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13/04/2023
Morning

Answers & Solutions

for

JEE (Main)-2023 (Online) Phase-2

(Mathematics, Physics and Chemistry)

Time : 3 hrs. **M.M. : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Mathematics, Physics** and **Chemistry** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The number of symmetric matrices of order 3, with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} is

   (1) 6^{10}

   (2) 10^6

   (3) 9^{10}

   (4) 10^9

Answer (2)

Sol. A = {0, 1, 2, 3, …8, 9}

$$M = \begin{bmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{bmatrix}$$

$\therefore$ Total symmetric matrix = 10^6

2. Let $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$, $\alpha > 2$ be the adjoint of a matrix A and $|A| = 2$. Then $\begin{bmatrix} \alpha - 2\alpha & \alpha \end{bmatrix} B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$ is equal to

   (1) 0 (2) 16

   (3) –16 (4) 32

Answer (3)

Sol. $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$, $\alpha > 2$

And adj $(A) = B$, $|A| = 2$

$\Rightarrow$ $|\text{adj}(A)| = |B|$

$\Rightarrow$ $2^2 = (8 - 3\alpha) - 3(4 - 3\alpha) + \alpha(-\alpha)$

$\Rightarrow$ $\alpha^2 - 6\alpha + 8 = 0$

$\Rightarrow$ $(\alpha - 4)(\alpha - 2) = 0$

$\alpha = 4, 2$ but $\alpha > 2$ so $\boxed{\alpha = 4}$

Now

$$\begin{bmatrix} \alpha - 2\alpha & \alpha \end{bmatrix} B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix} = \begin{bmatrix} 4 - 8 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 12 & 8 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 48 - 96 + 32 \end{bmatrix} = -16$$

3. Among

$$(S1): \lim_{n\to\infty} \frac{1}{n^2}(2 + 4 + 6 + ... + 2n) = 1$$

$$(S2): \lim_{n\to\infty} \frac{1}{n^{16}}\left(1^{15} + 2^{15} + 3^{15} + ... + n^{15}\right) = \frac{1}{16}$$

   (1) Both ($S1$) and ($S2$) are true

   (2) Only ($S1$) is true

   (3) Both ($S1$) and ($S2$) are false

   (4) Only ($S2$) is true

Answer (1)

Sol. $S_1: \lim_{n\to\infty} \frac{1}{n^2}[2 + 4 + 6 + ... + 2n]$

$$\lim_{n\to\infty} 2\frac{n(n+1)}{2n^2} = 1$$

$$S_2: \lim_{n\to\infty} \frac{1}{n^{16}}\left(1^{15} + 2^{15} + 3^{15} + ... + n^{15}\right)$$

$\because$ $\lim\limits_{n\to\infty} \dfrac{\sum\limits_{r=1}^{n} r^k}{n^{k+1}} = \dfrac{1}{k+1}$

Hence $k = 15$

$\therefore$ $\lim\limits_{n\to\infty} \dfrac{\sum\limits_{r=1}^{n} r^{15}}{n^{16}} = \dfrac{1}{16}$

$\therefore$ Both S_1 and S_2 are correct

4. Fractional part of the number $\frac{4^{2022}}{15}$ is equal to

(1) $\frac{8}{15}$ (2) $\frac{4}{15}$

(3) $\frac{14}{15}$ (4) $\frac{1}{15}$

Answer (4)

Sol. $\frac{4^{2022}}{15}$

$\because\ 4 \equiv 4 \pmod{15}$

$4^2 \equiv 1 \pmod{15}$

$4^{2022} \equiv 1 \pmod{15}$

$\therefore$ Fractional part of $\frac{4^{2022}}{15} = \frac{1}{15}$

5. $\max\limits_{0\le x\le\pi}\left\{x - 2\sin x\cos x + \frac{1}{3}\sin 3x\right\} =$

(1) $\frac{\pi + 2 - 3\sqrt{3}}{6}$ (2) π

(3) 0 (4) $\frac{5\pi + 2 + 3\sqrt{3}}{6}$

Answer (4)

Sol. $f'(x) = 1 - 2\cos 2x + \cos 3x$

$f''(x) = 4\sin 2x - 3\sin 3x$

$f'(x) = 0$

$\Rightarrow\ 1 - 2\left(2\cos^2 x - 1\right) + 4\cos^3 x - 3\cos x = 0$

$\left(2\cos x + \sqrt{3}\right)\left(2\cos x - \sqrt{3}\right)(\cos x - 1) = 0$

$\cos x = \frac{-\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 1$

$x = \frac{5\pi}{6}, \frac{\pi}{6}, 0$

$f''\left(\frac{5\pi}{6}\right) = -2\sqrt{3} - \sqrt{3} < 0$

$f''\left(\frac{\pi}{6}\right) = 2\sqrt{3} - \sqrt{3} > 0$

$f''(0) = 0$

So $x = \frac{5\pi}{6}$ is local maxima point

Maximum value of $f(x) = f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3}$

$= \frac{5\pi + 2 + 3\sqrt{3}}{6}$

6. The negation of the statement $\big((A \wedge (B \vee C))$ $\Rightarrow (A \vee B)) \Rightarrow A$ is

(1) equivalent to $\sim C$

(2) equivalent to $B \vee \sim C$

(3) a fallacy

(4) equivalent to $\sim A$

Answer (4)

Sol. $\big(A \wedge (B \vee C)\big) \Rightarrow (A \vee B) \Rightarrow A$

$\sim\big(\sim\big(A \wedge (B \vee C)\big) \vee (A \vee B)\big) \vee A$

$\big(A \wedge (B \vee C)\big) \wedge \sim (A \vee B) \vee A$

$= A$

$\therefore$ Negation of statement = $\sim A$

7. For $x \in \mathbb{R}$, two real valued functions $f(x)$ and $g(x)$ are such that, $g(x) = \sqrt{x} + 1$ and $fog(x) = x + 3 - \sqrt{x}$. Then $f(0)$ is equal to

(1) 1 (2) 5

(3) 0 (4) –3

Answer (2)

Sol. $f\left(g(x)\right) = x + 3 - \sqrt{x}$

$f\left(\sqrt{x} + 1\right) = x + 3 - \sqrt{x}$

Put $\sqrt{x} + 1 = t$

$\sqrt{x} = t - 1$

$x = (t - 1)^2$

$f(t) = (t - 1)^2 + 3 - (t - 1)$

$f(0) = 1 + 3 + 1 = 5$

8. Let $s_1, s_2, s_3 ...., s_{10}$ respectively be the sum of 12 terms of 10 A.Ps whose first terms are 1, 2, 3,....,10 and the common differences are 1, 3, 5,...,19 respectively. Then $\sum_{i=1}^{10} s_i$ is equal to

(1) 7220

(2) 7360

(3) 7260

(4) 7380

Answer (3)

Sol. $1 \le r \le 10$

$$S_i = \frac{12}{2}\left[2r + (12-1)(2r-1)\right] = 6\left(2r + 22r - 11\right)$$

$= 6(24r - 11)$

$$\sum_{r=1}^{10} S_r = 6\sum_{r=1}^{10}(24r - 11) = 6 \times \frac{10}{2}\left[24 - 11 + 240 - 11\right]$$

$= 30\ (242) = 7260$

9. Let the equation of plane passing through the line of intersection of the planes $x + 2y + az = 2$ and $x - y + z = 3$ be $5x - 11y + bz = 6a - 1$. For $c \in \mathbb{Z}$, if the distance of this plane from the point $(a, -c, c)$ is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

(1) 2 (2) 4

(3) – 4 (4) – 2

Answer (3)

Sol. Let the equation of plane passing through the intersection of two given planes:

$x + 2y + az - 2 + \lambda(x - y + z - 3) = 0$

$\Rightarrow\ x(\lambda + 1) + y(2 - \lambda) + z(a + \lambda) - 2 - 3\lambda = 0$

This is same as $5x - 11y + bz = 6a - 1$

$$\frac{\lambda+1}{5} = \frac{2-\lambda}{-11} = \frac{a+\lambda}{b} = \frac{2+3\lambda}{6a-1}$$

$\Rightarrow\ -11\lambda - 11 = 10 - 5\lambda$

$\Rightarrow\ 6\lambda = -21 \Rightarrow \lambda = \frac{-7}{2}$

$$\frac{2-\lambda}{-11} = \frac{2+3\lambda}{6a-1} \Rightarrow \frac{2+\frac{7}{2}}{-11} = \frac{2-\frac{21}{2}}{6a-1}$$

$$\Rightarrow\ -\frac{1}{2} = \frac{-17}{2(6a-1)} \Rightarrow 6a - 1 = 17 \Rightarrow a = 3$$

$$\frac{2-\lambda}{-11} = \frac{a+\lambda}{b} \Rightarrow -\frac{1}{2} = \frac{3-\frac{7}{2}}{b}$$

$$\Rightarrow\ -\frac{b}{2} = -\frac{1}{2} \Rightarrow b = 1$$

$\therefore$ point $(a, -c, c) \equiv (3, -c, c)$

Distance $= \frac{2}{\sqrt{a}} = \frac{2}{\sqrt{3}}$

Plane: $5x - 11y + z = 17$

$$\left|\frac{15 + 11c + c - 17}{\sqrt{147}}\right| = \frac{2}{\sqrt{3}}$$

$\Rightarrow\ |12c - 2| = 14$

$\Rightarrow\ c = -1,\ \frac{4}{3}$

$c = -1 \qquad (c \in \mathbb{Z})$

$\therefore\ \frac{a+b}{c} = \frac{3+1}{-1} = -4$

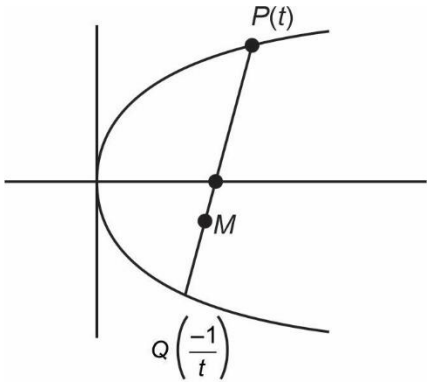
10. Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that $PM : MQ = 3 : 1$. Then which of the following points does **NOT** lie on the line passing through M and perpendicular to the line PQ?

