times 10^{-8}$ T

$= 6.28 \times 10^{-6}$ T

55. A person driving car at a constant speed of 15 m/s is approaching a vertical wall. The person notices a change of 40 Hz in the frequency of his car's horn upon reflection from the wall. The frequency of horn is ________ Hz.

(Given: Speed of sound : 30 m/s)

**Answer (420)**

**Sol.** $f_{app} = \left(\frac{330+15}{330-15}\right)f$

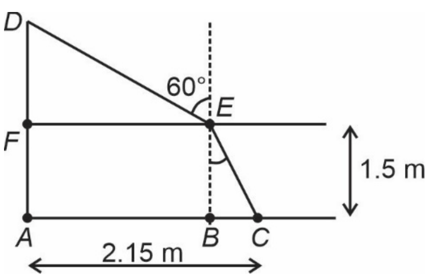
$\Rightarrow \left(\frac{345}{315}-1\right)f = 40$

$\Rightarrow f = \frac{40\times 315}{30} = 420$

56. A pole is vertically submerged in swimming pool, such that it gives a length of shadow 2.15 m within water when sunlight is incident at an angle of 30° with the surface of water. If swimming pool is filled to a height of 1.5 m, then the height of the pole above the water surface in centimeters is ($n_w = 4/3$) ________

**Answer (50)**

**Sol.**




$\sin 60° = \sin\theta \times \frac{4}{3}$

$\sin\theta = \frac{3\sqrt{3}}{8} \quad \Rightarrow \quad \tan\theta = \frac{3\sqrt{3}}{\sqrt{37}}$

$BC = 1.5 \times \tan\theta$

$AB = 2.15 - BC = 0.8686$ m

$DF = AB \tan 30° = 0.5015 \text{ m} \approx 50$ cm

57. A steel rod has a radius of 20 mm and a length of 2.0 m. A force of 62.8 kN stretches it along its length. Young's modulus of steel is $2.0 \times 10^{11}$ N/m$^2$. The longitudinal strain produced in the wire is _____ $\times 10^{-5}$

**Answer (25)**

**Sol.** Strain = $\frac{62.8\times 10^3}{3.14\times(0.02)^2} \times \frac{1}{2\times 10^{11}}$

$= \frac{2\times 10^4}{8\times 10^7} = 25\times 10^{-5}$

58. An ideal transformer with purely resistive load operates at 12 kV on the primary side. It supplies electrical energy to a number of nearby houses at 120 V. The average rate of energy consumption in the houses served by the transformer is 60 kW. The value of resistive load (Rs) required in the secondary circuit will be_______ mΩ.

**Answer (240)**

**Sol.** $P = 60 \times 10^3$ W

$\frac{V^2}{R} = 60\times 10^3$ W

$R = \frac{120\times 120}{60\times 10^3}\Omega$ = 240 mΩ

59. The length of a metallic wire is increased by 20% and its area of cross section is reduced by 4%. The percentage change in resistance of the metallic wire is _________.

**Answer (25)**

**Sol.** $R = \frac{\rho l}{A}$ and $R' = \frac{1.2\rho l}{0.96A}$

$R' = 1.25R$

$\Rightarrow \Delta R = 0.25R$

$\frac{\Delta R}{R} = 25\%$

60. Two identical solid spheres each of mass 2 kg and radii 10 cm are fixed at the ends of a light rod. The separation between the centres of the spheres is 40 cm. The moment of inertia of the system about an axis perpendicular to the rod passing through its middle point is ______ $\times 10^{-3}$ kg-m$^2$.

**Answer (176)**

**Sol.** $I = 2\left(2\times(0.2)^2\right) + 2\left(\frac{2}{5}\right)\left(2\times(0.1)^2\right)$

$= 0.16 + 0.016$

$I = 0.176$

$176 \times 10^{-3}$ kgm$^2$

# CHEMISTRY

## SECTION - A

**Multiple Choice Questions:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

**Choose the correct answer:**

61. Which of the following options are correct for the reaction?

$2[Au(CN)_2]^-(aq) + Zn(s) \rightarrow 2Au(s) + [Zn(CN)_4]^{2-}(aq)$

A. Redox reaction

B. Displacement reaction

C. Decomposition reaction

D. Combination reaction

Choose the correct answer from the options given below:

(1) A only (2) A and D only

(3) A and B only (4) C and D only

**Answer (3)**

**Sol.** (A) Oxidation state of zinc is changing from 0 to +2.

(B) Zinc is displacing Au from complex.

