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24/01/2023
Morning

Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A travelling wave is described by the equation

   $y(x, t) = [0.05 \sin (8x - 4t)]$ m

   The velocity of the wave is : [all the quantities are in SI unit]

   (1) 8 ms^{-1} (2) 0.5 ms^{-1}

   (3) 4 ms^{-1} (4) 2 ms^{-1}

Answer (2)

Sol. $\because$ $y(x, t) = [0.05\sin(8x - 4t)]$ m

$$\text{Speed of wave} = \left|\frac{\text{Coefficient of } t}{\text{Coefficient of } x}\right|$$

$$= \frac{4}{8} = 0.5 \text{ ms}^{-1}$$

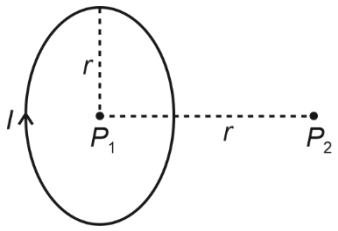
2. A circular loop of radius r is carrying current I A. The ratio of magnetic field at the center of circular loop and at a distance r from the center of the loop on its axis is:

   (1) $2\sqrt{2} : 1$ (2) $1 : 3\sqrt{2}$

   (3) $3\sqrt{2} : 2$ (4) $1 : \sqrt{2}$

Answer (1)

Sol.



$$B_{P_1} = \frac{\mu_0 I}{2r}$$

$$B_{P_2} = \frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}} = \frac{\mu_0 I}{2^{5/2} r}$$

$$\therefore \quad \frac{B_{P_1}}{B_{P_2}} = \frac{\frac{\mu_0 I}{2r}}{\frac{\mu_0 I}{2^{5/2} r}} = \frac{2\sqrt{2}}{1}$$

3. Consider the following radioactive decay process

   $$^{218}_{84}A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\gamma} A_3 \xrightarrow{\alpha} A_4 \xrightarrow{\beta^+} A_5 \xrightarrow{\gamma} A_6$$

   The mass number and the atomic number of A_6 are given by:

   (1) 210 and 80 (2) 210 and 82

   (3) 210 and 84 (4) 211 and 80

Answer (1)

Sol. $^{218}_{84}A \xrightarrow{\alpha} {}^{214}_{82}A_1 \xrightarrow{\beta^-} {}^{214}_{83}A_2 \xrightarrow{\gamma} {}^{214}_{83}A_3 \xrightarrow{\alpha} {}^{210}_{81}A_4 \xrightarrow{\beta^+} {}^{210}_{80}A_5 \xrightarrow{\gamma} {}^{210}_{80}A_6$

Mass number = 210

Atomic number = 80

4. The weight of a body at the surface of earth is 18 N. The weight of the body at an altitude of 3200 km above the earth's surface is (given, radius of earth R_e = 6400 km):

   (1) 8 N (2) 19.6 N

   (3) 9.8 N (4) 4.9 N

Answer (1)

Sol. W_{earth} = 18 N

mg_{earth} = 18

$$\text{Also } mg_h = mg_{\text{earth}}\left(\frac{R}{R+h}\right)^2$$

$$= 18\left(\frac{6400}{6400 + 3200}\right)^2$$

$$= 18 \times \frac{4}{9} = 8$$

5. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

   Assertion A : Photodiodes are preferably operated in reverse bias condition for light intensity measurement.

   Reason R : The current in the forward bias is more than the current in the reverse bias for a $p - n$ junction diode.

   In the light of the above statements, choose the ***correct*** answer from the options given below:

   (1) Both **A** and **R** are true and **R** is the correct explanation of **A**

   (2) **A** is false but **R** is true

   (3) **A** is true but **R** is false

   (4) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

Answer (4)

Sol. Photodiodes are preferably operated in reverse bias condition for light intensity measurement because it increases the width of depletion layer, therefore both are correct but not the correct explanation.

6. Given below are two statements :

Statement I : If the Brewster's angle for the light propagating from air to glass is θ_B, then the Brewster's angle for the light propagating from glass to air is $\frac{\pi}{2} - \theta_B$

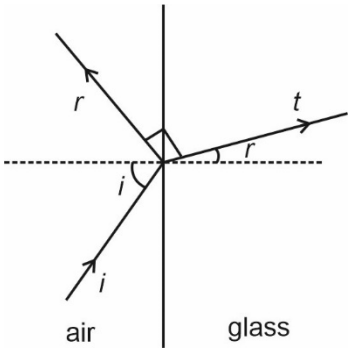
Statement II : The Brewster's angle for the light propagating from glass to air is $\tan^{-1}(\mu_g)$ where μ_g is the refractive index of glass.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both statement I and Statement II are true
(2) Both statement I and statement II are false
(3) Statement I is true but statement II is false
(4) Statement I is false but statement II is true

Answer (3)

Sol. Case I :



Transmitted is $\perp$ to reflected.

$i + r = 90°$

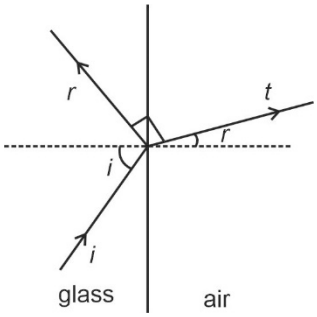
Snell's law

$\mu_a \sin i = \mu_g \sin r$

$$\tan i = \frac{\mu_g}{\mu_a}$$

$$i = \tan^{-1}\left(\frac{\mu_g}{\mu_a}\right) = \theta_B$$

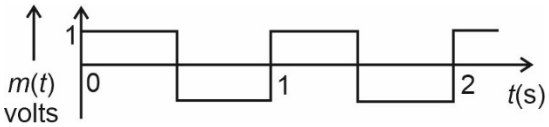
Case II :



$i + r = 90°$ as transmitted is $\perp$ to reflected.

$$\tan i = \frac{\mu_a}{\mu_g} \Rightarrow i = \tan^{-1}\frac{\mu_a}{\mu_g} = \frac{\pi}{2} - \theta_B$$

7. A modulating signal is a square wave, as shown in the figure.



If the carrier wave is given as $c(t) = 2\sin(8\pi t)$ volts, the modulation index is:

(1) $\frac{1}{2}$

(2) $\frac{1}{4}$

(3) 1

(4) $\frac{1}{3}$

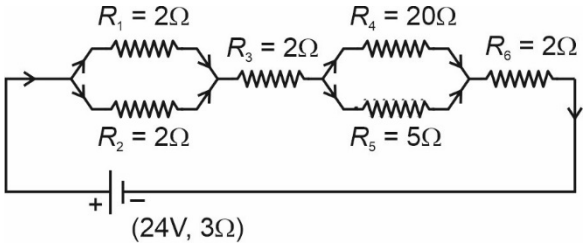
Answer (1)

Sol. A_m = Amplitude of modulating wave = 1 volt

A_c = Amplitude of carrier wave = 2 volt

Modulation index = $\frac{A_m}{A_c} = \frac{1}{2} = \frac{1}{2}$

8. As shown in the figure, a network of resistors is connected to a battery of 24V with an internal resistance of 3Ω. The currents through the resistors R_4 and R_5 are I_4 and I_5 respectively. The values of I_4 and I_5 are:



(1) $I_4 = \frac{8}{5}$A and $I_5 = \frac{2}{5}$A

(2) $I_4 = \frac{24}{5}$A and $I_5 = \frac{6}{5}$A

(3) $I_4 = \frac{2}{5}$A and $I_5 = \frac{8}{5}$A

(4) $I_4 = \frac{6}{5}$A and $I_5 = \frac{24}{5}$A

Answer (3)

Sol. $R_{eq} = \frac{R_1R_2}{R_1+R_2} + R_3 + \frac{R_4R_5}{R_5+R_4} + R_6 + r$

= (1 + 2 + 4 + 2 + 3)

= 12 Ω

$I = \frac{24}{12} = 2$ A

$I_4 = \left(\frac{R_5}{R_4+R_5}\right)I = \frac{5}{25}\times 2 = \frac{2}{5}$A

$I_5 = \frac{R_4 I}{R_4+R_5} = \frac{20}{25}\times I = \frac{8}{5}$A

9. Given below are two statements :

Statement I : The temperature of a gas is –73°C. When the gas is heated to 527°C, the root mean square speed of the molecules is doubled.