(1) (–6, 45) (2) (6, 29)

(3) (3, 33) (4) (–3, 43)

Answer (4)

Sol.



Length of focal chord at $(t) = a\left(t + \frac{1}{t}\right)^2 = 100$

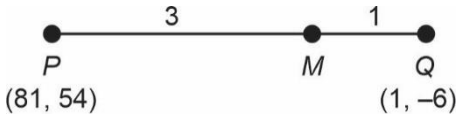
Where $a = 9$

$t + \frac{1}{t} = \pm\frac{10}{3}$

$\therefore \quad t = 3, \frac{1}{3}, -3, \frac{-1}{3}$

Since ordinate of P is +ve

$\therefore \quad t = 3$ or $\frac{1}{3}$



($t = 3$ as slope of PQ is positive)

$\therefore \quad M(21, 9)$

Required line : $y - 9 = \frac{-80}{60}(x - 21)$

$\Rightarrow \quad 4x + 3y = 111$

(–3, 43) does not lie on line

$\therefore$ Option (4) is correct.

11. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d} \cdot \vec{a} = 24$, then $\left|\vec{d}\right|^2$ is equal to

(1) 323

(2) 423

(3) 313

(4) 413

Answer (4)

Sol. $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$

$\Rightarrow \quad \left(\vec{d} - \vec{c}\right) \times \vec{b} = 0$

$\Rightarrow \quad \vec{d} - \vec{c} = \lambda\vec{b} \qquad \left(\because \vec{d} \neq \vec{c} \text{ as } \vec{c} \cdot \vec{a} = b\right)$

$\Rightarrow \quad \vec{d} = \vec{c} + \lambda\vec{b}$

$\vec{a} \cdot \vec{d} = 24 \qquad$ (given)

$\vec{a} \cdot \vec{c} + \lambda\vec{b} \cdot \vec{a} = 24$

$\Rightarrow \quad 6 + \lambda(3 - 8 + 14) = 24$

$\Rightarrow \quad 9\lambda = 18$

$\Rightarrow \quad \lambda = 2$

$\therefore \quad \vec{d} = \vec{c} + 2\vec{b}$

$= 8\hat{i} - 5\hat{j} + 18\hat{k}$

$\left|\vec{d}\right|^2 = 64 + 25 + 324$

$= 413$

$\therefore$ Option (4) is correct.

12. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is

(1) $\frac{37}{16}$

(2) $\frac{15}{16}$

(3) $\frac{21}{16}$

(4) $\frac{81}{64}$

Answer (3)

Sol. $P(H) = 3P(T)$

$\therefore \quad P(H) = \frac{3}{4}, P(T) = \frac{1}{4}$

Since coin is tossed till either 1H or 3T occurs. So, process will end in the maximum of 3 throws.

Xi	1	2	3
$P(Xi)$	$\frac{3}{4}$	$\frac{1}{4}\cdot\frac{3}{4}$	$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}$

Mean $= 1\times\frac{3}{4} + 2\times\frac{3}{16} + 3\times\frac{1}{16}$

$= \frac{21}{16}$

Option (3) is correct.

13. The area of the region enclosed by the curve $f(x)=\max\{\sin x, \cos x\}, -\pi \le x \le \pi$ and the x-axis is

(1) $2\sqrt{2}\left(\sqrt{2}+1\right)$

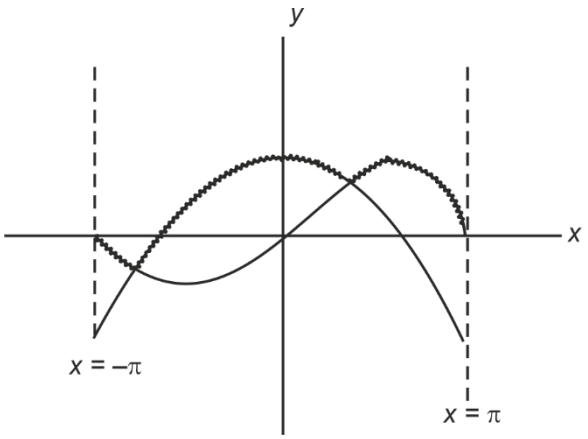
(2) 4

(3) $4\left(\sqrt{2}\right)$

(4) $2\left(\sqrt{2}+1\right)$

Answer (2)

Sol.



$$\left|\int_{-\pi}^{-3\pi/4}\sin x dx\right|+\left|\int_{\pi/4}^{\pi}\sin x dx\right|+\left|\int_{-3\pi/4}^{-\pi/2}\cos x\,dx\right|$$

$$+\left|\int_{-\pi/2}^{\pi/4}\cos x\,dx\right|$$

$$\left|\frac{1}{\sqrt{2}}-1\right|+\left|1+\frac{1}{\sqrt{2}}\right|+\left|-1+\frac{1}{\sqrt{2}}\right|+\left|\frac{1}{\sqrt{2}}+1\right|$$

$$2+\frac{2}{\sqrt{2}}+2-\frac{2}{\sqrt{2}}=4$$

14. For the differentiable function $f:\mathbb{R}-\{0\}-\mathbb{R}$, let $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$, then $\left|f(3)+f'\left(\frac{1}{4}\right)\right|$ is equal to

(1) $\frac{33}{5}$ (2) 7

(3) $\frac{29}{5}$ (4) 13

Answer (4)

Sol. $3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10$ ...(i)

$x \to \frac{1}{x}$

$3f\left(\frac{1}{x}\right)+2f(x)=x-10$ ...(ii)

From (i) & (ii)

$5f(x)=\frac{3}{x}-2x-10$

Now, $f(3) = -3$

$5f'(x)=\frac{-3}{x^2}-2$

$5f'\left(\frac{1}{4}\right)=-48-2$

$f'\left(\frac{1}{4}\right)=-10$

$\left|f(3)+f'\left(\frac{1}{4}\right)\right|=13$

15. Let the tangent and normal at the point $\left(3\sqrt{3},1\right)$ on the ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line $x=2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α, β), then $\alpha^2-\beta^2$ is equal to

(1) 61 (2) 60

(3) $\frac{304}{5}$ (4) $\frac{314}{5}$

Answer (3)

Sol. $\frac{x^2}{36}+\frac{y^2}{4}=1$

$T: \frac{3\sqrt{3}x}{36}+\frac{y}{4}=1$

$T: \frac{\sqrt{3}x}{12}+\frac{y}{4}=1$

$N: \frac{x-3\sqrt{3}}{\frac{3\sqrt{3}}{36}}=\frac{y-1}{\frac{1}{4}}$

$\frac{12x-36\sqrt{3}}{\sqrt{3}}=4y-4$

$3x-9\sqrt{3}=\sqrt{3}y-\sqrt{3}$

$N: 3x-\sqrt{3}y=8\sqrt{3}$

$A(0, 4)$ $B(0, -8)$

$C: x^2+(y-4)(y+8)=0$

Line $x=2\sqrt{5}$

$20+y^2+4y-32=0$

$y^2+4y-12=0$

$(y+6)(y-2)=0$

$P\left(2\sqrt{5}, -6\right)$ $Q\left(2\sqrt{5}, 2\right)$

$C: x^2+y^2+4y-32=0$

$P\left(2\sqrt{5}, -6\right)$ $Q\left(2\sqrt{5}, 2\right)$

$T: xx_1+yy_1+2y+2y_1-32=0$

$T_1: 2\sqrt{5}x-6y+2y-12-32=0$

$2\sqrt{5}x-4y=44$

$T_1: \sqrt{5}x-2y=22$ ...(i)

$T_2: 2\sqrt{5}x+2y+2y+4-32=0$

$2\sqrt{5}x+4y=28$

$T_2: \sqrt{5}x+2y=14$ ...(ii)

From (i) & (ii)

$\alpha=\frac{18}{\sqrt{5}}$ $\beta=-2$

$\alpha^2-\beta^2=\frac{304}{5}$

16. For the system of linear equations

$2x+4y+2az=b$

$x+2y+3z=4$

$2x-5y+2z=8$

which of the following is NOT correct?

(1) It has unique solution if $a=b=6$

(2) It has infinitely many solutions if $a=3$, $b=6$

(3) It has infinitely many solutions if $a=3$, $b=8$

(4) It has unique solution if $a=b=8$

Answer (2)

Sol. $2x+4y+2az=b$

$x+2y+3z=4$

$2x-5y+2z=8$

$\Delta=\begin{vmatrix}2 & 4 & 2a\\ 1 & 2 & 3\\ 2 & -5 & 2\end{vmatrix}=-18(a-3)$

$\Delta_3=\begin{vmatrix}b & 2 & 4\\ 4 & 1 & 2\\ 8 & 2 & -5\end{vmatrix}=9(b-8)$

If $a=3$ and $b=8$ we'll have infinite solution

For $a=8$, $b=8$, $\Delta\neq 0$, $\Delta_3=0\Rightarrow$ unique solution

For $a=b=6$, $\Delta\neq 0$, $\Delta_3\neq 0\Rightarrow$ unique solution

$\therefore$ For $a=3$, $b=6$ we'll have no solution

$\therefore$ Option 2 is incorrect

17. The distance of the point (–1, 2, 3) from the plane $\vec{r}\cdot\left(\hat{i}-2\hat{j}+3\hat{k}\right)=10$ parallel to the line of the shortest distance between the lines $\vec{r}=\left(\hat{i}-\hat{j}\right)+\lambda\left(2\hat{i}+\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}-\hat{j}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$ is

(1) $3\sqrt{6}$ (2) $2\sqrt{5}$

(3) $2\sqrt{6}$ (4) $3\sqrt{5}$

Answer (3)

Sol. Line of shortest distance is parallel to

$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 1\\ 1 & -1 & 1\end{vmatrix}=\hat{i}-\hat{j}-2\hat{k}$

Line passing through $A(-1, 2, 3)$ and having direction ratio (1, –1, –2) is

$\frac{x+1}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}=\lambda$

Let the line intersect the plane at P

$P\equiv\left(\lambda-1,\ -\lambda+2,\ -2\lambda+3\right)$

P lies on the plane $x-2y+3z=10$

$\therefore$ $(\lambda-1)-2(-\lambda+2)+3(-2\lambda+3)=10$

$\lambda=-2$

$P\equiv(-3,\ 4,\ 7)$

$AP=\sqrt{4+4+16}=2\sqrt{6}$

18. $\int_0^{\infty} \frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}dx =$