62. The setting time of Cement is increased by adding

(1) Clay (2) Silica

(3) Gypsum (4) Limestone

**Answer (3)**

**Sol.** Gypsum is added to slow down the process of setting of the cement so that it gets sufficiently hardened.

**Ref.:** NCERT Class XI_Page No. 312_(s-Block Elements)

63. A compound is formed by two elements X and Y. The element Y forms cubic close packed arrangement and those of element X occupy one third of the tetrahedral voids. What is the formula of the compound?

(1) $X_2Y_3$

(2) $X_3Y_2$

(3) $X_3Y$

(4) $XY_3$

**Answer (1)**

**Sol.** $Y_4X_{\frac{8}{3}}$

$Y_4X_{\frac{8}{3}}$

$\Rightarrow Y_{12}X_8$

$\Rightarrow X_2Y_3$

64. The standard electrode potential of $M^+/M$ in aqueous solution does not depend on

(1) Hydration of a gaseous metal ion

(2) Sublimation of a solid metal

(3) Ionisation of a solid metal atom

(4) Ionisation of a gaseous metal atom

**Answer (3)**

**Sol.** Ionisation energy is defined for gaseous atom and not solid atom.

65. Match List I and List II

| LIST I<br>Vitamin | | LIST II<br>Deficiency disease | |
|---|---|---|---|
| A. | Vitamin A | I. | Beri-Beri |
| B. | Thiamine | II. | Cheilosis |
| C. | Ascorbic acid | III. | Xerophthalmia |
| D. | Riboflavin | IV. | Scurvy |

Choose the correct answer from the options given below

(1) A-III, B-I, C-IV, D-II

(2) A-IV, B-I, C-III, D-II

(3) A-IV, B-II, C-III, D-I

(4) A-III, B-II, C-IV, D-I

**Answer (1)**

**Sol.** Vitamin A : Xerophthalmia

Thiamine : Beri Beri

Ascorbic acid : Scurvy

Riboflavin : Cheilosis

66. The difference between electron gain enthalpies will be maximum between :

(1) Ne and F  (2) Ar and F

(3) Ne and Cl  (4) Ar and Cl

**Answer (3)**

**Sol.** $\Delta H_{eg}$ for chlorine = –349 kJ $mole^{-1}$

$\Delta H_{eg}$ for Neon = +116 kJ $mole^{-1}$

67. The possibility of photochemical smog formation is more at

(1) Marshy lands

(2) Industrial areas

(3) Himalayan villages in winter

(4) The places with healthy vegetation

**Answer (2)**

**Sol.** Photochemical smog formation will be more at Industrial areas.

68. Match List I with List II

| List I<br>Oxide | | List II<br>Type of bond | |
|---|---|---|---|
| A. | $N_2O_4$ | I. | 1 N = O bond |
| B. | $NO_2$ | II. | 1 N – O –N bond |
| C. | $N_2O_5$ | III. | 1 N – N bond |
| D. | $N_2O$ | IV. | 1 N = N / N ≡ N bond |

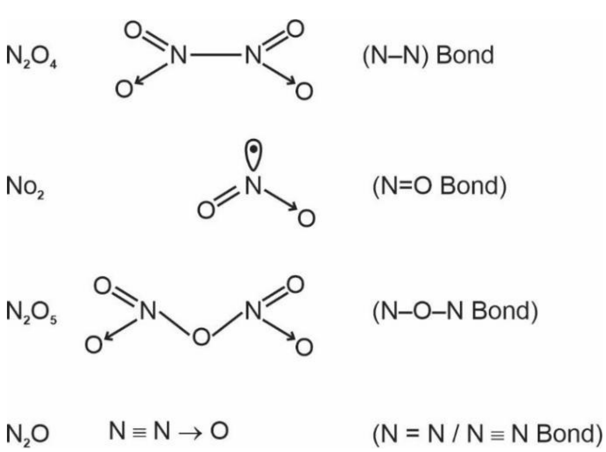
Choose the correct answer from the options given below :