Statement II : The product of pressure and volume of an ideal gas will be equal to translational kinetic energy of the molecules.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both statement I and statement II are true

(2) Statement I is false but statement II is true

(3) Both statement I and statement II are false

(4) Statement I is true but statement II is false

Answer (4)

Sol. $T_i = 200$K $\quad v_{rms} \propto \sqrt{T}$

$T_f = 800$K

$\frac{V_i}{V_f} = \sqrt{\frac{T_i}{T_f}} = \sqrt{\frac{200}{800}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$V_f = 2V_i$

Translational K.E. $= \left(\frac{3}{2}PV\right)$

10. Two long straight wires *P* and *Q* carrying equal current 10A each were kept parallel to each other at 5 cm distance. Magnitude of magnetic force experienced by 10 cm length of wire *P* is F_1. If distance between wires is halved and currents on them are doubled, force F_2 on 10 cm length of wire *P* will be:

(1) $10F_1$ (2) $8F_1$

(3) $\frac{F_1}{10}$ (4) $\frac{F_1}{8}$

Answer (2)

Sol. $\frac{F_1}{l} = \frac{\mu_0 i_1}{2\pi r} i_2 = \frac{\mu_0 i_1 i_2}{2\pi r}$

If i_1 and i_2 both are doubled, if $r_f = \left(\frac{r}{2}\right)$

$\frac{F_f}{l} = \frac{\mu_0(2i_1)(2i_2)}{2\pi\left(\frac{r}{2}\right)} = \frac{8\mu_0 i_1 i_2}{2\pi r}$

$F_f = 8F_1$

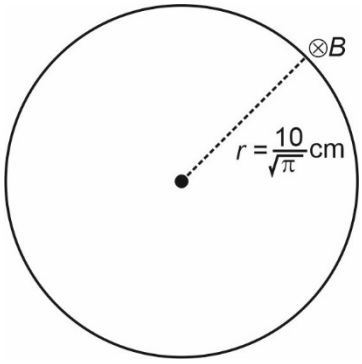
11. A conducting circular loop of radius $\frac{10}{\sqrt{\pi}}$cm is placed perpendicular to a uniform magnetic field of 0.5 T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is:

(1) emf = 1 mV (2) emf = 5 mV

(3) emf = 100 mV (4) emf = 10 mV

Answer (4)

Sol.



$\varepsilon_{ind} = \left|-\frac{d\phi}{dt}\right|$

$= \left(\frac{dB}{dt}\right)A$

$= \left(\frac{0.5}{0.5}\right)\pi\left(\frac{1}{10\sqrt{\pi}}\right)^2$

$= 1\times\frac{1}{100}$V

= 0.01 V

= 10 mV

12. 1 g of a liquid is converted to vapour at 3×10^5 P_a pressure. If 10% of the heat supplied is used for increasing the volume by 1600 cm^3 during this phase change, then the increase in internal energy in the process will be:

(1) 4800 J (2) 432000 J

(3) 4.32×10^8 J (4) 4320 J

Answer (4)

Sol. $\Delta Q = \Delta U + \Delta W$

$10\,\Delta W = \Delta W + \Delta W$

$\Delta U = 9\,\Delta W$

$= 9 \times (3 \times 10^5)\left[1600 \times \frac{1}{10^6}\right]$

= 4320 J

13. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force?

(1) $2d\sqrt{k}$ (2) $d\sqrt{k}$

(3) $1.5d\sqrt{k}$ (4) $k\sqrt{d}$

Answer (2)

Sol. q_1 ——— d ——— q_2

dielectric constant = K

$$F_{\text{medium}} = \frac{1}{4\pi(K\varepsilon_0)} \times \frac{q_1 q_2}{d^2}$$

$\because$ $F_{\text{air}} = F_{\text{medium}}$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(d_{\text{air}})^2} = \frac{1}{4\pi(K\varepsilon_0)}\frac{q_1 q_2}{d^2}$$

$\therefore$ $\boxed{d_{\text{air}} = \sqrt{K}d}$

14. Given below are two statements:

Statement I: An elevator can go up or down with uniform speed when its weight is balanced with the tension of its cable.

Statement II: Force exerted by the floor of an elevator on the foot of a person standing on it is more than his/her weight when the elevator goes down with increasing speed.

In the light of the above statements, choose the ***correct*** answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Statement I is false but Statement II is true

(4) Both statement I and Statement II are false

Answer (1)

Sol. **Statement I** is correct, because lift is moving with zero acceleration.

Statement II is incorrect as force exerted will be less than the weight.

15. In $\vec{E}$ and $\vec{K}$ represent electric field and propagation vectors of the EM waves in vacuum, then magnetic field vector is given by:

(ω – angular frequency):

(1) $\frac{1}{\omega}(\vec{K} \times \vec{E})$ (2) $\omega(\vec{E} \times \vec{K})$

(3) $\vec{K} \times \vec{E}$ (4) $\omega(\vec{K} \times \vec{E})$

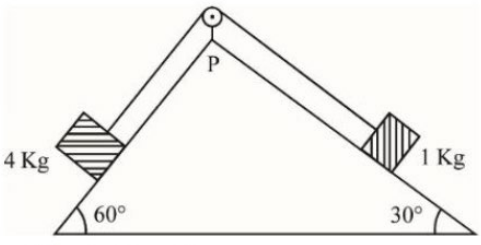
Answer (1)

Sol. $\vec{E} \Rightarrow$ electric field

$\vec{K} \Rightarrow$ propagation vector

$$\boxed{\vec{B} = \frac{1}{\omega}(\vec{K} \times \vec{E})}$$

16. As per given figure, a weightless pulley P is attached on a double inclined frictionless surfaces. The tension in the string (massless) will be (if g = 10 m/s^2)

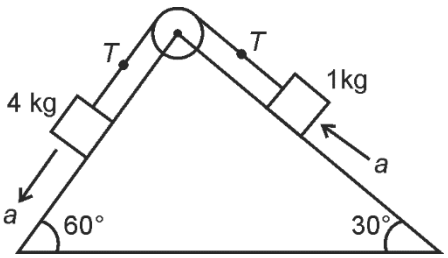


(1) $(4\sqrt{3}+1)N$ (2) $(4\sqrt{3}-1)N$

(3) $4(\sqrt{3}+1)N$ (4) $4(\sqrt{3}-1)N$

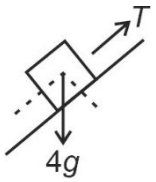
Answer (3)

Sol.



Let's consider *T* tension in string and *a* acceleration of blocks.

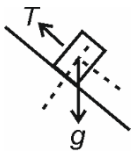
FBD of 4kg



$4g\sin\theta - T = 4a$

$$\frac{4g\sqrt{3}}{2} - T = 4a \qquad \ldots(1)$$

FBD of 1kg



$T - g\sin30° = a$

$$T - \frac{g}{2} = a \qquad \ldots(2)$$

From (1) and (2)

$T = 4\left(1+\sqrt{3}\right)N$

17. Match List I with List II

LIST I		LIST II	
A.	Planck's constant (h)	I.	$[M^1L^2T^{-2}]$
B.	Stopping potential (V_s)	II.	$[M^1L^1T^{-1}]$
C.	Work function (ϕ)	III.	$[M^1L^2T^{-1}]$
D.	Momentum (p)	IV.	$[M^1L^2T^{-3}A^{-1}]$

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II

(2) A-I, B-III, C-IV, D-II

(3) A-II, B-IV, C-III, D-I

(4) A-III, B-I, C-II, D-IV

Answer (1)

Sol. $[h] = [ML^2T^{-1}] \rightarrow$ A-III

$[V_s] = [ML^2T^{-3}A^{-1}] \rightarrow$ B-IV

$[\phi] = [ML^2T^{-2}] \rightarrow$ C-I

$[p] = [MLT^{-1}] \rightarrow$ D-II

18. From the photoelectric effect experiment, following observations are made. Identify which of these are correct.

A. The stopping potential depends only on the work function of the metal.

B. The saturation current increases as the intensity of incident light increases.

C. The maximum kinetic energy of a photo electron depends on the intensity of the incident light.

D. Photoelectric effect can be explained using wave theory of light.

Choose the correct answer from the options given below:

(1) A, B, D only

(2) A, C, D only

(3) B only

(4) B, C only

Answer (3)

Sol. (A) From Einstein's equation

$K_{max} = eV_s = h\nu - \phi$

Form the stopping potential (V_s) depends on ϕ & ν.

(B) Saturation current is proportional to intensity, *i.e.*, number of incident photons.

(C) K_{max} only depends on nature of photon and ϕ.

(D) Einstein used particle behaviour of photon to explain photon electric effect.

Only B is correct.