(1) $\log_e\left(\frac{32}{27}\right)$

(2) $\log_e\left(\frac{512}{81}\right)$

(3) $\log_e\left(\frac{256}{81}\right)$

(4) $\log_e\left(\frac{64}{27}\right)$

Answer (1)

Sol. Let $e^x = t \Rightarrow x = \ln t,\ dx = \frac{1}{t}dt$

$$I = \int_1^{\infty} \frac{6}{t(t+1)(t+2)(t+3)}dt$$

$$= \int_1^{\infty} \frac{1}{t} - \frac{3}{t+1} + \frac{3}{t+2} - \frac{1}{t+3}dt$$

$$= \ln(t) - 3\ln|t+1| + 3\ln|t+2| - \ln(t+3)\Big]_1^{\infty}$$

$$= \ln\left|\frac{t(t+2)^3}{(t+3)(t+1)^3}\right|\Bigg]_1^{\infty}$$

$$= -\ln\left(\frac{3^3}{4\cdot 2^3}\right) = \ln\left(\frac{32}{27}\right)$$

19. The set of all $a \in IR$ for which the equation $x|x-1|+|x+2|+a=0$ has exactly one real root, is

(1) $(-6, \infty)$

(2) $(-\infty, \infty)$

(3) $(-6, -3)$

(4) $(-\infty, -3)$

Answer (2)

Sol. $x\,|\,x-1\,|+|\,x+2\,|+a=0$

Case I : $x < -2$

$-x^2 + x - x - 2 + a = 0$

$a = x^2 + 2$

$y = x^2 + 2$ is decreasing $\forall\ x \in (-\infty, -2)$

Case II : $-2 \le x < 1$

$-x^2 + x + x + 2 + a = 0$

$a = x^2 - 2x - 2$

$y = x^2 - 2x - 2$ is decreasing $\forall\ x \in [-2, 1)$

Case III : $x \ge 1$

$x^2 - x + x + 2 + a = 0$

$a = -(x^2 + 2)$

$y = -(x^2 + 2)$ is decreasing $\forall\ x \in [1, \infty)$

$\therefore$ Exactly one real root $\forall\ x \in R$

20. Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7$ with initial conditions $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at

(1) no point

(2) two points

(3) one point

(4) infinite number of points

Answer (1)

Sol. $\frac{dy}{dx} = y + 7 \ \Rightarrow\ \frac{dy}{y+7} = dx$

$\Rightarrow\ \ln|\,y+7\,| = x + c$

$\Rightarrow\ |y+7| = k.e^x$

$\Rightarrow\ y = k.e^x - 7$

$y_1(0) = 0 \ \Rightarrow k = 7 \ \Rightarrow y_1(x) = 7(e^x - 1)$

$y_2(0) = 1 \Rightarrow 1 = k - 7 \Rightarrow 8 = k$

$\Rightarrow\ y_2(x) = 8e^x - 7$

$y_1(x) = y_2(x) \ \Rightarrow 8e^x - 7 = 7e^x - 7$

$\Rightarrow$ No point of intersection

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. The number of seven digit positive integers formed using the digits 1, 2, 3 and 4 only and sum of the digits equal to 12 is _____.

Answer (413)

Sol. $x_1 + x_2 + x_3 + ...x_7 = 12$

Number of solutions

= Coefficient of x^{12} in $(x^1 + x^2 + x^3 + x^4)^7$

= Coefficient of x^5 in $(1 + x + x^2 + x^3)^7$

= Coefficient of x^5 in $(1 - x^4)^7 (1 - x)^{-7}$

= Coefficient of x^5 in $(1 - 7x^4)(1 - x)^{-7}$

= Coefficient of x^5 in $(1 - 7x^4) \sum_{r=0}^{\infty} {}^{7+r-1}C_r \cdot x^r$

$= {}^{11}C_5 - 7 \times {}^{7}C_1$

$= 462 - 49 = 413$

22. Let m_1 and m_2 be the slopes of the tangents drawn from the point $P(4, 1)$ to the hyperbola $H : \frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to _____.

Answer (8)

Sol. Tangent : $y = mx + \sqrt{25 - 16m^2}$

Passes through (4, 1).

$1 = 4m + \sqrt{25 - 16m^2}$

$(4m - 1)^2 = 25 - 16m^2$

$4m^2 - m - 3$

$m_1 = 1, \; m_2 = \frac{-3}{4}$

$|m_1| = 1, \quad |m_2| = \frac{3}{4}$

Equation of tangents with slopes 1 & $\frac{17}{9}$

$y = x - 3$ …(I)

$y = \frac{3}{4}x - 4$ …(II)

(I) & (II) $\Rightarrow$ Q ≡ (–4, –7)

$P \equiv (4, 1)$

$(PQ)^2 = 8^2 + 8^2 = 128$

(I) $\Rightarrow \alpha = 3$, (II) $\Rightarrow \beta = \frac{16}{3}$

$\frac{(PQ)^2}{\alpha\beta} = \frac{128}{3 \times \frac{16}{3}} = 8$

23. Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} = \vec{b} \times \vec{c}$ and $|\vec{b}|^2 = 50$, then $\left|72 - |\vec{b} + \vec{c}|^2\right|$ is equal to _____.

Answer (66)

Sol. $|\vec{a}| = |\vec{b} \times \vec{c}|$

$= \sqrt{11} = |\vec{b}| . \sqrt{22} . \sin\theta$

$\therefore \quad \sin\theta = \frac{1}{10}$

or $\cos\theta = \frac{\sqrt{99}}{10}$

$\left|72 - |\vec{b} + \vec{c}|^2\right| = \left|72 - \left(50 + 22 + 2 \times 5\sqrt{2} . \sqrt{22} \frac{\sqrt{99}}{10}\right)\right|$

$= \left|72 - \left(72 + \frac{2 \times 5 \times 2 \times 11 \times 3}{10}\right)\right|$

= |66| = 66

24. Let the image of the point $\left(\frac{5}{3},\frac{5}{3},\frac{8}{3}\right)$ in the plane $x - 2y + z - 2 = 0$ be P. If the distance of the point $Q(6, -2, \alpha)$, $\alpha > 0$, from P is 13, then α is equal to _____.

Answer (15)

Sol. $\frac{x-\frac{5}{3}}{1} = \frac{y-\frac{5}{3}}{-2} = \frac{2-\frac{8}{3}}{1} = \frac{-2(-1)}{6}$

$(x, y, z) = (2, 1, 3)$

$PQ = 13$

$4^2 + 3^2 + (\alpha - 3)^2 = 169$

$|\alpha - 3| = 12$

$\alpha = 15$

25. The sum to 20 terms of the series $2\cdot2^2 - 3^2 + 2\cdot4^2 - 5^2 + 2\cdot6^2 - \ldots\ldots$ is equal to _____.

Answer (1310)

Sol. $\sum_{r=1}^{10}\left(2\cdot(2r)^2 - (2r+1)^2\right)$

$= \sum_{r=1}^{10}\left(8r^2 - 4r^2 - 4r - 1\right)$

$= \sum_{r=1}^{10}\left(4r^2 - 4r - 1\right)$

$= \frac{4\cdot10\cdot11\cdot21}{6} - 4\frac{10\cdot11}{2} - 10$

$= 44\cdot35 - 220 - 10$

$= 1540 - 230 = 1310$

26. Let $w = z\bar{z} + k_1 z + k_2 iz + \lambda(1+i), k_1, k_2 \in \mathbb{R}$. Let $\text{Re}(w) = 0$ be the circle C of radius 1 in the first quadrant touching the line $y = 1$ and the y-axis. If the curve $\text{Im}(w) = 0$ intersects C at A and B, then $30(AB)^2$ is equal to _____.

Answer (24)

Sol. $\text{Re}(z) = 0 \Rightarrow x^2 + y^2 + k_1 x - k_2 y + \lambda = 0$

$\text{Im}(z) = 0 \Rightarrow k_1 y + k_2 x + \lambda = 0$

Touching $y = 1$ and y-axis, with radius 1

we have equation of circle

$x^2 + y^2 - 2x - 4y + 4 = 0$

$k_1 = -2, k_2 = 4, \lambda = 4$

which intersects $-2y + 4x + 4 = 0$

at $(0, 2)$ and $\left(\frac{2}{5}, \frac{14}{5}\right)$

$30(AB)^2 = 24$

27. Let α be the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{6}{x^{\frac{3}{2}}}\right)^n$, $n \le 15$. If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is $\lambda\alpha$, then λ is equal to _____.

Answer (36)

Sol. $T_{r-1} = {}^nC_r \left(x^{\frac{1}{2}}\right)^{n-r}\left(-6x^{\frac{-3}{2}}\right)^r$

$= (-6)^r \cdot {}^nC_r \cdot x^{\frac{n-r}{2}} \cdot x^{\frac{-3r}{2}}$

$n = 4r$ for constant term so n is divisible by 4.

${}^nC_{\frac{n}{4}}(-6)^{\frac{n}{4}} = \alpha$

$(-5)^n - {}^nC_{\frac{n}{4}}(-6)^{n/4} = 649$

$n = 4$

$\alpha = -24$

For coefficient of x^{-4}

$\frac{n-4r}{2} = -4$

$n = 4r - 8 \Rightarrow \boxed{r = 3}$

$\lambda(-24) = (-6)^3 \cdot {}^4C_3$

$\lambda = 36$

28. If $S=\left\{x\in\mathbb{R}:\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}}\right)-\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}}\right)=\frac{\pi}{4}\right\}$,

then $\sum_{x\in S}\left(\sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right)-\cos\left(\left(x^2+x+5\right)\pi\right)\right)$ is

equal to _____.