(1) A-III, B-I, C-II, D-IV  (2) A-II, B-IV, C-III, D-I

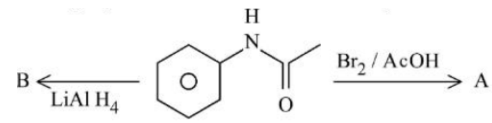
(3) A-III, B-I, C-IV, D-II  (4) A-II, B-I, C-III, D-IV

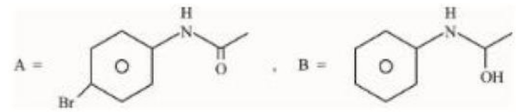
**Answer (1)**

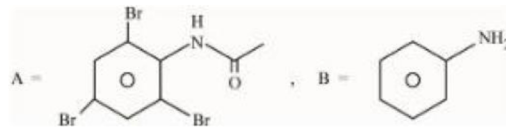
**Sol.**

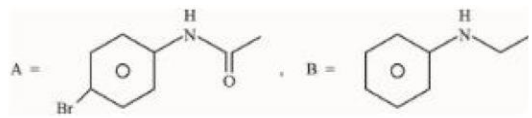


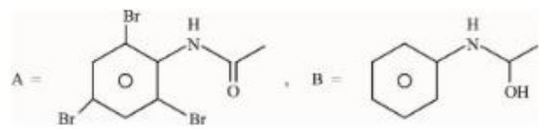

69. The major products A and B from the following reactions are :




(1) 

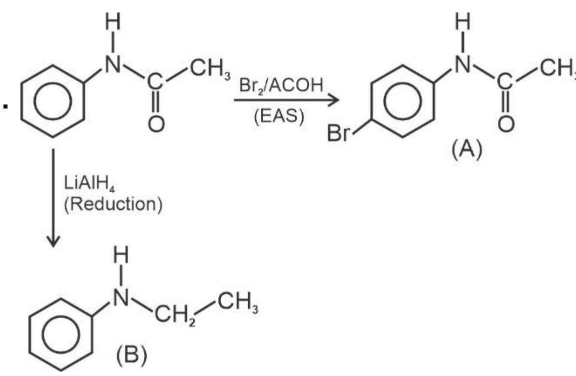
(2) 

(3) 

(4) 

**Answer (3)**

**Sol.**




70. Strong reducing and oxidizing agents among the following , respectively, are

(1) $Ce^{3+}$ and $Ce^{4+}$

(2) $Ce^{4+}$ and $Tb^{4+}$

(3) $Ce^{4+}$ and $Eu^{2+}$

(4) $Eu^{2+}$ and $Ce^{4+}$

**Answer (4)**

**Sol.** $E^{\circ}_{Ce^{4+}|Ce^{3+}} = +1.74\,V$

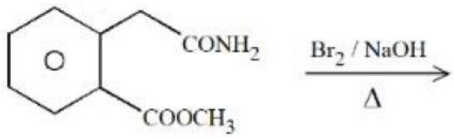
Lanthanoids have +3 oxidation state as stable oxidation state

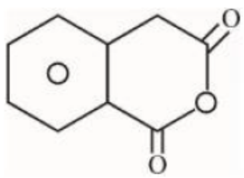
+4 oxidation state : Oxidising agents

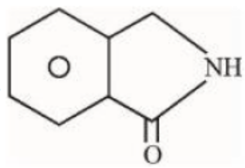
+2 oxidation state : Reducing agents

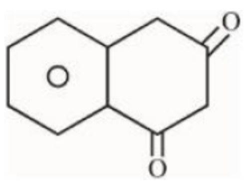
(Reference : NCERT class XII^th page 235)

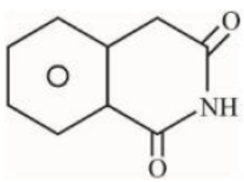
71. The major product formed in the following reaction is