19. The maximum vertical height to which a man can throw a ball is 136 m. The maximum horizontal distance upto which he can throw the same ball is:

(1) 272 m

(2) 68 m

(3) 192 m

(4) 136 m

Answer (1)

Sol. For vertical throw,

$$h = \frac{v^2}{2g}$$

$$\nu = \sqrt{2gh} = \sqrt{2g \times 136} \qquad \ldots(1)$$

For max range, $\theta = 45°$

$$R_{max} = \frac{\nu^2}{g} \qquad \ldots(2)$$

From (1) and (2)

$$R_{max} = \frac{\nu^2}{g} = \frac{2g \times 136}{g}$$

$= 272$ m

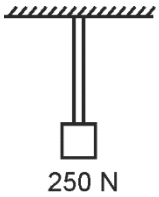
20. A 100 m long wire having cross-sectional area 6.25×10^{-4} m^2 and Young's modulus is 10^{10} Nm^{-2} is subjected to a load of 250 N, then the elongation in the wire will be:

(1) 6.25×10^{-6} m (2) 6.25×10^{-3} m

(3) 4×10^{-3} m (4) 4×10^{-4} m

Answer (3)

Sol. $\Delta l = \frac{Fl}{AY}$



F= 250 N

l = 100 m

$A = 6.25 \times 10^{-4}$ m^2

$Y = 10^{10}$

$\therefore \quad \Delta l = 4 \times 10^{-3}$ m

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A hole is drilled in a metal sheet. At 27°C, the diameter of hole is 5 cm. When the sheet is heated to 177°C, the change in the diameter of hole is $d \times 10^{-3}$ cm. The value of d will be _____ if coefficient of linear expansion of the metal is 1.6×10^{-5}/°C.

Answer (12)

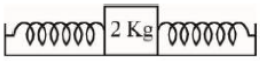
Sol. $\Delta D = D\alpha\Delta t$

$= 5 \times 1.6 \times 10^{-5} (177 - 27)$

$= 0.012$ cm

$= 12 \times 10^{-3}$ cm

so, d = 12

22. 

A block of a mass 2 kg is attached with two identical springs of spring constant 20 N/m each. The block is placed on a frictionless surface and the ends of the springs are attached to rigid supports (see figure). When the mass is displaced from its equilibrium position, it executes a simple harmonic motion. The time period of oscillation is $\frac{\pi}{\sqrt{x}}$ in SI unit. The value of x is _____.

Answer (5)

Sol. Both the springs are in parallel so net spring constant is $K_{net} = K_1 + K_2 = 40$ N/m

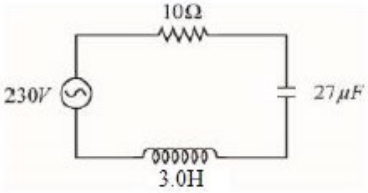
So $T = 2\pi\sqrt{\frac{m}{K_{net}}}$

$= 2\pi\sqrt{\frac{2}{40}}$

$= \frac{\pi}{\sqrt{5}}$

$x = 5$

23. In the circuit shown in the figure, the ratio of the quality factor and the band width is _____ s.



Answer (10)

Sol. Bandwidth $\Delta\omega = \frac{R}{L}$

Quality factor $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

So $\frac{Q}{\Delta\omega} = \frac{\frac{1}{R}\sqrt{\frac{L}{C}}}{\frac{R}{L}}$

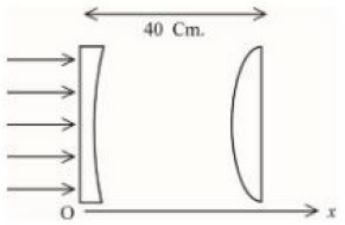
$= \frac{L^{\frac{3}{2}}}{R^2\sqrt{C}}$

$= \frac{3^{\frac{3}{2}}}{10^2(27\times10^{-6})^{\frac{1}{2}}}$

$= \frac{3\sqrt{3}}{100(3\sqrt{3}\times10^{-3})}$

$= 10$

24. As shown in the figure, a combination of a thin plano concave lens and a thin plano convex lens is used to image an object placed at infinity. The radius of curvature of both the lenses is 30 cm and refraction index of the material for both the lenses is 1.75. Both the lenses are placed at distance of 40 cm from each other. Due to the combination, the image of the object is formed at distance x = ____ cm, from concave lens.



Answer (120)

Sol. $\frac{1}{f_{concave}} = (1.75-1)\left(\frac{1}{\infty}-\frac{1}{+30}\right) = -\frac{0.75}{30}$

$f_{concave} = -40$ cm

$\frac{1}{f_{convex}} = (1.75-1)\left(\frac{1}{30}-\frac{1}{\infty}\right) = \frac{0.75}{30}$

$f_{convex} = 40$ cm

Let the first image is formed at v_1 so

$\frac{1}{v_1}-\frac{1}{\infty} = \frac{1}{f_{concave}} = -\frac{1}{40}$

$\Rightarrow \quad v_1 = -40$ cm

for second image

$\frac{1}{x-40}-\frac{1}{-80} = \frac{1}{40}$

$\Rightarrow$ x = 120 cm

25. A spherical body of mass 2 kg starting from rest acquires a kinetic energy of 10000 J at the end of 5th second. The force acted on the body is _____ N.

Answer (40)

Sol. Let the force be F so acceleration $a = \frac{F}{m}$

So displacement $S = \frac{1}{2}at^2 = \frac{Ft^2}{2m}$

So work done W = F.S $= \frac{F^2t^2}{2m}$

From work energy Theorem

$\Delta KE = W$

$W = \frac{F^2t^2}{2m} = 10000$

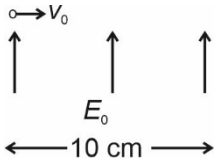
$F = \sqrt{\frac{10000\times 2\times 2}{5^2}}$

$F = 40$ N

26. A stream of a positively charged particles having $\frac{q}{m} = 2\times 10^{11}\frac{C}{kg}$ and velocity $\vec{v}_0 = 3\times 10^7 \hat{i}\, m/s$ is deflected by an electric field $1.8\hat{j}$ kV/m. The electric field exists in a region of 10 cm along x direction. Due to the electric field, the deflection of the charge particles in the y direction is ____ mm.

Answer (2)

Sol.



$F_y = \frac{qE_y}{m}$

$a_y = 2\times 10^{11}\times 1800$

$= 36\times 10^{13}$ m/s^2

Time = $= \frac{10\times 10^{-2}}{v_0} = \frac{0.1}{3\times 10^7} = \left(\frac{1}{3}\times 10^{-8}\right)$sec.

$\therefore \quad y = \frac{1}{2}at^2$

$\Rightarrow y = \frac{1}{2}\times 36\times 10^{13}\times\left(\frac{1}{3}\times 10^{-8}\right)^2$

$= 2\times 10^{-3}$ m

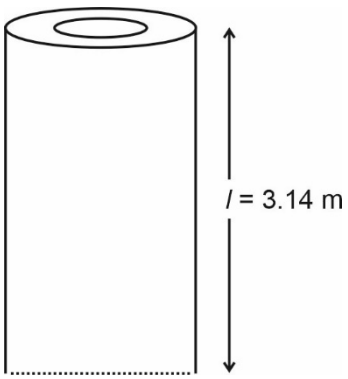
$= 2$ mm

27. A hollow cylindrical conductor has length of 3.14 m, while its inner and outer diameters are 4 mm and 8 mm respectively. The resistance of the conductor is $n\times 10^{-3}\,\Omega$.

If the resistivity of the material is $2.4\times 10^{-8}\,\Omega$m. The value of n is _____.

Answer (2)

Sol.



$$R = \frac{\rho l}{A} = \frac{2.4\times10^{-8}\times3.14}{\pi(16-4)\times10^{-6}}$$

$$= \frac{2.4}{12}\times10^{-2}$$

$$= \frac{24}{12}\times10^{-3}$$

$$= 2\times10^{-3}\,\Omega$$

value of n is 2.

28. Assume that protons and neutrons have equal masses. Mass of a nucleon is 1.6×10^{-27} kg and radius of nucleus is $1.5 \times 10^{-15}\ A^{1/3}$ m. The approximate ratio of the nuclear density and water density is $n \times 10^{13}$. The value of n is _____.

Answer (11)

Sol. Radius = $1.5\times10^{-15}A^{1/3}$

Volume $= \frac{4\pi}{3}r^3$

Mass of nucleus = $(1.6 \times 10^{-27})\ A$ kg

Density of nucleus $= \frac{1.6\times10^{-27}\times A}{\frac{4}{3}\times\pi\times\left(1.5\times10^{-15}\ A^{\frac{1}{3}}\right)^3}$

$$= \frac{1.6\times3\times8\times10^{18}}{4\pi\times27}$$

$$= \frac{32}{9\pi}\times10^{17}$$

Density of water = 1000 kg/m^3

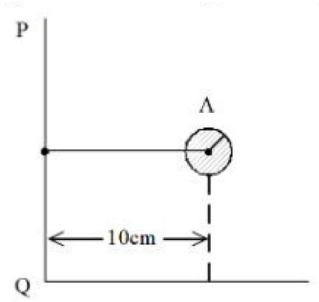
$$\frac{\text{Density of nucleus}}{\text{Density of water}} = \frac{\frac{32}{9\pi}\times10^{17}}{1000}$$

$$= \frac{320}{9\pi}\times10^{13}$$

$= 11.32 \times 10^{13}$

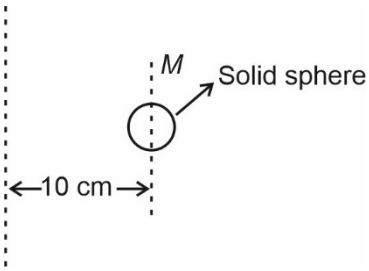
$\boxed{\text{value of } n = 11}$

29. Solid sphere A is rotating about an axis PQ. If the radius of the sphere is 5 cm then its radius of gyration about PQ will be $\sqrt{x}$ cm. The value of x is _____.