Answer (4)

Sol. $\sin^{-1}\left(\frac{x+1}{\sqrt{x^2+x+2}}\right)=\frac{\pi}{4}+\sin^{-1}\frac{x}{\sqrt{x^2+1}}$

$\Rightarrow \frac{x+1}{\sqrt{x^2+x+2}}=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{x^2+1}}+\frac{x}{\sqrt{2}\sqrt{x^2+1}}$

$\Rightarrow \frac{x+1}{\sqrt{x^2+x+2}}=\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}$

$\Rightarrow x=-1$ OR $\sqrt{x^2+x+2}=\sqrt{2}\cdot\sqrt{x^2+1}$

$\Rightarrow x = 0, x = 2$ (Rejected)

$S = \{0, 2\}$

$\sum_{n\in S}\left(\sin(x^2+x+5)\frac{\pi}{2}-\cos(x^2+x+5)\pi\right)=4$

29. Let for $x\in\mathbb{R}, S_0(x)=x, S_k(x)=C_kx+k\int_0^x S_{k-1}(t)\,dt$,

where $C_0=1, C_k=1-\int_0^1 S_{k-1}(x)\,dx, k=1,2,3,...$ Then

$S_2(3)+6C_3$ is equal to _____.

Answer (18)

Sol. $S_0(x) = x, C_0 = 1$

$\Rightarrow S_1(x)=\frac{x}{2}+1\cdot\int_0^x t\,dt=\frac{x}{2}+\frac{x^2}{2}$

$C_1=1-\int_0^1 x\,dx=\frac{1}{2}$

$C_2=1-\int_0^1\left(\frac{x}{2}+\frac{x^2}{2}\right)dx=\frac{7}{12}$

$S_2(x)=\frac{7}{12}x+2\int_0^x\left(\frac{t}{2}+\frac{t^2}{2}\right)dt$

$=\frac{7x}{12}+\frac{x^2}{2}+\frac{x^3}{3}$

$C_3=1-\int_0^1\left(\frac{7x}{12}+\frac{x^2}{2}+\frac{x^3}{3}\right)dx=\frac{11}{24}$

Now, $S_2(3)+6\cdot C_3=\frac{61}{4}+6\cdot\frac{11}{24}=18$

30. Let the mean of the data

x	1	3	5	7	9
Frequency (f)	4	24	28	α	8

be 5. If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m+\sigma^2}$ is equal to _____.

Answer (8)

Sol. $\frac{4+72+28\times5+7\alpha+72}{64+\alpha}=5$

$\Rightarrow \alpha = 16$

$\sum f_i = 80$

$\text{M.D}=\frac{\sum f_i\,|x_i-5|}{\sum f_i}$

$=\frac{4+4+24\times2+0+16\times2+8\times4}{80}$

$=\frac{8}{5}$

$\sigma^2=\frac{\sum f_i\,|x_i-5|}{\sum f_i}$

$=\frac{4+16+24\times4+0+16\times4+8\times16}{80}=\frac{22}{5}$

$\frac{3\alpha}{m+\sigma^2}=\frac{3\times16}{\frac{8}{5}+\frac{22}{5}}=8$

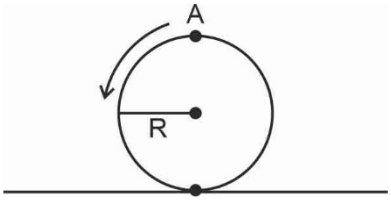
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

31. A disc is rolling without slipping on a surface. The radius of the disc is R. At t = 0, the top most point on the disc is A as shown in figure. When the disc completes half of its rotation, the displacement of point A from its initial position is



(1) 2R (2) $R\sqrt{(\pi^2+4)}$

(3) $R\sqrt{(\pi^2+1)}$ (4) $2R\sqrt{(1+4\pi^2)}$

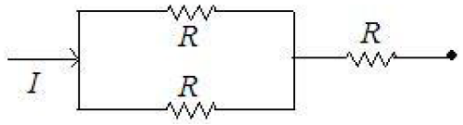
Answer (2)

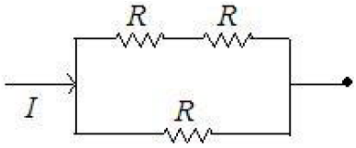
Sol. Displacement of $CM = R\omega\frac{T}{2} = \pi r$

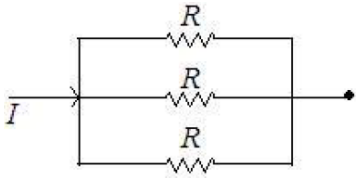
$\Rightarrow$ Displacement of $A = \sqrt{(2R)^2+(\pi R)^2}$

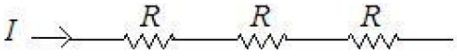
$= R\sqrt{\pi^2+4}$

32. Different combination of 3 resistors of equal resistance R are shown in the figures. The increasing order for power dissipation is:

(A) 

(B) 

(C) 

(D) 

(1) $P_B < P_C < P_D < P_A$

(2) $P_C < P_D < P_A < P_B$

(3) $P_C < P_B < P_A < P_D$

(4) $P_A < P_B < P_C < P_D$

Answer (3)

Sol. $R_A = \frac{3R}{2}$

$R_B = \frac{2R}{3}$

$R_C = \frac{R}{3}$

$R_D = 3R$

$\Rightarrow$ $P_D > P_A > P_B > P_C$ [$\because$ $P = I^2R_{eff}$]

33. A planet having mass 9 M_e and radius $4R_e$, where M_e and R_e are mass and radius of earth respectively, has escape velocity in km/s given by:

(Given escape velocity on earth $V_e = 11.2 \times 10^3$ m/s)

(1) 67.2 (2) 16.8

(3) 11.2 (4) 33.6

Answer (2)

Sol. $V_{esc} = \sqrt{\frac{2GM}{R}}$

$\Rightarrow$ $\frac{V'_{esc}}{V_{esc}} = \sqrt{\frac{9}{4}} = 1.5$

$\Rightarrow$ $V'_{esc} = 16.8$ km/s

34. ${}^{238}_{92}A \rightarrow {}^{234}_{90}B + {}^{4}_{2}D + Q$

In the given nuclear reaction, the approximate amount of energy released will be:

[Given, mass of ${}^{238}_{92}A = 238.05079 \times 931.5\ MeV/c^2$,

mass of ${}^{234}_{90}B = 234.04363 \times 931.5\ MeV/c^2$,

mass of ${}^{4}_{2}D = 4.00260 \times 931.5\ MeV/c^2$]

(1) 3.82 MeV

(2) 5.9 MeV

(3) 2.12 MeV

(4) 4.25 MeV

Answer (4)

Sol. $\phi = (\Delta m)\cdot c^2$

$= 4.56 \times 10^{-3} \times 931.5$ MeV

$= 4.24$ MeV

35. The difference between threshold wavelengths for two metal surfaces A and B having work function $\phi_A = 9$ eV and $\phi_B = 4.5$ eV in nm is:

{Given, hc = 1242 eV nm}

(1) 540 (2) 276

(3) 264 (4) 138

Answer (4)

Sol. $\frac{hc}{\lambda} = \phi$

$\Rightarrow \lambda = \frac{hc}{\phi}$

$\Rightarrow \lambda = \frac{1242}{4.5} - \frac{1242}{9}$ nm

$= \frac{1242}{9}$ nm

$= 138$ nm

36. Two trains 'A' and 'B' of length '*l*' and '4*l*' are travelling into a tunnel of length 'L' in parallel tracks from opposite directions with velocities 108 km/h and 72 km/h, respectively. If train 'A' take 35s less time than train 'B' to cross the tunnel then, length 'L' of tunnel is:

(Given L = 60 *l*)

(1) 1200 m

(2) 900 m

(3) 1800 m

(4) 2700 m

Answer (3)

Sol. $\Delta t_A = \frac{l + L}{V_A}$

$\Delta t_B = \frac{4l + L}{V_B}$

$\Rightarrow \frac{4l + L}{20} - \frac{l + L}{30} = 35$

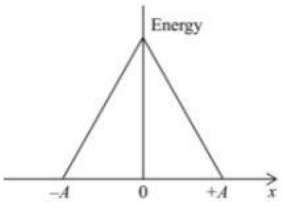
$\Rightarrow \frac{64l}{20} - \frac{61l}{30} = 35$

$\Rightarrow l = \frac{35 \times 600}{700} = 30$ m

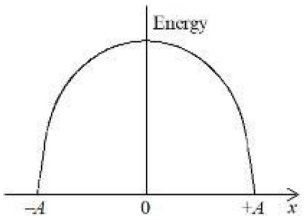
$\Rightarrow L = 1800$ m

37. Which graph represents the difference between total energy and potential energy of a particle executing SHM vs it's distance from mean position?

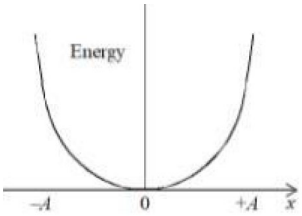
(1)



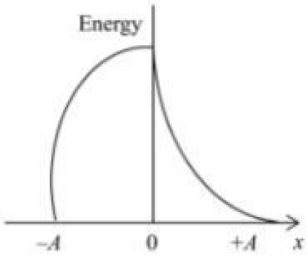
(2)



(3)



(4)



Answer (2)

Sol. T – U = k

$\Rightarrow T - U = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

38. The rms speed of oxygen molecule in a vessel at particular temperature is $\left(1 + \frac{5}{x}\right)^{\frac{1}{2}} \nu$, where ν is the average speed of the molecule. The value of *x* will be:

(take $\pi = \frac{22}{7}$)

(1) 27

(2) 8

(3) 28

(4) 4

Answer (3)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{avg} = \sqrt{\frac{8RT}{\pi M}}$

$\Rightarrow \frac{v_{rms}}{v_{avg}} = \sqrt{\frac{3\pi}{8}} = \sqrt{\frac{3}{8} \times \frac{22}{7}}$

$= \sqrt{1 + \frac{5}{28}}$

39. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x+1}}$, that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of x will be

(1) 16

(2) 2

(3) 2.4

(4) 4

Answer (1)

Sol. $\frac{3\sqrt{x}}{\sqrt{x}+1} = \frac{\frac{300 \times g \times 100}{5 \text{ minutes}}}{\frac{50 \times g \times 100}{2 \text{ minutes}}}$

$= \frac{60}{25} = 2.4$

$\Rightarrow 3\sqrt{x} = 2.4\sqrt{x} + 2.4$

$\Rightarrow x = 16$

40. A vessel of depth 'd' is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be-

(1) $\frac{2d(n_1 + n_2)}{n_1 + n_2}$

(2) $\frac{d(n_1 + n_2)}{2n_1 n_2}$

(3) $\frac{dn_1 n_2}{2(n_1 + n_2)}$

(4) $\frac{dn_1 n_2}{(n_1 + n_2)}$

Answer (2)