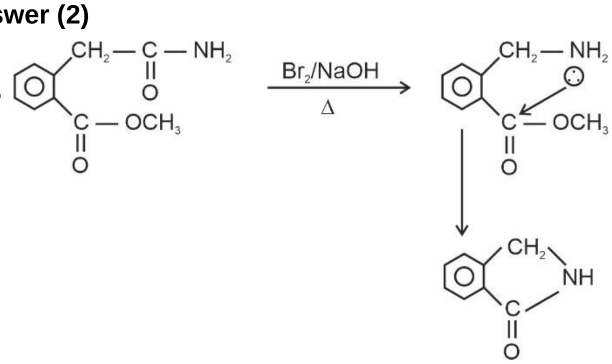

(1) 

(2) 

(3) 

(4) 

**Answer (2)**

**Sol.** 


72. For the reaction

$$RCH_2Br + I^- \xrightarrow{\text{Acetone}} \underset{\text{major}}{RCH_2I} + Br^-$$

The correct statement is

(1) $Br^-$ can act as competing nucleophile.

(2) The reaction can occur in acetic acid also.

(3) The transition state formed in the above reaction is less polar than the localised anion.

(4) The solvent used in the reaction solvates the ions formed in rate determining step

**Answer (3)**

**Sol.** $R-CH_2-Br+I^- \xrightarrow{\text{Acetone}} R-CH_2-I+Br^-$

NaBr is not soluble in acetone and hence reaction shifts in forward reaction and reaction is called Finkelstein reaction which proceeds through $S_N2$ mechanism

For $S_N2$ reactions, transition state formed is less polar than the localised anion.

73. Polymer used in orlon is:

(1) Polyethene

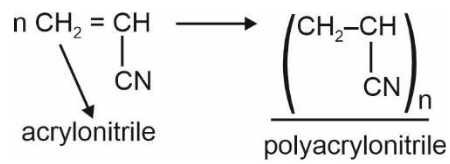
(2) Polycarbonate

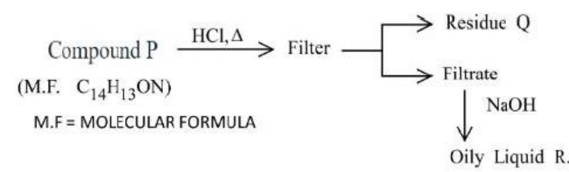
(3) Polyamide

(4) Polyacrylonitrile

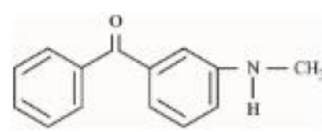
**Answer (4)**

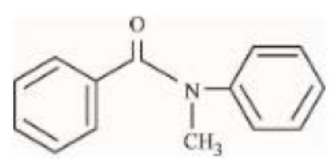
**Sol.** Orlon is polyacrylonitrile

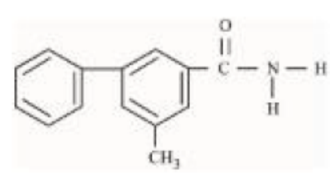


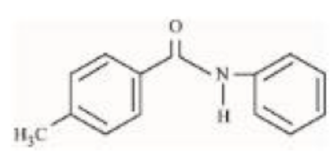

74. 


Compound P is neutral, Q gives effervescence with $NaHCO_3$ while R reacts with Hinsberg's reagent to give solid soluble in NaOH. Compound P is

(1) 

(2) 

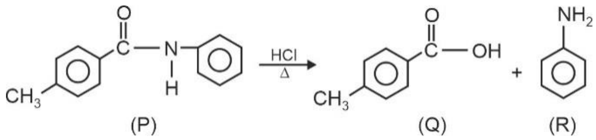
(3) 

(4) 

**Answer (4)**

**Sol.** R is 1° Amine

Q is Carboxylic Acid




75. Match List I with List II

| LIST I – Enzymatic reaction | | LIST II - Enzyme | |
|---|---|---|---|
| A. | Sucrose → Glucose and Fructose | I. | Zymase |
| B. | Glucose → ethyl alcohol and $CO_2$ | II. | Pepsin |
| C. | Starch → Maltose | III. | Invertase |
| D. | Proteins → Amino acids | IV. | Diastase |

Choose the correct answer from the options given below.