Answer (110)

Sol.



$I = I_{cm} + Md^2$

$\Rightarrow \quad MK^2 = \frac{2}{5}MR^2 + Md^2$

$K = \sqrt{\frac{2}{5}R^2 + d^2}$

$K = \sqrt{\frac{2}{5}\times5^2 + 10^2}$ cm

$K = \sqrt{110}$ cm

Value of $\boxed{x = 110}$.

30. Vectors $a\hat{i} + b\hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $\frac{x}{2}$. The value of x is _____.

Answer (1)

Sol. $\left(a\hat{i} + b\hat{j} + \hat{k}\right)\cdot\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) = 0$

$2a - 3b + 4 = 0$

$(2a - 3b = -4 \times) 7$ … (1)

also, $(3a + 2b = 7 \times) 4$ … (2)

adding (1) and (2)

$14a + 12a - 21b + 8b = 0$

$26a - 13b = 0$

$\frac{a}{b} = \frac{13}{26} = \frac{1}{2}$

Value of $\boxed{x = 1}$.

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

31. Given below are two statements:

Statement I : Noradrenaline is a neurotransmitter.

Statement II : Low level of noradrenaline is not the cause of depression in human.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both statement I and Statement II are incorrect

(4) Both statement I and Statement II are correct

Answer (1)

Sol.
- Noradrenaline is a neurotransmitter.
- Low level of noradrenaline is a cause for depression in human.

32. Reaction of BeO with ammonia and hydrogen fluoride gives A which on thermal decomposition gives BeF_2 and NH_4F. What is 'A'?

(1) H_3NBeF_3 (2) $(NH_4)BeF_3$

(3) $(NH_4)Be_2F_5$ (4) $(NH_4)_2BeF_4$

Answer (4)

Sol.

$$BeO + NH_3 + HF \longrightarrow \underset{(A)}{(NH_4)_2BeF_4} \xrightarrow{\Delta} BeF_2 + NH_4F$$

Compound A is $(NH_4)_2BeF_4$

33. **Statement I :** For colloidal particles, the values of colligative properties are of small order as compared to values shown by true solutions at same concentration.

Statement II : For colloidal particles, the potential difference between the fixed layer and the diffused layer of same charges is called the electrokinetic potential or zeta potential.

In the light of the above statements, choose the correct answer from the options given below

(1) Both statement I and Statement II are true

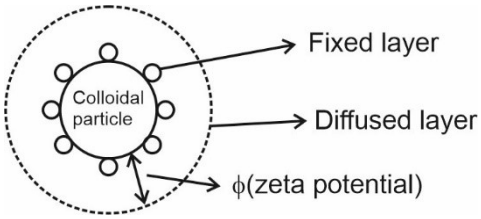
(2) Both statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Answer (3)

Sol. For colloidal particles value of colligative properties is less as compared to true solutions at same concentration as number of particles are less.



But fixed layer and diffused layer have opposite charges.

34. In the depression of freezing point experiment

A. Vapour pressure of the solution is less than that of pure solvent

B. Vapour pressure of the solution is more than that of pure solvent

C. Only solute molecules solidify at the freezing point

D. Only solvent molecules solidify at the freezing point

Choose the most appropriate answer from the options given below:

(1) A and D only (2) B and C only

(3) A only (4) A and C only

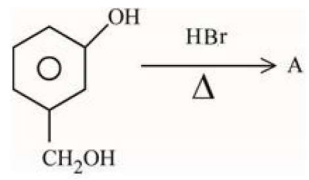
Answer (1)

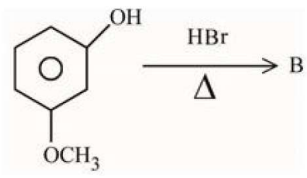
Sol. A and D are correct

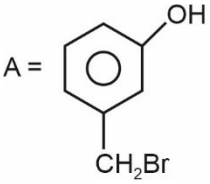
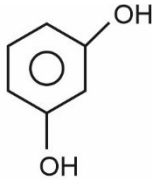
as $(vp)_{solution} < (vp)_{solvent}$

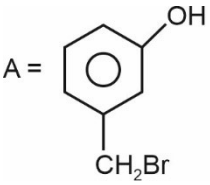
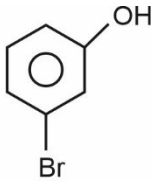
and only solvent particles undergoes solidification.

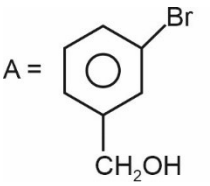
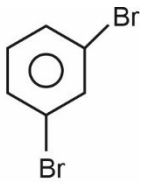
35. 'A' and 'B' formed in the following set of reactions are:

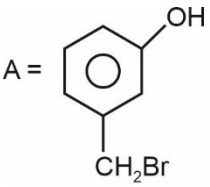
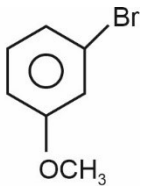




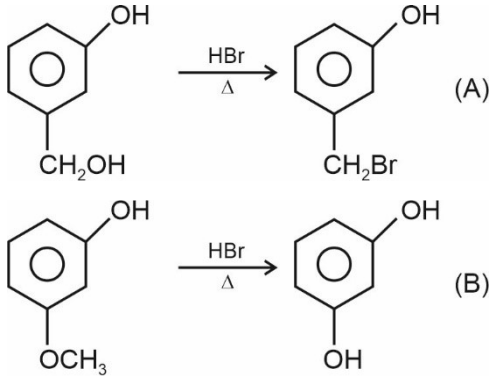
(1) A =  , B = 

(2) A =  , B = 

(3) A =  , B = 

(4) A =  , B = 

Answer (1)

Sol. 

36. The primary and secondary valencies of cobalt respectively in $[Co(NH_3)_5Cl]Cl_2$ are :

(1) 3 and 5 (2) 2 and 6

(3) 2 and 8 (4) 3 and 6

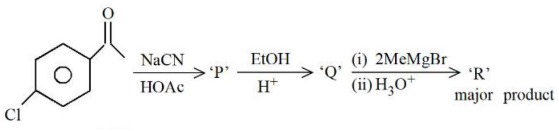
Answer (4)

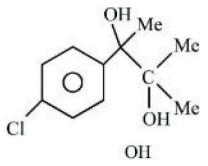
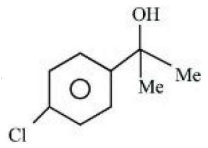
Sol. $[Co(NH_3)_5Cl]Cl_2$

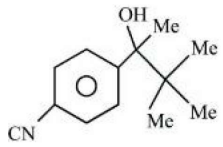
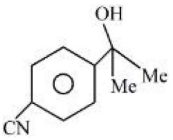
Oxidation no. = primary valencies = 3

Co-ordination no. = secondary valencies = 6

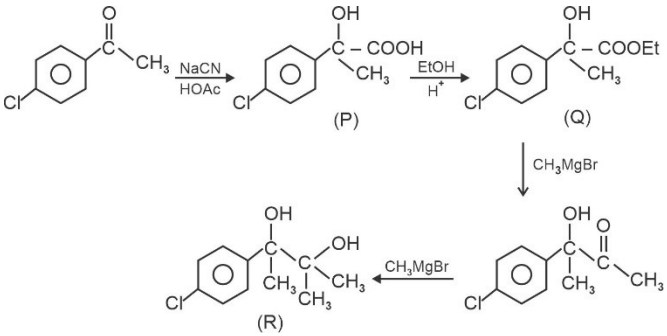
37. 'R' formed in the following sequence of reactions is



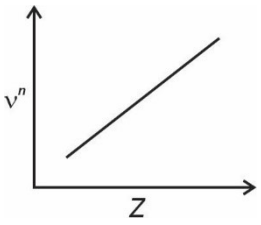
(1)  (2) 

(3)  (4) 

Answer (1)

Sol. 

38. It is observed that characteristic X-ray spectra of elements show regularity. When frequency to the power "n" i.e. ν^n of X-rays emitted is plotted against atomic number "Z", following graph is obtained.