Sol. Total shift = $\frac{d}{2}\left[1 - \frac{1}{n_1} + 1 - \frac{1}{n_2}\right]$

$\Rightarrow$ Apparent depth = $d - \frac{d}{2}\left[2 - \frac{1}{n_1} - \frac{1}{n_2}\right]$

$= \frac{d(n_1 + n_2)}{2n_1 n_2}$

41. A body of mass (5 ± 0.5) kg is moving with a velocity of (20 ± 0.4) m/s. Its kinetic energy will be

(1) (1000 ± 0.14) J

(2) (500 ± 0.14) J

(3) (500 ± 140) J

(4) (1000 ± 140) J

Answer (4)

Sol. $K = \frac{1}{2}mv^2$

$\Rightarrow \frac{dK}{K} = \frac{dm}{m} + \frac{2dv}{v}$

$\Rightarrow \frac{dK}{1000} = \frac{0.5}{5} + \frac{2 \times 0.4}{20}$

$\Rightarrow dK = 100 + 40 = 140$

$\Rightarrow$ KE = (1000 ± 140) J

42. Under isothermal condition, the pressure of a gas is given by $P = aV^{-3}$, where a is a constant and V is the volume of the gas. The bulk modulus at constant temperature is equal to

(1) $3P$

(2) P

(3) $2P$

(4) $\frac{P}{2}$

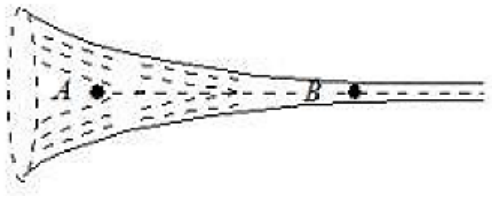
Answer (1)

Sol. $B = -\frac{dP}{dV/V} = -V\frac{dP}{dV}$

$= -V\frac{d}{dV}\left\{\frac{a}{V^3}\right\}$

$= -V\left[\frac{-3a}{V^4}\right]$

$= \frac{3a}{V^3} = 3P$

43. 

The figure shows a liquid of given density flowing steadily in horizontal tube of varying cross-section. Cross-sectional areas at A is 1.5 cm^2, and B is 25 mm^2, if the speed of liquid at B is 60 cm/s then $(P_A - P_B)$ is

(Given P_A and P_B are liquid pressures at A and B points.

Density ρ = 1000 kg m^{-3}

A and B are on the axis of tube)

(1) 135 Pa (2) 27 Pa

(3) 175 Pa (4) 36 Pa

Answer (3)

Sol. $P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2 \ldots(1)$

Also, 1.5 × cm^2 × V_A = 25 mm^2 × V_B

$$V_A = \frac{25\times10^{-6}}{1.5\times10^{-4}}\times 0.6 \text{ m/s}$$

$$= \frac{50}{3}\times\frac{1}{100}\times 0.6 \text{ m/s}$$

$$= \frac{1}{10}\text{m/s}$$

$$\Rightarrow \quad P_A - P_B = \frac{1}{2}[0.6^2 - 0.1^2] = \frac{1000}{2}\times 0.7\times 0.5$$

= 175 Pa

44. The source of time varying magnetic field may be

(A) a permanent magnet

(B) an electric field changing linearly with time

(C) direct current

(D) a decelerating charge particle

(E) an antenna fed with a digital signal

Choose the correct answer from the options given below.

(1) (C) and (E) only

(2) (D) only

(3) (A) only

(4) (B) and (D) only

Answer (2)

Sol. A decelerating charged particle can result in a time. varying magnetic field by emitting radiation.

45. Match **List-I** with **List-II**

	List-I (Layer of atmosphere)		**List-II (Approximate height over earth's surface)**
(A)	F_1 - Layer	(I)	10 km
(B)	D – Layer	(II)	170-190 km
(C)	Troposphere	(III)	100 km
(D)	E - Layer	(V)	65-75 km

Choose the correct answer from the options given below.

(1) A – II, B – IV, C – I, D – III

(2) A – II, B – IV, C – III, D – I

(3) A – III, B – IV, C – I, D – II

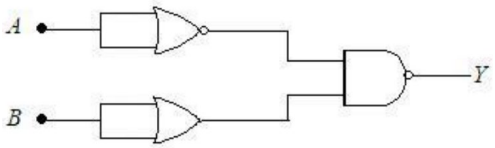
(4) A – II, B – I, C – IV, D – III

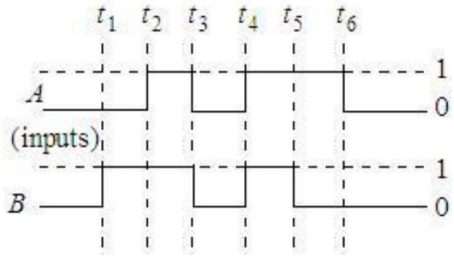
Answer (1)

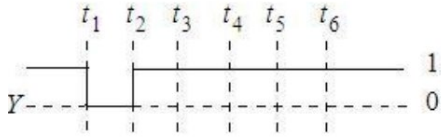
Sol. D layer : 60 – 90 km

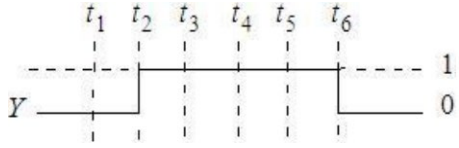
E layer : 100 – 125 km

46. For the following circuit and given inputs A and B, choose the correct option for output 'Y'

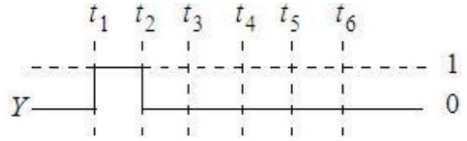




(1) 

(2) 

(3)



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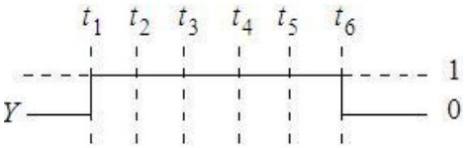


Answer (1)

Sol. $Y = (A' \cdot B)'$

$= A + B'$

$\Rightarrow$ Option 1.

47. Two charges of each magnitude 0.01 C and separated by a distance of 0.4 mm constitute an electric dipole. If the dipole is placed in an uniform electric field '$\vec{E}$' of 10 dyne/C making 30° angle with $\vec{E}$, the magnitude of torque acting on dipole is:

(1) 4.0×10^{-10} Nm (2) 1.0×10^{-8} Nm

(3) 1.5×10^{-9} Nm (4) 2.0×10^{-10} Nm

Answer (4)

Sol. $|\vec{\tau}| = |\vec{p} \times \vec{E}|$

$= 0.01 \times 0.4 \times 10^{-3} \times 10 \times 10^{-5} \times \frac{1}{2}$ Nm

$= 2 \times 10^{-10}$ Nm

48. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is:

(1) 3 : 4 (2) 9 : 16

(3) 16 : 9 (4) 4 : 3

Answer (2)

Sol. $K = \frac{p^2}{2m}$

$\Rightarrow \frac{K}{K'} = \frac{16}{9} = \frac{m'}{m} \quad \Rightarrow \frac{m}{m'} = \frac{9}{16}$

49. Which of the following Maxwell's equation is valid for time varying conditions but not valid for static conditions:

(1) $\oint \vec{B} \cdot \vec{dl} = \mu_0 I$ (2) $\oint \vec{E} \cdot \vec{dl} = 0$

(3) $\oint \vec{D} \cdot \vec{dA} = Q$ (4) $\oint \vec{E} \cdot \vec{dl} = -\frac{\partial \phi_B}{\partial t}$

Answer (4)

Sol. Equation 4 is used for time-varying conditions.

50. A bullet of 10 g leaves the barrel of gun with a velocity of 600 m/s. If the barrel of gun is 50 cm long and mass of gun is 3 kg, then value of impulse supplied to the gun will be:

(1) 6 Ns (2) 3 Ns

(3) 36 Ns (4) 12 Ns

Answer (1)

Sol. Impulse = $\Delta p = \frac{10}{1000} \times 600$ Ns

= 6 Ns

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. A potential V_0 is applied across a uniform wire of resistance R. The power dissipation is P_1. The wire is then cut into two equal halves and a potential of V_0 is applied across the length of each half. The total power dissipation across two wires is P_2. The ratio of $P_2 : P_1$ is $\sqrt{x}:1$. The value of x is _____.

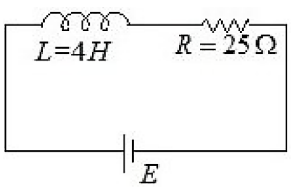
Answer (16)

Sol. $P_1 = \frac{V_0^2}{R}$

$P_2 = \frac{V_0^2}{R/2} \times 2 = 4P_1$

$\Rightarrow \frac{P_2}{P_1} = 4 \quad \Rightarrow \quad x = 16$

52. In the given figure, an inductor and resistor are connected in series with a battery of emf E volt. $\frac{E^a}{2b}$ J/s represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\frac{b}{a}$ will be _____.