(1) A-I, B-II, C-IV, D-III

(2) A-III, B-I, C-IV, D-II

(3) A-III, B-I, C-II, D-IV

(4) A-I, B-IV, C-III, D-II

**Answer (2)**

**Sol.** (A) Sucrose $\xrightarrow{\text{Invertase}}$ Glucose + Fructose

(B) Glucose $\xrightarrow{\text{Zymase}}$ Ethyl alcohol and $CO_2$

(C) Starch $\xrightarrow{\text{Diastase}}$ Maltose

(D) Proteins $\xrightarrow{\text{Pepsin}}$ Amino Acids

76. For a concentrated solution of a weak electrolyte ($K_{eq}$ = equilibrium constant) $A_2B_3$ of concentration 'c', the degree of dissociation 'α' is

(1) $\left(\frac{K_{eq}}{5c^4}\right)^{\frac{1}{5}}$

(2) $\left(\frac{K_{eq}}{108c^4}\right)^{\frac{1}{5}}$

(3) $\left(\frac{K_{eq}}{25c^2}\right)^{\frac{1}{5}}$

(4) $\left(\frac{K_{eq}}{6c^5}\right)^{\frac{1}{5}}$

**Answer (2)**

**Sol.**

$$\begin{array}{cccc} A_2B_3 & \rightleftharpoons & 2A^{3+} & + \; 3B^{2-} \\ C & & - & - \\ \downarrow & & \downarrow & \downarrow \\ C(1-\alpha) & & 2C\alpha & 3C\alpha \end{array}$$

$$K_{eq} = \frac{(2C\alpha)^2(3C\alpha)^3}{C(1-\alpha)} = 108C^4\alpha^5$$

$$\frac{K_{eq}}{108C^4} = \alpha^5 \Rightarrow \alpha = \left(\frac{K_{eq}}{108C^4}\right)^{\frac{1}{5}}$$

77. Match List-I with List-II.

| **List-I Element detected** | | **List-II Reagent used/Product formed** | |
|---|---|---|---|
| A. | Nitrogen | I. | $Na_2[Fe(CN)_5NO]$ |
| B. | Sulphur | II. | $AgNO_3$ |
| C. | Phosphorus | III. | $Fe_4[Fe(CN)_6]_3$ |
| D. | Halogen | IV | $(NH_4)_2MoO_4$ |

Choose the correct answer from the options given below:

(1) A → III; B → I; C → IV; D → II

(2) A → II; B → IV; C → I; D → III

(3) A → IV; B → II; C → I; D → III

(4) A → II; B → I; C → IV; D → III

**Answer (1)**

**Sol.** Nitrogen : $Fe_4[Fe(CN)_6]_3$ (Prussian Blue)

Sulphur : $Na_2[Fe(CN)_5NO]$

Phosphorous : $(NH_4)_2MoO_4$

Halogen : $AgNO_3$

78. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A:** The spin only magnetic moment value for $[Fe(CN)_6]^{3-}$ is 1.74 BM, whereas for $[Fe(H_2O)_6]^{3+}$ is 5.92 BM.

**Reason B:** In both complexes, Fe is present in +3 oxidation state.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R are true but R is NOT the correct explanation of A

(4) Both A and R are true and R is the correct explanation of A

**Answer (3)**

**Sol.** → $CN^-$ is strong field ligand and hence pairing will take place.

→ $H_2O$ is a weak field ligand and hence pairing will not take place.