(1) 1 (2) 2

(3) 3 (4) $\frac{1}{2}$

Answer (4)

Sol. $h\nu = \Delta E = 13.6 \times Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow \nu \propto Z^2$

$\Rightarrow (\nu)^{1/2} \propto Z \quad \left(n = \frac{1}{2}\right)$

39. The magnetic moment of a transition metal compound has been calculated to be 3.87 B.M. The metal ion is

(1) Ti^{2+} (2) Mn^{2+}

(3) Cr^{2+} (4) V^{2+}

Answer (4)

Sol. $\mu = 3.87$

$n = 3$

$V_{23} = 4s^2 3d^3$

$V^{2+} = 4s^0 3d^3$ (n = 3)

40. Which of the following is true about freons?

(1) These are chemicals causing skin cancer

(2) These are radicals of chlorine and chlorine monoxide

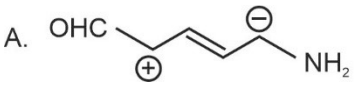
(3) All radicals are called freons

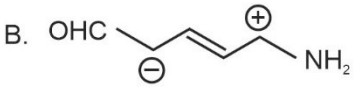
(4) These are chlorofluorocarbon compounds

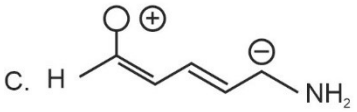
Answer (4)

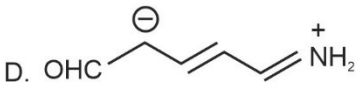
Sol. Freons are chlorofluorocarbons

41. Increasing order of stability of the resonance structures is:

A. 

B. 

C. 

D. 

Choose the correct answer from the options given below:

(1) C, D, A, B (2) D, C, B, A

(3) D, C, A, B (4) C, D, B, A

Answer (Not given in the options)

Sol. Correct stabilising order is

C < A < B < D

(This question should be given bonus)

42. **Assertion A:** Hydrolysis of an alkyl chloride is a slow reaction but in the presence of NaI, the rate of the hydrolysis increases.

Reason R: I^- is a good nucleophile as well as a good leaving group.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) **A** is false but **R** is true

(3) **A** is true but **R** is false

(4) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

Answer (1)

Sol. $CH_3 - Cl \xrightarrow{H_2O} CH_3OH \quad (\text{slow process}) \ldots (r_1)$

$$CH_3 - Cl \xrightarrow[\substack{I^- \text{being a Good} \\ \text{Nucleophile}}]{NaI} CH_3 - I \xrightarrow[\substack{I^- \text{being a Good} \\ \text{leaving group}}]{H_2O} CH_3 - OH \ldots (r_2)$$

$r_2 >> r_1$

as I^- is a good nucleophile as well as good leaving group.

43. Match List I with List II

	LIST I		LIST II
A.	Chlorophyll	I.	Na_2CO_3
B.	Soda ash	II.	$CaSO_4$
C.	Dentistry, Ornamental work	III.	Mg^{2+}
D.	Used in white washing	IV.	$Ca(OH)_2$

Choose the correct answer from the options given below:

(1) A-II, B-I, C-III, D-IV

(2) A-III, B-IV, C-I, D-II

(3) A-II, B-III, C-IV, D-I

(4) A-III, B-I, C-II, D-IV

Answer (4)

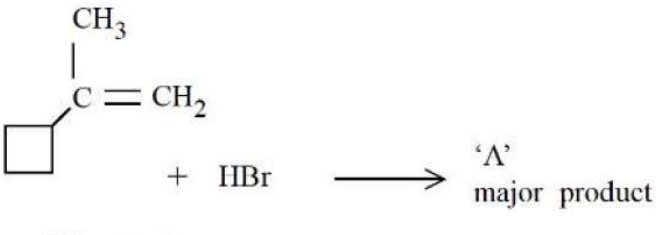
Sol. Chlorophyll contains Mg^{2+} Ions (A – III)

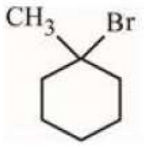
Soda ash is Na_2Co_3 (B – I)

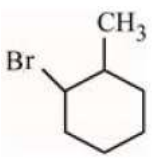
Dentistry; ornamental work – $CaSO_4$ (C – II)

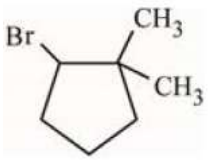
Used in white washing – $Ca(OH)_2$ (D – IV)

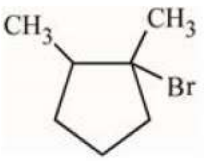
44. In the following given reaction, 'A' is



(1) 

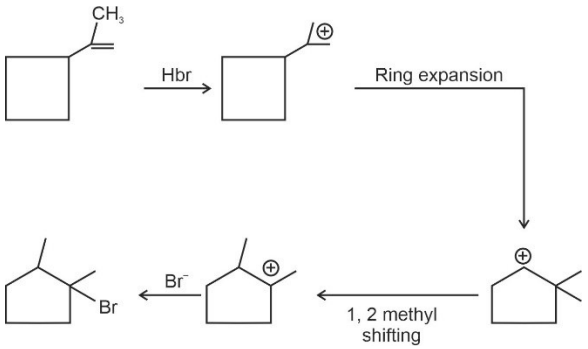
(2) 

(3) 

(4) 

Answer (4)

Sol.



45. Match List I with List II

LIST I		LIST II	
A.	Reverberatory furnace	I.	Pig Iron
B.	Electrolytic cell	II.	Aluminum
C.	Blast furnace	III.	Silicon
D.	Zone refining furnace	IV.	Copper

Choose the correct answer from the options given below:

(1) A-I, B-IV, C-II, D-III (2) A-I, B-III, C-II, D-IV

(3) A-IV, B-II, C-I, D-III (4) A-III, B-IV, C-I, D-II

Answer (3)

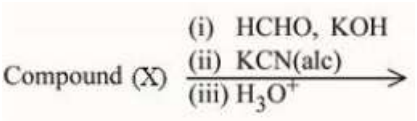
Sol. (A) Reverberatory furnance is used for extraction of copper.

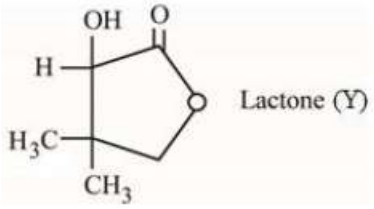
(B) Electrolytic cell is used for obtaining highly reactive metals like aluminium.

(C) Blast furnace is used for extraction of Iron.

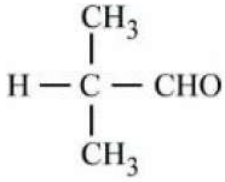
(D) Zone refining furnace is used for silicon.

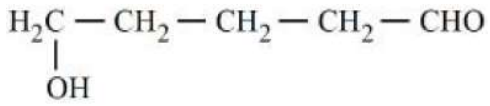
46. Compound (X) undergoes following sequence of reactions to give the Lactone (Y).

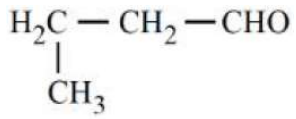


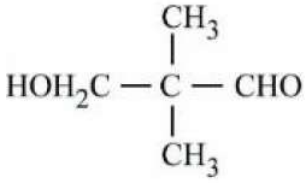


Compound (X) is

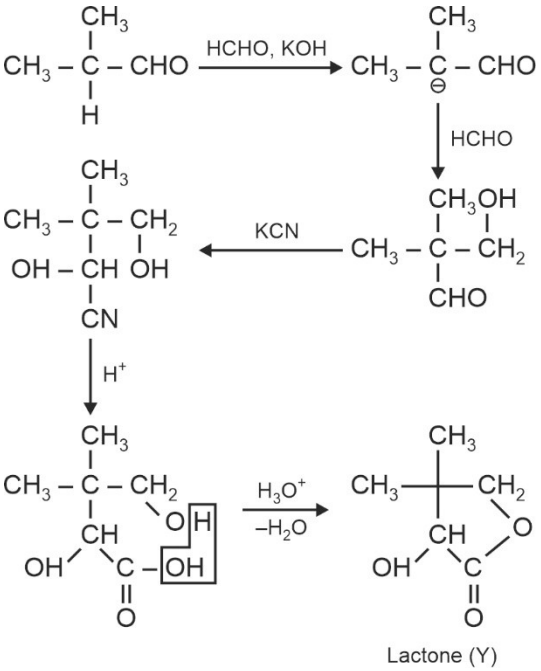
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

47. Which of the phosphorus oxoacid can create silver mirror from $AgNO_3$ solution?

(1) $H_4P_2O_5$

(2) $(HPO_3)_n$

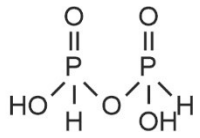
(3) $H_4P_2O_7$

(4) $H_4P_2O_6$

Answer (1)

Sol. $H_4P_2O_5$ can act as a reducing agent due to (P – H) bond.

$H_4P_2O_5$



48. Decreasing order of the hydrogen bonding in following forms of water is correctly represented by

A. Liquid water

B. Ice

C. Impure water

Choose the correct answer from the options given below:

(1) A > B > C

(2) B > A > C

(3) A = B > C

(4) C > B > A

Answer (2)

Sol. Extent of hydrogen bonding :

Ice > liquid water > impure water

- In ice, 4 molecules of H_2O are connected to H_2O molecule.
- Impure water will have less hydrogen bonding.