Answer (25)

Sol. $i = \frac{E}{R}\left[1 - e^{-Rt/L}\right]$

$$E_{\text{induction}} = \frac{1}{2}Li^2 = \frac{1}{2}L\frac{E^2}{R^2}\left[1 - e^{-Rt/L}\right]^2 = x \text{ (let)}$$

$$\Rightarrow \frac{dx}{dt} = 2\left[1 - e^{-Rt/L}\right]\cdot e^{-Rt/L}\cdot\frac{R}{L}\cdot\frac{E^2L}{2R^2}$$

$$= \frac{E^2}{R}\left[e^{-Rt/L} - e^{-2Rt/L}\right]$$

For maximum rate, $\frac{d^2x}{dt^2} = 0$

$$\Rightarrow e^{-Rt/L} = 2e^{-2Rt/L} \qquad \Rightarrow e^{Rt/L} = 2$$

$$\Rightarrow \left(\frac{dx}{dt}\right)_{\text{max.}} = \frac{E^2}{R}\left[2^{-1} - 2^{-2}\right] = \frac{E^2}{4R}$$

$\Rightarrow a = 2$ and $b = 2R = 50$

$$\Rightarrow \frac{b}{a} = 25$$

53. The elastic potential energy stored in a steel wire of length 20 m stretched through 2 cm is 80 J. The cross sectional area of the wire is _____ mm^2. (Given, $y = 2.0 \times 10^{11}$ Nm^{-2})

Answer (40)

Sol. Potential energy

$$= \frac{1}{2}\times \text{stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2}\times Y \times \left(\frac{2/100}{20}\right)^2 \times A \times 20$$

$$\Rightarrow 80 = \frac{1}{2}\times Y \times \frac{1}{10^6}\times A \times 20$$

$$\Rightarrow A = \frac{160\times 10^6}{20\times 2\times 10^{11}}\text{m}^2$$

$= 4 \times 10^{-5}$ m^2 = 40 mm^2

54. At a given point of time the value of displacement of a simple harmonic oscillator is given as $y = A\cos(30°)$. If amplitude is 40 cm and kinetic energy at that time is 200 J, the value of force constant 1.0×10^x Nm^{-1}. The value of x is _____.

Answer (4)

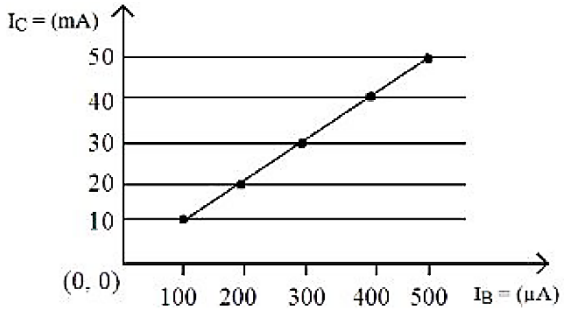
Sol. $\text{KE} = \frac{1}{2}k\left(A^2 - x^2\right)$

$$\Rightarrow 200 = \frac{1}{2}\times 10^x \times 0.4^2 \times \frac{1}{4}$$

$$\Rightarrow 10^x = \frac{400\times 4}{0.16} = 10^4$$

$\Rightarrow x = 4$

55. From the given transfer characteristic of a transistor in CE configuration, the value of power gain of this configuration is 10^x, for $R_B = 10$ kΩ, and $R_C = 1$ kΩ. The value of x is _____.



Answer (3)

Sol. Power gain = $\beta^2 \cdot \frac{R_C}{R_B}$

$$= \left[\frac{10\times 10^{-3}}{100\times 10^{-6}}\right]^2 \times \frac{1}{10} = 1000$$

$\Rightarrow$ x = 3

56. The radius of 2nd orbit of He$^+$ of Bohr's model is r_1 and that of fourth orbit of Be^{3+} is represented as r_2. Now the ratio $\frac{r_2}{r_1}$ is x : 1. The value of x is _____.

Answer (2)

Sol. $r = r_0\frac{n^2}{Z}$

$$\Rightarrow \frac{r_2}{r_1} = \frac{\frac{4^2}{4}}{\frac{2^2}{2}} = 2$$

57. When a resistance of 5 Ω is shunted with a moving coil galvanometer, it shows a full scale deflection for a current of 250 mA, however when 1050 Ω resistance is connected with it in series, it gives full scale deflection for 25 volt. The resistance of galvanometer is _____ Ω.

Answer (50)

Sol. $(i_G)_{max} = \frac{5}{5+R_G} i_{max} = \frac{5}{5+R_G} \times 250 \text{ mA}$

Also, $25 = (i_G)_{max}[R_G + 1050]$

$\Rightarrow 25 = \frac{5}{5+R_G} \times \frac{1}{4}[R_G + 1050]$

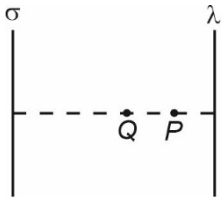
$\Rightarrow 500 + 100R_G = 5R_G + 5250$

$\Rightarrow R_G = \frac{4750}{95} = 50\,\Omega$

58. A thin infinite sheet charge and an infinite line charge of respective charge densities + σ and + λ are placed parallel at 5 m distance from each other. Points 'P' and 'Q' are at $\frac{3}{\pi}$m and $\frac{4}{\pi}$m perpendicular distances from line charge towards sheet charge, respectively. 'E_P' and 'E_Q' are the magnitudes of resultant electric field intensities at point 'P' and 'Q', respectively. If $\frac{E_P}{E_Q} = \frac{4}{a}$ for $2|\sigma| = |\lambda|$, then the value of *a* is _____

Answer (6)

Sol.



$$E_P = \left|\frac{\sigma}{2\varepsilon_0} - \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{3/\pi}\right| = \left|\frac{\sigma}{2\varepsilon_0} - \frac{\lambda}{6\varepsilon_0}\right| = \frac{\sigma}{6\varepsilon_0}$$

$$E_Q = \left|\frac{\sigma}{2\varepsilon_0} - \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{4/\pi}\right| = \left|\frac{\sigma}{2\varepsilon_0} - \frac{\lambda}{8\varepsilon_0}\right| = \frac{\sigma}{4\varepsilon_0}$$

$$\Rightarrow \frac{E_P}{E_Q} = \frac{2}{3} \Rightarrow a = 6$$

59. A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of moving sphere is π : 22 then, the value of its angular speed will be _____ rad/s.

Answer (4)

Sol. $\frac{L}{E} = \frac{I_{CM}\omega}{\frac{1}{2}Mv^2 + \frac{1}{2}\cdot\frac{2}{5}MR^2\omega^2} = \frac{\frac{2}{5}MR^2\omega}{\frac{7}{10}MR^2\omega^2}$

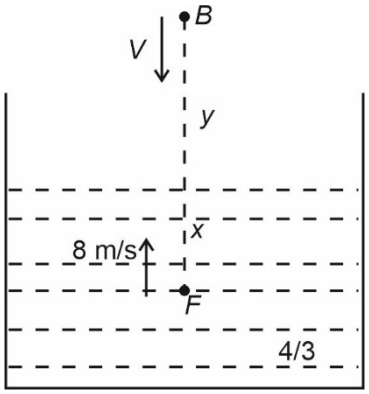
$\Rightarrow \frac{\pi}{22} = \frac{4}{7}\cdot\frac{1}{\omega}$

$\Rightarrow \omega = 4$ rad/s

60. A fish rising vertically upward with a uniform velocity of 8 ms^{-1}, observes that a bird is diving vertically downward towards the fish with the velocity of 12 ms^{-1}. If the refractive index of water is $\frac{4}{3}$, then the actual velocity of the diving bird to pick the fish, will be _____ ms^{-1}.

Answer (3)

Sol.



w.r.t. fish: Separation = $x + \frac{4y}{3}$

$\Rightarrow \frac{d}{dt}\left[x + \frac{4y}{3}\right] = -12$

$\Rightarrow \frac{4}{3}\cdot\frac{dy}{dt} = -12 + 8 = -4$

$\Rightarrow \frac{dy}{dt} = -3$ m/s

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

61. Match the following

	Column-A		Column-B
a.	Nylon 6	I.	Natural Rubber
b.	Vulcanized Rubber	II.	Cross Linked
c.	cis-1, 4-polyisoprene	III.	Caprolactam
d.	Polychloroprene	IV.	Neoprene

Choose the correct answer from options given below:

(1) a→III, b→II, c→I, d→IV

(2) a→III, b→IV, c→I, d→II

(3) a→IV, b→III, c→II, d→I

(4) a→II, b→III, c→IV, d→I

Answer (1)

Sol. Nylon 6 (III) Caprolactam

Vulcanized Rubber (II) Cross Linked

cis-1, 4-polyisoprene (I) Natural Rubber

Polychloroprene (IV) Neoprene

62. The mismatched combinations are

A. Chlorophyll – Co

B. Water hardness – EDTA

C. Photography – $[Ag(CN)_2]^-$

D. Wilkinson catalyst – $[(Ph_3P)_3RhCl]$

E. Chelating ligand – D-Penicillamine

Choose the correct answer from the options given below.

(1) A, C and E only

(2) A and C only

(3) A and E only

(4) D and E only

Answer (2)

Sol. A. Chlorophyll – Mg

B. Water hardness – EDTA

C. Photography – AgBr (not $[Ag(CN)_2^-]$)

D. Wilkinson catalyst – $[(Ph_3P)_3RhCl]$

E. D-Penicillamine is chelating ligand

Hence, A and C are incorrect.

63. In which of the following processes, the bond order increases and paramagnetic character changes to diamagnetic one?

(1) $O_2 \rightarrow O_2^+$

(2) $O_2 \rightarrow O_2^{2-}$

(3) $NO \rightarrow NO^+$

(4) $N_2 \rightarrow N_2^+$

Answer (3)

Sol. $\underset{\text{Paramagnetic}}{NO} \rightarrow \underset{\text{Diamagnetic}}{NO^+}$

64. ClF_5 at room temperature is a

(1) Colourless liquid with trigonal bipyramidal geometry

(2) Colourless gas with square pyramidal geometry

(3) Colourless gas with trigonal bipyramidal geometry

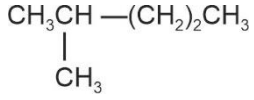
(4) Colourless liquid with square pyramidal geometry

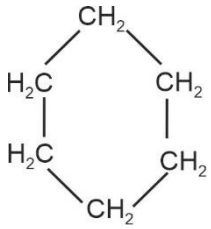
Answer (4)

Sol. ClF_5 is colourless liquid with square pyramidal geometry.