79. Match List-I with List-II.

| List-I<br>Name of reaction | | List-II<br>Reagent used | |
|---|---|---|---|
| A. | Hell-Volhard-Zelinsky reaction | I. | NaOH + $I_2$ |
| B. | Iodoform reaction | II. | (i) $CrO_2Cl_2$, $CS_2$<br>(ii) $H_2O$ |
| C. | Etard reaction | III. | (i) $Br_2$/red phosphorus<br>(ii) $H_2O$ |
| D. | Gatterman-Koch reaction | IV | CO, HCl, anhyd. $AlCl_3$ |

Choose the correct answer from the options given below:

(1) A → III; B → I; C → II; D → IV

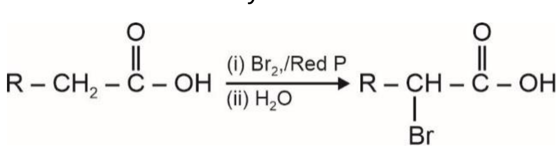
(2) A → I; B → II; C → III; D → IV

(3) A → III; B → II; C → I; D → IV

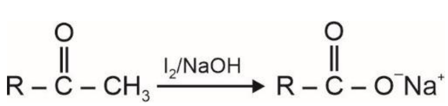
(4) A → III; B → I; C → IV; D → II

**Answer (1)**

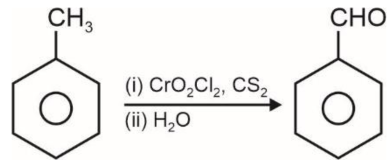
**Sol.** (A) Hell-Volhard-Zelinsky reaction:



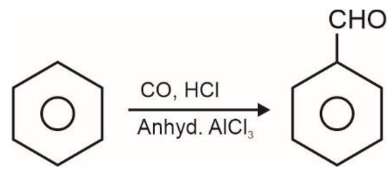

(B) Iodoform reaction:




(C) Etard reaction:




(D) Gatterman-Koch reaction




80. Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.

**Assertion A:** Loss of electron from hydrogen atom results in nucleus of ~$1.5 \times 10^{-3}$ pm size.

**Reason R**: Proton ($H^+$) always exists in combined form.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct and R is the correct explanation of A

(2) A is correct but R is not correct

(3) A is not correct but R is correct

(4) Both A and R are correct but R is NOT the correct explanation of A

**Answer (4)**

**Sol.** Size of nucleus is of order

$1.5 \times 10^{-15}$ m or $1.5 \times 10^{-3}$ pm

$H^+$ always exists in combined form. There is no relation between two statements and hence option (4) is the answer.

## SECTION - B

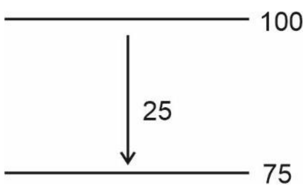
**Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Mass of Urea ($NH_2CONH_2$) required to be dissolved in 1000 g of water in order to reduce the vapour pressure of water by 25% is _______ g. (Nearest integer)

Given : Molar mass of N, C, O and H are 14, 12, 16 and 1 g $mol^{-1}$ respectively.

**Answer (1111)**

**Sol.**




$$\frac{25}{75} = \frac{\text{moles of urea}}{\left(\frac{1000}{18}\right)}$$

$$\frac{1}{3} \times \frac{1000}{18} = \text{moles of urea}$$

Mass of urea = $\frac{1}{3} \times \frac{1000}{18} \times 60$

= 1111.11 gm

82. The value of log K for the reaction $A \rightleftharpoons B$ at 298 K is _______. (Nearest integer)

Given : $\Delta H° = -54.07$ kJ $mol^{-1}$

$\Delta S° = 10$ $JK^{-1}mol^{-1}$

(Take 2.303 × 8.314 × 298 = 5705)

**Answer (10)**

**Sol.** $\Delta G° = \Delta H° - T\Delta S°$

$$= -54.07 - \frac{298(10)}{1000}$$

= –57.05 kJ/mole

$\Delta G° = -2.303 \, RT \log K_{eq}$

$-57.05 \times 1000 = -2.303 \times 8.314 \times 298 \log K_{eq}$

$-57.05 \times 1000 = -5705 \log K_{eq}$

$10 = \log K_{eq}$

83. In ammonium – phosphomolybdate, the oxidation state of Mo is + _______

**Answer (6)**

**Sol.** Ammonium phosphomolybdate is

$(NH_4)_3PO_4 \cdot 12\,MoO_3$

$MoO_3$
↓
(+6)