49. An ammoniacal metal salt solution gives a brilliant red precipitate on addition of dimethylglyoxime. The metal ion is

(1) Ni^{2+}

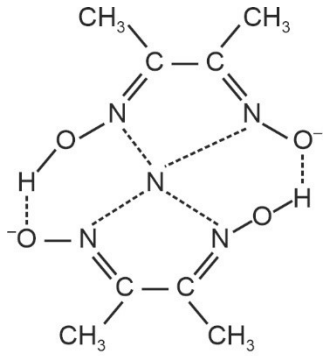
(2) Cu^{2+}

(3) Fe^{2+}

(4) Co^{2+}

Answer (1)

Sol. Ni^{2+} forms cherry red ppt with dmg



$[Ni(dmg)_2]$

50. Order of covalent bond :

A. KF > KI; LiF > KF

B. KF < KI; LiF > KF

C. SnCl4 > SnCl2; CuCl > NaCl

D. LiF > KF; CuCl < NaCl

E. KF < KI; CuCl > NaCl

Choose the correct answer from the options given below:

(1) B, C only (2) C, E only

(3) B, C, E only (4) A, B only

Answer (3)

Sol. B is correct KF < KI; LiF > KF

C is correct $\underset{+4}{SnCl_4} > \underset{+2}{SnCl_2}$; CuCl > NaCl

E is correct KF < KI; CuCl > NaCl

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

51. The d-electronic configuration of $[CoCl_4]^{2-}$ in tetrahedral crystal field is $e^m t_2^n$. Sum of "m" and "number of unpaired electrons" is _____.

Answer (7)

Sol. $[CoCl_4]^{2-}$

$Co^{2+} = 4s^0 3d^7$ $\uparrow\downarrow$ $\uparrow$ $\uparrow$ t_{2g}

m = 4 $\uparrow\downarrow$ $\uparrow\downarrow$ e_g

= $e_g^4 t_{2g}^3$

Number of unpaired electrons = 3

52. The dissociation constant of acetic acid is $x \times 10^{-5}$. When 25 mL of 0.2 M CH_3COONa solution is mixed with 25 mL of 0.02 M CH_3COOH solution, the pH of the resultant solution is found to be equal to 5. The value of *x* is _____.

Answer (10)

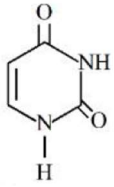
Sol. $pH = pK_a + \log\left(\frac{25\times0.2}{25\times0.02}\right)$

$5 = pK_a + \log 10$

$pK_a = 4 \Rightarrow K_a = 10^{-4} = 10 \times 10^{-5}$

$x = 10$

53. Uracil is a base present in RNA with the following structure. % of N in uracil is _____.



Given :

Molar mass N = 14 g mol^{-1}

O = 16 g mol^{-1}

C = 12 g mol^{-1}

H = 1 g mol^{-1}

Answer (25)

Sol. Uracil is $C_4H_4N_2O_2$

$$\% \text{ by mass of N} = \frac{14\times2}{112}\times100$$

$= 25\%$

54. The number of correct statement/s from the following is _____.

A. Larger the activation energy, smaller is the value of the rate constant.

B. The higher is the activation energy, higher is the value of the temperature coefficient.

C. At lower temperatures, increase in temperature causes more change in the value of k than at higher temperature.

D. A plot of ln k vs $\frac{1}{T}$ is a straight line with slope equal to $-\frac{E_a}{R}$.

Answer (3)

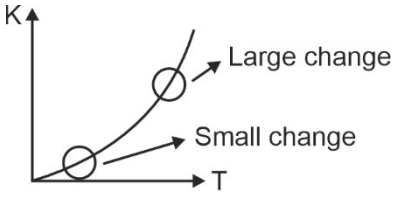
Sol. (A) $k = Ae^{-\frac{E_a}{RT}}\left(E_a\uparrow \quad k\downarrow\right)$

(B) $\ln k = \ln A - \frac{E_a}{RT}$

$$\frac{1}{k}\cdot\frac{dk}{dT} = \frac{+E_a}{RT^2}$$

$E_a \uparrow$ temp. coefficient $\uparrow$

(C)



(D) $\ln k = \ln A - \frac{E_a}{RT}$

Slope of ln k vs $\frac{1}{T}$ is $\left(\frac{-E_a}{R}\right)$

55. At 298 K, a 1 litre solution containing 10 mmol of $Cr_2O_7^{2-}$ and 100 mmol of Cr^{3+} shows a pH of 3.0.

Given : $Cr_2O_7^{2-} \rightarrow Cr^{3+}$; E° = 1.330V

and $\frac{2.303\,RT}{F} = 0.059$ V

The potential for the half cell reaction is $x \times 10^{-3}$ V. The value of x is _____.

Answer (917)

Sol. $6e^- + \underset{10^{-3}m}{14H^+} + \underset{10^{-2}m}{Cr_2O_7^{2-}} \rightarrow \underset{10^{-1}m}{2Cr^{3+}} + 7H_2O$

$$E_{cell} = 1.330 - \frac{0.059}{6}\log\frac{10^{-2}}{(10^{-2})(10^{-42})}$$

$$= 1.330 - \frac{0.059}{6}(42)$$

$= 1.330 - 0.413$

$= 0.917 = 917 \times 10^{-3}$

$x = 917$

56. 5 g of NaOH was dissolved in deionized water to prepare a 450 mL stock solution. What volume (in mL) of this solution would be required to prepare 500 mL of 0.1 M solution?_____

Given : Molar Mass of Na, O and H is 23, 16 and 1 g mol^{-1} respectively

Answer (180)

Sol. Molarity of solution = $\frac{5}{(40)}\frac{(1000)}{(450)}$

$\Rightarrow M\times V = 500\times.1$

$\Rightarrow \frac{5}{450}\times\frac{1000}{450}\times V = 500\times.1$

$\boxed{V = 180 \text{ mL}}$

57. For independent processes at 300 K

Process	ΔH/kJ mol^{-1}	ΔS/J K^{-1}
A	–25	–80
B	–22	40
C	25	–50
D	22	20

The number of non-spontaneous processes from the following is_____

Answer (2)

Sol. – C is non – spontaneous as $(\Delta H > 0, \Delta S < 0)$

D is Non - spontaneous

For D,

$\Delta G = \Delta H - T\Delta S$

= 22,000 – (300) (20)

= (22,000 – 6000) > 0

Non-spontaneous as ($\Delta G > 0$)

58. When $Fe_{0.93}O$ is heated in presence of oxygen, it converts to Fe_2O_3. The number of **correct** statement/s from the following is_____

A. The equivalent weight of $Fe_{0.93}O$ is $\frac{\text{Molecular weight}}{0.79}$

B. The number of moles of Fe^{2+} and Fe^{3+} in 1 mole of $Fe_{0.93}O$ is 0.79 and 0.14 respectively.

C. $Fe_{0.93}O$ is metal deficient with lattice comprising of cubic closed packed arrangement of O^{2-} ions.

D. The % composition of Fe^{2+} and Fe^{3+} in $Fe_{0.93}O$ is 85% and 15% respectively.

Answer (4)

Sol. A : $Fe_{0.93}O \longrightarrow Fe_2O_3$

$nF = \left(3 - \frac{200}{93}\right) \times .93$

= .79

$\boxed{\text{eq. wt} = \frac{mw}{.79}}$

B : $Fe_{0.93}O$ $\qquad$ $Fe_{93}O_{100}$

$Fe_x^{+2}\ Fe_{(93-x)}^{+3}O_{100}$

$2x + 3(93 - x) = 200$

$x = 79$

% of $Fe^{2+} = \frac{79}{93} \times 100 = 85\%$

% of $Fe^{3+} = \frac{14}{93} \times 100 = 15\%$

= $Fe_{0.93}O$ is metal deficient compound

A, B, C, D are correct.

59. If wavelength of the first line of the Paschen series of hydrogen atom is 720 nm, then the wavelength of the second line of this series is_____ nm. (Nearest integer)

Answer (492)

Sol. $\frac{1}{720} = R \times \left(\frac{1}{9} - \frac{1}{16}\right)$

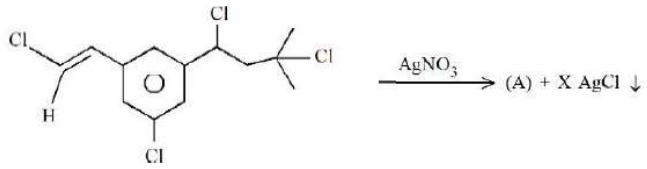
$\Rightarrow R = \frac{9 \times 16}{720 \times 7}$

$\frac{1}{\lambda'} = \frac{9 \times 16}{720 \times 7} \times \left(\frac{1}{9} - \frac{1}{25}\right)$

λ' = 492.18 nm

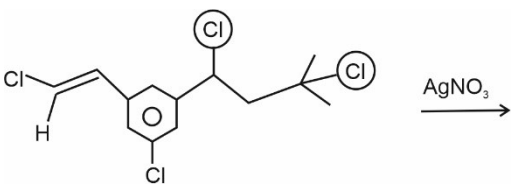
$\boxed{\lambda' = 492\ \text{nm}}$ (nearest integer)

60. Number of moles of AgCl formed in the following reaction is_____



Answer (2)

Sol.