65. In the following reaction 'X' is

$$CH_3(CH_2)_4CH_3 \xrightarrow[\substack{HCl\\ \triangle}]{\text{Anhy. } AlCl_3} \underset{\text{major product}}{'X'}$$

(1) 

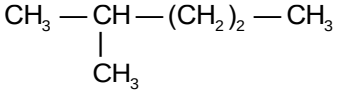
(2) 

(3) $CH_3(CH_2)_4CH_2Cl$

(4) $Cl–CH_2–(CH_2)_4–CH_2–Cl$

Answer (1)

Sol. $CH_3(CH_2)_4CH_3 \xrightarrow[\substack{HCl\\ \triangle}]{\text{Anhy. } AlCl_3}$



(isomerisation reaction)

66. $D-(+)-$ Glyceraldehyde $\xrightarrow[\text{ii)}H_2O/H^+ \\ \text{iii)}HNO_3]{\text{i)HCN}}$

The products formed in the above reaction are

(1) One optically active and one meso product

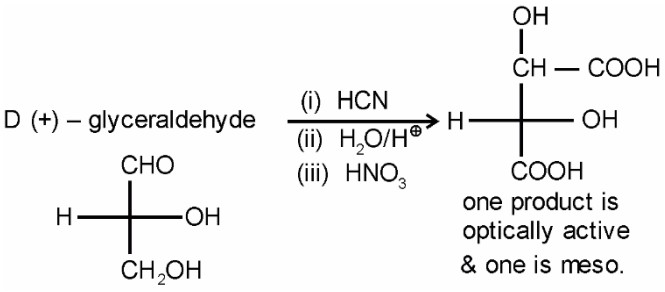
(2) Two optically inactive products

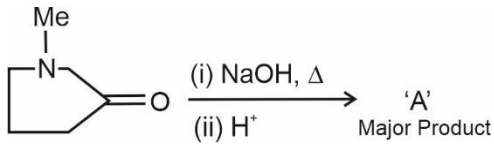
(3) Two optically active products

(4) One optically inactive and one meso product

Answer (1)

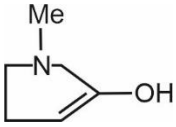
Sol.

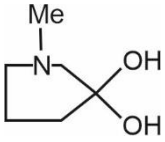


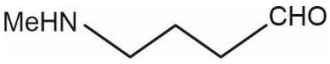
67. 

'A' is

(1) 

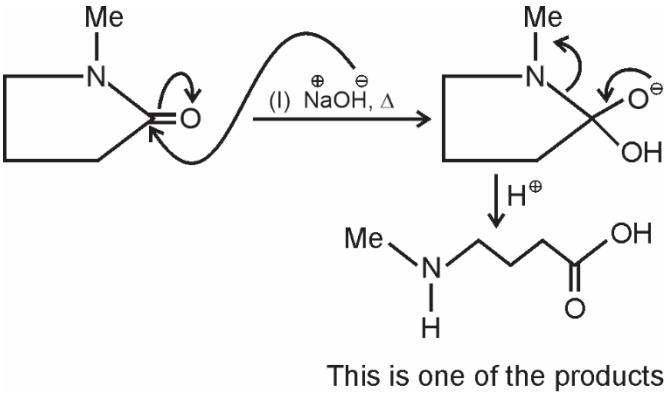
(2) 

(3) 

(4) 

Answer (1)

Sol.



68. $Be(OH)_2$ reacts with $Sr(OH)_2$ to yield an ionic salt. Choose the incorrect option related to this reaction from the following

(1) Both Sr and Be elements are present in the ionic salt

(2) Be is tetrahedrally coordinated in the ionic salt

(3) The element Be is present in the cationic part of the ionic salt

(4) The reaction is an example of acid-base neutralization reaction

Answer (3)

Sol. $Be(OH)_2 + Sr(OH)_2 \longrightarrow Sr^{+2}[Be(OH)_4]^{-2}$

As Be is present in anionic part.

69. Given below are two statements:

Statement I: Permutit process is more efficient compared to the synthetic resin method for the softening of water.

Statement II: Synthetic resin method results in the formation of soluble sodium salts.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both the statements I and II are incorrect

(2) Statement I is incorrect but statement II is correct

(3) Both the statements I and II are correct

(4) Statement I is correct but statement II is incorrect

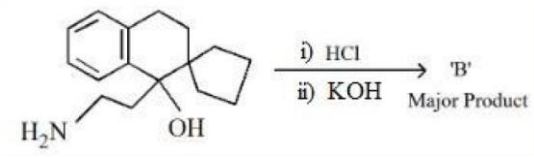
Answer (1)

Sol. I. Synthetic resin is more efficient as cations as well as anions responsible for removal of permanent hardness.

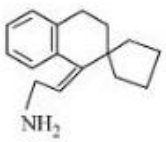
II. In synthetic resin method, cations and anions are removed in the form of ppt.

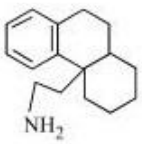
Hence, both statements are incorrect.

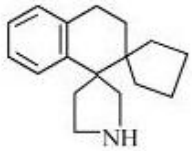
70. In the reaction given below

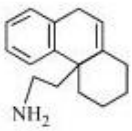


'B' is:

(1) 

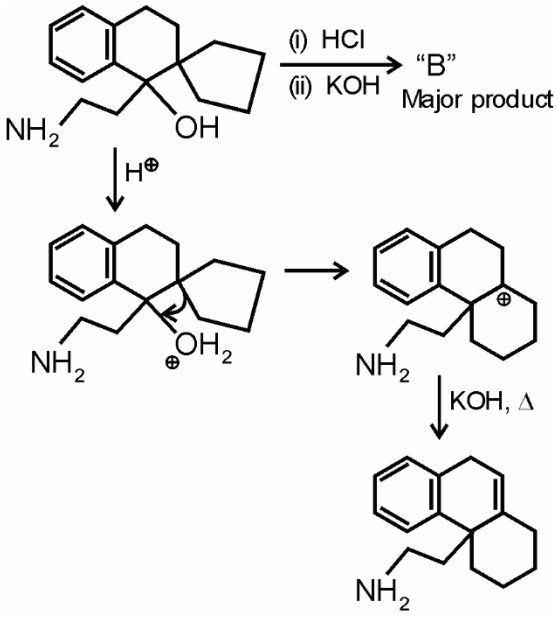
(2) 

(3) 

(4) 

Answer (4)

Sol.



71. What happens when a lyophilic sol is added to a lyophobic sol?

(1) Film of lyophilic sol is formed over lyophobic sol
(2) Lyophilic sol is dispersed in lyophobic sol
(3) Film of lyophobic sol is formed over lyophilic sol
(4) Lyophobic sol is coagulated

Answer (1)

Sol. Film of lyophilic sol is formed over lyophobic sol as lyophilic sol acts as a protective colloid.

72. The energy of an electron in the first Bohr orbit of hydrogen atom is -2.18×10^{-18} J. Its energy in the third Bohr orbit is _____.

(1) $\frac{1}{9}$th of this value
(2) $\frac{1}{27}$ of this value
(3) Three times of this value
(4) One third of this value

Answer (1)

Sol. $E_3 = \frac{E_1}{n^2}$

$= \frac{E_1}{9}$

73. The pair of lanthanides in which both elements have high third-ionization energy is:

(1) Dy, Gd (2) Lu, Yb
(3) Eu, Yb (4) Eu, Gd

Answer (3)

Sol. Eu has a configuration of $4f^7 6s^2$ & Yb has a configuration of $4f^{14} 6s^2$

Hence, both have high 3rd ionisation energy due to help filled & fully filled configuration respectively.

74. Which one of the following is most likely a mismatch?

(1) Zinc - Liquation
(2) Copper - Electrolysis
(3) Nickel - Mond process
(4) Titanium - van Arkel Method

Answer (1)

Sol. zinc is not refined using liquation.

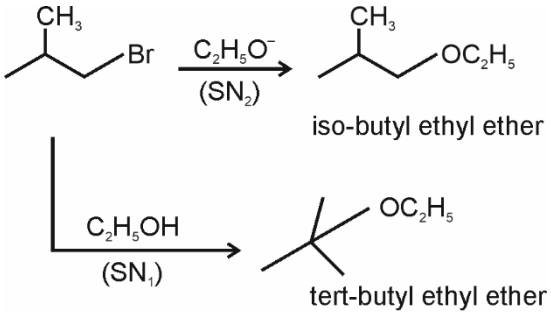
Distillation is used instead.

75. 2-Methyl propyl bromide reacts with $C_2H_5O^-$ and gives 'A' whereas on reaction with C_2H_5OH it gives 'B'. The mechanism followed in these reactions and the products 'A' and 'B' respectively are:

(1) S_N2, A = iso-butyl ethyl ether; S_N1, B = tert-butyl ethyl ether
(2) S_N1, A = tert-butyl ethyl ether; S_N1, B = 2-butyl ethyl ether
(3) S_N2, A = 2-butyl ethyl ether; S_N2, B = iso-butyl ethyl ether
(4) S_N1, A = tert-butyl ethyl ether; S_N2, B = iso-butyl ethyl ether

Answer (1)

Sol.



76. Which of the following statements are **not** correct?

A. The electron gain enthalpy of F is more negative than that of Cl.
B. Ionization enthalpy decreases in a group of periodic table.
C. The electronegativity of an atom depends upon the atoms bonded to it.
D. Al_2O_3 and NO are examples of amphoteric oxides.

Choose the most appropriate answer from the options given below:

(1) A, B, C and D (2) A, B and D only
(3) B and D only (4) A, C and D only

Answer (4)

Sol. A is incorrect ($|\Delta H_{eg}| \rightarrow Cl > F$)

B is correct (ionisation energy decreases down the group).

C is incorrect (As E.N. has fixed value on Pauling scale).

D is incorrect as NO is a neutral oxide.

Hence, A, C and D are incorrect.

77. The radical which mainly causes ozone depletion in the presence of UV radiations is :

(1) $Cl^{\bullet}$ (2) $NO^{\bullet}$

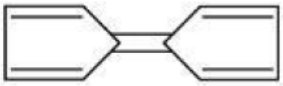
(3) $\dot{O}H$ (4) $CH_3^{\bullet}$

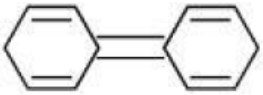
Answer (1)

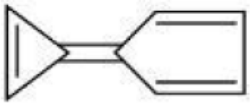
Sol. $Cl^{\bullet}$ causes ozone depletion.