84. Number of bromo derivatives obtained on treating ethane with excess of $Br_2$ in diffused sunlight is _________

**Answer (9)**

**Sol.** 6 Bromine atoms : 1 product possible

5 Bromine atoms : 1 product possible

4 Bromine atoms : 2 products possible

3 Bromine atoms : 2 products possible

2 Bromine atoms : 2 products possible

1 Bromine atom : 1 product possible

Total 9 products possible

85. For the adsorption of hydrogen on platinum, the activation energy is 30 kJ $mol^{-1}$ and for the adsorption of hydrogen on nickel, the activation energy is 41.4 kJ $mol^{-1}$. The logarithm of the ratio of the rates of chemisorption on equal areas of the metals at 300 K is ______ (Nearest integer)

Given : ln 10 = 2.3

R = 8.3 J $K^{-1}$ $mol^{-1}$

**Answer (2)**

**Sol.** $\log\left(\frac{K_2}{K_1}\right) = \frac{\Delta E}{2.3\,RT}$

$$= \frac{11.4 \times 1000}{2.3 \times 8.3 \times 300}$$

$$= 1.990 \approx 2$$

86. The number of species from the following which have square pyramidal structure is______

$PF_5$, $BrF_4^-$, $IF_5$, $BrF_5$, $XeOF_4$, $ICl_4^{-4}$

**Answer (3)**

**Sol.** $IF_5$; $BrF_5$ and $XeOF_4$ have square pyramidal structure.

87. If 5 moles of $BaCl_2$ is mixed with 2 moles of $Na_3PO_4$, the maximum number of moles of $Ba_3(PO_4)_2$ formed is______ (Nearest integer)

**Answer (1)**

**Sol.** $3BaCl_2 + 2Na_3PO_4 \longrightarrow Ba_3(PO_4)_2 + 6NaCl$

| $BaCl_2$ | $Na_3PO_4$ | $Ba_3(PO_4)_2$ |
|---|---|---|
| 5 | 2 | (–) |
| ↓ 3 | ↓ | ↓ |
| 2 | 0 | 1 mole |

mole of $Ba_3(PO_4)_2$ formed = 1

88. Number of ambidentate ligands in a representative metal complex $[M(en)(SCN)_4]$ is_____________.
[en = ethylenediamine]

**Answer (4)**

**Sol.** 4 $SCN^-$ ligands are ambidentate.

89. The wavelength of an electron of kinetic energy $4.50 \times 10^{-29}$ J is _________ $\times 10^{-5}$ m. (Nearest integer) Given: mass of electron is $9 \times 10^{-31}$ kg, $h = 6.6 \times 10^{-34}$ Js

**Answer (7)**

**Sol.** $\lambda = \frac{h}{\sqrt{2mKE}}$

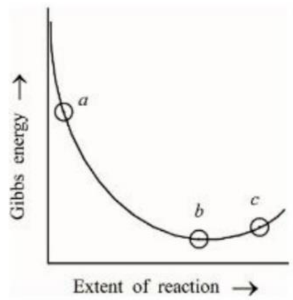
$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9 \times 10^{-31} \times 4.5 \times 10^{-29}}}$$

$$= \frac{6.6 \times 10^{-34}}{9 \times 10^{-30}} = \frac{6.6}{9} \times 10^{-4}$$

$$= \frac{66}{9} \times 10^{-5}$$

$$= 7.33 \times 10^{-5}$$

90. Consider the graph of Gibbs free energy G vs extent of reaction. The number of statement/s from the following which are true with respect to points (a), (b) and (c) is_______




A. Reaction is spontaneous at (a) and (b)
B. Reaction is at equilibrium at point (b) and non-spontaneous at point (c)
C. Reaction is spontaneous at (a) and non-spontaneous at (c)
D. Reaction is non-spontaneous at (a) and (b)

**Answer (2)**

**Sol.** At point a : Slope = –ve

dG = –ve

⇒ spontaneous

At point b : Slope = 0

dG = 0

⇒ equilibrium

At point c: Slope = +ve

dG = +ve

⇒ non-spontaneous

❑ ❑ ❑