Circled Cl will get precipitated

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. Let N denote the number that turns up when a fair die is rolled. If the probability that the system of equations

$x + y + z = 1$

$2x + Ny + 2z = 2$

$3x + 3y + Nz = 3$

has unique solution is $\frac{k}{6}$, then the sum of value of k and all possible values of N is

(1) 18 (2) 20

(3) 21 (4) 19

Answer (2)

Sol. For unique solution $\Delta \neq 0$

i.e. $\begin{vmatrix} 1 & 1 & 1 \\ 2 & N & 2 \\ 3 & 3 & N \end{vmatrix} \neq 0$

$\Rightarrow (N^2 - 6) - (2N - 6) + (6 - 3N) \neq 0$

$\Rightarrow N^2 - 5N + 6 \neq 0$

$\therefore N \neq 2$ and $N \neq 3$

$\therefore$ Probability of not getting 2 or 3 in a throw of dice $= \frac{2}{3}$

As given $\frac{2}{3} = \frac{k}{6} \Rightarrow k = 4$

$\therefore$ Required value = 1 + 4 + 5 + 6 + 4 = 20

62. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}, \vec{v} = 2\hat{i} + \hat{j} - \hat{k}, \vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to

(1) 2 (2) 1

(3) $\frac{3}{2}$ (4) $-\frac{2}{3}$

Answer (2)

Sol. Given $\vec{v} \times \vec{w} = \lambda\vec{v} + \vec{u}$ ...(i)

Taking dot with $\vec{v}$ we get

$[\vec{v}\,\vec{v}\,\vec{w}] = \lambda|\vec{v}|^2 + \vec{u} \cdot \vec{v}$

Substituting values we have

$6\lambda + 3 = 0 \Rightarrow \lambda = -\frac{1}{2}$

$\therefore$ Equation (i) becomes

$\vec{v} \times \vec{w} = \vec{u} - \frac{\vec{v}}{2}$ ...(ii)

Taking dot with $\vec{w}$ of (ii) we get

$0 = \vec{u} \cdot \vec{w} - \frac{\vec{v} \cdot \vec{w}}{2}$

$\Rightarrow \vec{u} \cdot \vec{w} = \frac{2}{2} = 1$ (as $\vec{v} \cdot \vec{w} = 2$ given)

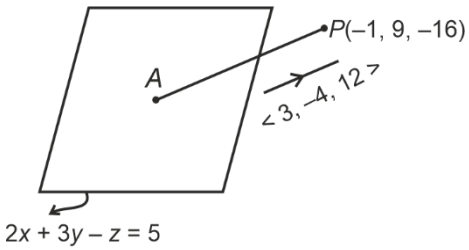
63. The distance of the point (–1, 9, –16) from the plane $2x + 3y - z = 5$ measured parallel to the line $\frac{x+4}{3} = \frac{2-y}{4} = \frac{z-3}{12}$ is

(1) $20\sqrt{2}$ (2) $13\sqrt{2}$

(3) 26 (4) 31

Answer (3)

Sol. $\frac{x+4}{3} = \frac{y-2}{-4} = \frac{z-3}{12}$



Equation of line AP

$\frac{x+1}{3} = \frac{y-9}{-4} = \frac{z+16}{12}$

Point $A(3\lambda - 1, -4\lambda + 9, 12\lambda - 16)$ lies on $2x + 3y - z = 5$, $\Rightarrow 6\lambda - 2 - 12\lambda + 27 - 12\lambda + 16 = 5 \Rightarrow \lambda = 2$

$\Rightarrow$ Point $A(5, 1, 8)$

$\Rightarrow AP^2 = 6^2 + 8^2 + 24^2 = 4(9 + 16 + 144) = 4 \times 169$

$AP = 26$

Option (3) is correct.

64. Let $y = y(x)$ be the solution of the differential equation $x^3\,dy + (xy - 1)dx = 0$, $x > 0$, $y\left(\frac{1}{2}\right) = 3 - e$. Then $y(1)$ is equal to

(1) e (2) $2 - e$

(3) 3 (4) 1

Answer (4)

Sol. $x^3dy + xy\,dx - dx = 0$

$\Rightarrow \frac{dy}{dx} = \frac{1 - xy}{x^3}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$

I.F. $= e^{\int \frac{dx}{x^2}} = e^{-\frac{1}{x}}$

$\therefore \quad ye^{-\frac{1}{x}} = \int \frac{e^{-\frac{1}{x}}}{x^3}dx$

For RHS put $-\frac{1}{x} = t \Rightarrow \frac{dx}{x^2} = dt$

$\therefore \quad ye^{-\frac{1}{x}} = -\int te^t dt$

$\Rightarrow \quad ye^{-\frac{1}{x}} = -\left[te^t - e^t\right] + c$

$\Rightarrow \quad ye^{-\frac{1}{x}} = \frac{e^{-\frac{1}{x}}}{x} + e^{-\frac{1}{x}} + c$ ...(i)

$\downarrow y\left(\frac{1}{2}\right) = 3 - e$

$\Rightarrow (3 - e)e^{-2} = 2e^{-2} + e^{-2} + c$

$\Rightarrow \quad c = -\frac{1}{e}$ ...(ii)

For $y(1)$ put $x = 1$, $c = -e^{-1}$ in equation (i) we get

$ye^{-1} = e^{-1} + e^{-1} - e^{-1}$

$\Rightarrow \quad y = 1$

65. If A and B are two non-zero $n \times n$ matrics such that $A^2 + B = A^2B$, then

(1) $A^2B = BA^2$ (2) $A^2B = I$

(3) $A^2 = I$ or $B = I$ (4) $AB = I$

Answer (1)

Sol. Given : $A^2 + B = A^2B$ ...(i)

$\Rightarrow \quad A^2 + B - I = A^2B - I$

$\Rightarrow \quad A^2B - A^2 - B + I = I$

$\Rightarrow \quad A^2(B - I) - I(B - I) = I$

$\Rightarrow \quad (A^2 - I)(B - I) = I$

$\therefore$ $A^2 - I$ is the inverse matrix of $B - I$ and vice versa.

So, $(B - I)(A^2 - I) = I$

$\Rightarrow \quad BA^2 - B - A^2 + I = I$

$\therefore \quad A^2 + B = BA^2$ ...(ii)

So, by (i) and (ii)

$A^2B = BA^2$

$\therefore$ Option (1) is correct.

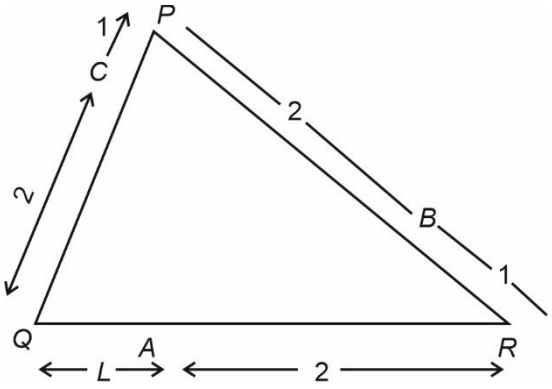
66. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{Area(\Delta PQR)}{Area(\Delta ABC)}$ is equal to

(1) $\frac{5}{2}$ (2) 4

(3) 3 (4) 2

Answer (3)

Sol.