78. Among the following compounds, the one which shows highest dipole moment is

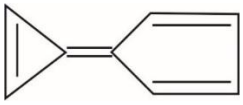
(1) 

(2) 

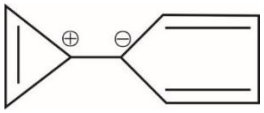
(3) 

(4) 

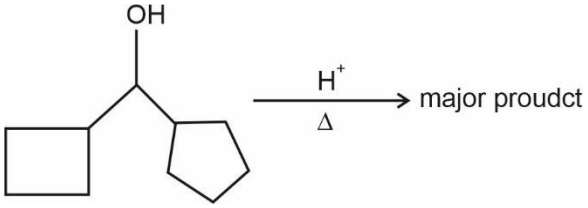
Answer (4)

Sol. 

exists as a dipole in the form of



High dipole moment as both the groups attached become aromatic

79. 

In the above reaction, left hand side and right hand side rings are named as 'A' and 'B' respectively. They undergo ring expansion. The correct statement for this process is:

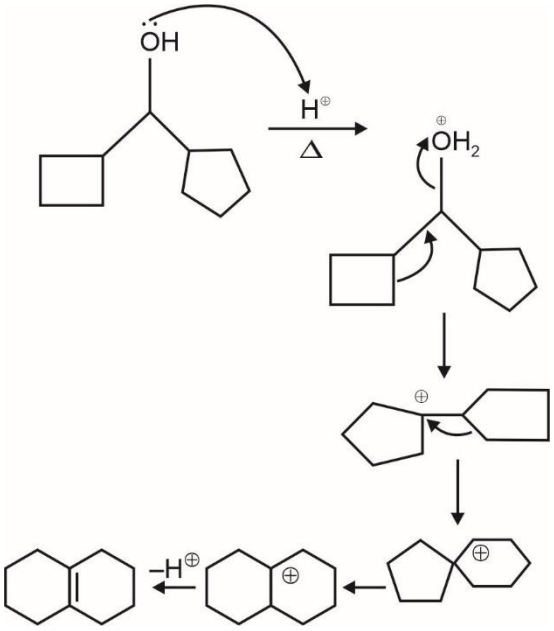
(1) Ring expansion can go upto seven membered rings

(2) Finally both rings will become six membered each.

(3) Finally both rings will become five membered each.

(4) Only A will become 6 membered.

Answer (2)

Sol. 

80. The incorrect statement from the following for borazine is:

(1) It contains banana bonds

(2) It can react with water

(3) It is a cyclic compound

(4) It has electronic delocalization

Answer (1)

Sol. Borazine is an aromatic compound with electron delocalization. It does not contain banana bond.

$B_3N_3H_6 + 9H_2O \rightarrow 3NH_3 + 3H_3BO_3 + 3H_2O$

On heating, borazine hydrolyses slowly.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. A certain quantity of real gas occupies a volume of 0.15 dm^3 at 100 atm and 500 K when its compressibility factor is 1.07. Its volume at 300 atm and 300 K (When its compressibility factor is 1.4) is ____ $\times 10^{-4}$ dm^3 (Nearest integer)

Answer (392)

Sol. $\frac{Z_1}{Z_2} = \frac{P_1V_1 \times T_2}{T_1 \times P_2V_2}$

$\frac{1.07}{1.4} = \frac{100 \times 0.15 \times 300}{500 \times 300 \times V_2}$

$V_2 = 0.03925\ dm^3$

$\simeq 392 \times 10^{-4} dm^3$

82. 25.0 mL of 0.050 M $Ba(NO_3)_2$ is mixed with 25.0 mL of 0.020 M NaF. K_{sp} of BaF_2 is 0.5×10^{-6} at 298 K. The ratio of $[Ba^{2+}]\ [F^-]^2$ and K_{sp} is _____ .

Answer (5)

Sol. $[Ba^{+2}] = \left(\frac{1.25}{50}\right)$

$[F^-] = \left(\frac{0.5}{50}\right)$

$Q_{sp} = [Ba^{+2}][F^-]^2 = \frac{1.25 \times 0.5 \times 0.5}{50 \times 50 \times 50}$

$K_{sp} = 0.5 \times 10^{-6}$

$\frac{Q_{sp}}{K_{sp}} = 5$

83. $KMnO_4$ is titrated with ferrous ammonium sulphate hexahydrate in presence of dilute H_2SO_4. Number of water molecules produced for 2 molecules of $KMnO_4$ is _____

Answer (68)

Sol. $8H^+ + MnO_4^- + 5\,Fe^{+2} \longrightarrow 5\,Fe^{+3} + Mn^{+2} + 4\,H_2O$

For 2 moles of MnO_4^-, 8 moles of H_2O are released

5 moles of Mohr's salt contain 30 moles of H_2O.

So, 10 moles of Mohr's salt contain 60 moles of H_2O

So, total moles of H_2O produced = 68

84. $t_{87.5}$ is the time required for the reaction to undergo 87.5% completion and t_{50} is the time required for the reaction to undergo 50% completion. The relation between $t_{87.5}$ and t_{50} for a first order reaction is

$t_{87.5} = x \times t_{50}$

The value of x is _____ . (Nearest integer)

Answer (3)

Sol. 87.5% consumption means 3 half lives.

So, X = 3

85. An organic compound gives 0.220 g of CO_2 and 0.126 g of H_2O on complete combustion. If the % of carbon is 24 then the % of hydrogen is _____ $\times 10^{-1}$. (Nearest integer)

Answer (56)

Sol. Mass of C = $\frac{0.220}{44} \times 12 = 0.06$ g

Mass of H = $\frac{0.126}{18} \times 2 = 0.014$ g

Given $\frac{24}{100} \times (y) = 0.06$

$y = \frac{6}{24} = \frac{1}{4}$

$= 0.014 \times 4 \times 100$

$= 5.6 = 56 \times 10^{-1}$

86. A metal surface of 100 cm^2 area has to be coated with nickel layer of thickness 0.001 mm. A current of 2 A was passed through a solution of $Ni(NO_3)_2$ for 'x' seconds to coat the desired layer. The value of x is _____. (Nearest integer)

(ρ_{Ni} (density of Nickel) is 10 g mL^{-1}, Molar mass of Nickel is 60 g mol^{-1} F = 96500 C mol^{-1})

Answer (16)

Sol. Millimoles of Ni coated

$= \frac{(2x)100}{96500 \times 2}$

$= \frac{10x}{965}$

Mass of Ni = $\left(\frac{x}{965}\right)600$

$= \frac{600x}{965}$ mg

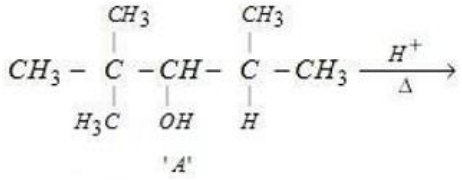
Volume = $\frac{60x}{965}\times10^{-3}cm^3$

$= 100 \times 10^{-4}$

$\Rightarrow x = 0.1608\times10^3$

≈ 161

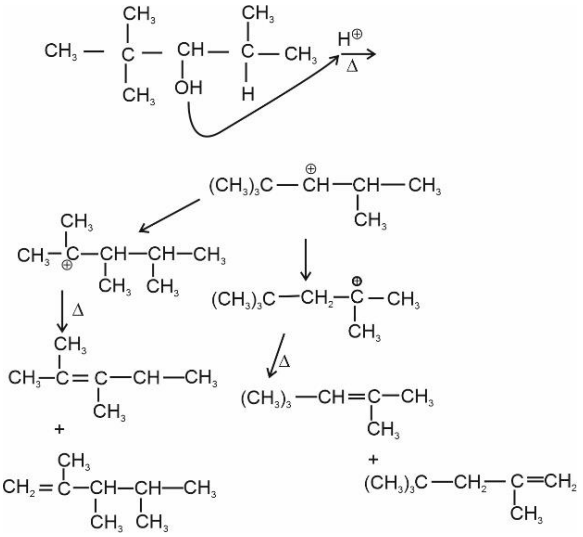
87. For the given reaction



The, total number of possible products formed by tertiary carbocation of A is _____.

Answer (4)

Sol.



(Total 4 products are possible through 3° carbocation)

88. $A_2 + B_2 \rightarrow 2AB$. $\Delta H_f = -200$ kJ mol^{-1}

AB, A_2 and B_2 are diatomic molecules. If the bond enthalpies of A_2, B_2 and AB are in the ratio 1 : 0.5 : 1, then the bond enthalpy of A_2 is _____ kJ mol^{-1} (Nearest integer)

Answer (400)

Sol. $A_2 + B_2 \longrightarrow 2AB,\ \Delta H_f^\circ = -200\ kJmol^{-1}$

B.E. of $A_2 : B_2 : AB = k : \frac{k}{2} : k$

$\Delta H_f^\circ = [(B.E\ of\ A_2) + (B.E.\ of\ B_2)$
$- 2(B.E.\ of\ AB)]$

$= \left[k + \frac{k}{2} - 2k\right]$

$= \frac{3k}{2} - 2k$

$-200 = \frac{-k}{2}$

$k = 400 = B.E.\ of\ A_2\ (in\ kJ/mol)$

89. 20 mL of calcium hydroxide was consumed when it was reacted with 10 mL of unknown solution of H_2SO_4. Also 20 mL standard solution of 0.5 M HCl containing 2 drops of phenolphthalein was titrated with calcium hydroxide, the mixture showed pink colour when burette displayed the value of 35.5 mL whereas the burette showed 25.5 mL initially. The concentration of H_2SO_4 is _____ M. (Nearest integer)

Answer (1)

Sol. As pink colour is obtained on consumption of 10 ml of $Ca(OH)_2$, we have

$20 \times 0.5 \times 1 = 10 \times M \times 2$

Molarity of $Ca(OH)_2 = \frac{10}{20} = \frac{1}{2}M$

Also, mmoles of $Ca(OH)_2$ = mm of H_2SO_4

$\frac{1}{2}\times 20 = 10\times M$

Molarity of H_2SO_4 = 1 M

90. Solution of 12 g of non-electrolyte (A) prepared by dissolving it in 1000 mL of water exerts the same osmotic pressure as that of 0.05 M glucose solution at the same temperature. The empirical formula of A is CH_2O. The molecular mass of A is _____ g. (Nearest integer)

Answer (240)

Sol. Osmotic pressure of A = O.P. of glucose

$\frac{12}{M\times(1)} = 0.05$

$M = \frac{12}{5}\times100$

$= \frac{1200}{9} = 240\ g/mol$

❑ ❑ ❑