By PSY formula

$\frac{\Delta ABC}{\Delta PQR} = \frac{(PC \times QA \times RB) + (CQ \times AR \times BP)}{PQ \times QR \times RP}$

$= \frac{8 + 1}{3 \times 3 \times 3} = \frac{1}{3}$

67. For three positive integers p, q, r, $x^{pq^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that 3, 3 $\log_y x$, 3$\log_z y$, 7 $\log_x z$ are in A.P with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to

(1) 12 (2) 2

(3) –6 (4) 6

Answer (2)

Sol. $x^{pq^2} = y^{qr} = z^{p^2r}$

$3\log_y x = \frac{7}{2}, 3\log_z y = 4, 7\log_x z = \frac{9}{2}$

$\Rightarrow \quad x = y^{\frac{7}{6}}, y = z^{\frac{4}{3}}, z = x^{\frac{9}{14}}$

$y^{\frac{7}{6}pq^2} = y^{qr} = y^{\frac{3}{4}p^2r}$

$\Rightarrow \quad \frac{7}{6}pq^2 = qr = \frac{3}{4}p^2r$

$\therefore \quad 7pq = 6r,\ 4q = 3p^2$

$r = pq + 1$

$r = \frac{6r}{7} + 1 \Rightarrow r = 7$

$pq = 6$

$p\left(\frac{3p^2}{4}\right) = 6$

$p = 2,\ q = 3$

$r - p - q = 7 - 5 = 2$

68. The relation

$R = \{(a,b) : gcd(a,b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is

(1) Reflexive but not symmetric

(2) Neither symmetric nor transitive

(3) Symmetric but not transitive

(4) Transitive but not reflexive

Answer (3)

Sol. $gcd\ (a, a) = a$ so (a, a) so $\in R \Rightarrow$ not reflexive

If $gcd\ (a, b) = 1 \Rightarrow gcd\ (b, a) = 1$

$\therefore \quad (b, a) \in R \Rightarrow$ Symmetric

If $gcd\ (a, b) = 1$ and $gcd\ (b, c) = 1$

$\Rightarrow \quad gcd\ (a, c) = 1$

$\therefore \quad R$ is not transitive

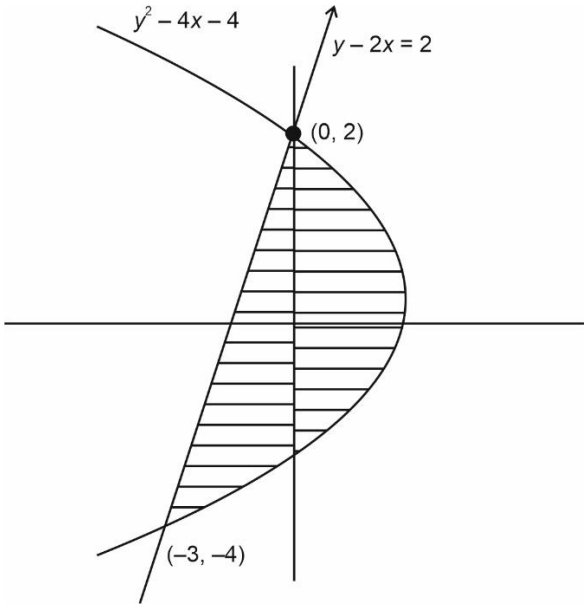
69. The area enclosed by the curves $y^2 + 4x = 4$ and $y - 2x = 2$ is

(1) $\frac{23}{3}$ (2) $\frac{22}{3}$

(3) $\frac{25}{3}$ (4) 9

Answer (4)

Sol.



Required area = $\int_{-4}^{2}\left(\frac{4-y^2}{4} - \frac{y-2}{2}\right)dy$

$= \int_{-4}^{2}\frac{8-2y-y^2}{4}dy$

$= \frac{1}{4}\left\{8y - y^2 - \frac{y^3}{3}\right\}_{-4}^{2}$

= 9 square units

70. $\lim\limits_{t\to 0}\left(1^{\frac{1}{\sin^2 t}} + 2^{\frac{1}{\sin^2 t}} + \ldots + n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(1) n^2 (2) $n^2 + n$

(3) $\frac{n(n+1)}{2}$ (4) n

Answer (4)

Sol. $I = \lim\limits_{t\to 0} n\left(\left(\frac{1}{n}\right)\text{cosec}^2 t + \left(\frac{2}{n}\right)^{\text{cosec}^2 t} + \ldots + 1\right)^{\sin^2 t}$

$I = n$

$\because \quad \lim\limits_{t\to 0}\left(\frac{r}{n}\right)^{\text{cosec}^2 t} = 0, \forall\ 1 \le r < n$

71. The value of $\sum_{r=0}^{22} {}^{22}C_r\ {}^{23}C_r$ is

(1) ${}^{44}C_{22}$ (2) ${}^{45}C_{24}$

(3) ${}^{44}C_{23}$ (4) ${}^{45}C_{23}$

Answer (4)

Sol. $(1+x)^{22} = {}^{22}C_0 + {}^{22}C_1x + {}^{22}C_2x^2 + ...$

$$+ {}^{22}C_{21}x^{21} + {}^{22}C_{22}x^{22} \quad ...(i)$$

$$(x+1)^{23} = {}^{23}C_0x^{23} + {}^{23}C_1x^{22} + {}^{23}C_2x^{21} + ...$$

$$+ {}^{23}C_{21}x^2 + {}^{23}C_{22}x + {}^{23}C_{23} \quad ...(ii)$$

Multiplying (i) & (ii) and comparing coefficients of x^{23} on both sides

$${}^{45}C_{23} = {}^{22}C_0 \cdot {}^{23}C_0 + {}^{22}C_1 \cdot {}^{23}C_1 + {}^{22}C_2 \cdot {}^{23}C_2 + ...$$

$$+ {}^{22}C_{22} \cdot {}^{23}C_{22}$$

$$\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r = {}^{45}C_{23}$$

72. Let a tangent to the curve $y^2 = 24x$ meet the curve $xy = 2$ at the points A and B. Then the mid points of such line segments AB lie on a parabola with the

(1) Directrix $4x = 3$

(2) Length of latus rectum $\frac{3}{2}$

(3) Length of latus rectum 2

(4) Directrix $4x = -3$

Answer (1)

Sol. $y^2 = 24x$, $xy = 2$

Let the equation of tangent to $y^2 = 24x$ is $ty = x + 6t^2$

$ty = x + 6t^2$ meet the curve $xy = 2$ at points A and B.

Let mid-point of AB is $P(h, k)$.

$ty = \frac{2}{y} + 6t^2$	$t \cdot \frac{2}{x} = x + 6t^2$
$ty^2 - 6t^2y - 2 = 0$	$x^2 + 6t^2x - 2t = 0$
$y_1 + y_2 = 6t$	$x_1 + x_2 = -6t^2$

$\Rightarrow$ Mid-point P is $(-3t^2, 3t)$

$\Rightarrow$ $h = -3t^2$, $k = 3t$

$\Rightarrow$ $\left(\frac{h}{-3}\right) = \left(\frac{k}{3}\right)^2$

$\Rightarrow$ $y^2 = -3x$

$\Rightarrow$ Length of L.R. = 3

Equation of directrix is $x = \frac{3}{4}$

73. The equation $x^2 - 4x + [x] + 3 = x[x]$, where $[x]$ denotes the greatest integer function, has

(1) Exactly two solutions in $(-\infty, \infty)$

(2) No solution

(3) A unique solution in $(-\infty, 1)$

(4) A unique solution in $(-\infty, \infty)$

Answer (4)

Sol. $x^2 - 4x + [x] + 3 = x[x]$

$(x - 1)(x - 3) = (x - 1)[x]$

$(x - 1)(x - 3 - [x]) = 0$

$x = 1$ or $x - 3 - [x] = 0$

$\{x\} = 3$

$x = \phi$

$\Rightarrow$ a unique solution in $(-\infty, \infty)$

74. The distance of a point $(7, -3, -4)$ from the plane passing through the points $(2, -3, 1)$, $(-1, 1, -2)$ and $(3, -4, 2)$ is

(1) $4\sqrt{2}$

(2) 4

(3) $5\sqrt{2}$

(4) 5

Answer (3)

Sol. $A(2, -3, 1)$, $B(-1, 1, -2)$, $C(3, -4, 2)$

$\overrightarrow{AB} = -3\hat{i} + 4\hat{j} - 3\hat{k}$ $\quad$ $\overrightarrow{AC} = \hat{i} - \hat{j} + \hat{k}$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & -3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} - \hat{k}$$

Let equation of plane is $x - z + \lambda = 0$ passes through point $A(2, -3, 1) \Rightarrow \lambda = -1$

Equation of plane is $x - z - 1 = 0$

Distance of point $(7, -3, -4)$ from the plane $x - z - 1 = 0$ is $5\sqrt{2}$

75. The compound statement $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$ is equivalent to

(1) $((\sim P) \vee Q) \wedge (\sim Q)$

(2) $(\sim Q) \wedge P$

(3) $(\sim P) \vee Q$

(4) $((\sim P) \vee Q) \wedge ((Q) \vee P)$

Answer (4)

Sol. $(\sim(P \wedge Q)) \vee ((\sim P) \wedge Q) \Rightarrow ((\sim P) \wedge (\sim Q))$

$(\sim P \vee \sim Q) \vee (\sim P \wedge Q) \Rightarrow (\sim P \wedge \sim Q)$

$\Rightarrow (\sim P \vee \sim Q \vee \sim P) \wedge (\sim P \vee \sim Q \vee Q) \Rightarrow (\sim P \wedge \sim Q)$

$\Rightarrow (\sim P \vee \sim Q) \wedge (T) \Rightarrow (\sim P \wedge \sim Q)$

$\Rightarrow (\sim P \vee \sim Q) \Rightarrow (\sim P \wedge \sim Q)$

$\Rightarrow \sim(\sim P \vee \sim Q) \vee (\sim P \wedge \sim Q)$

$\Rightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q) \Rightarrow (\sim P \vee Q) \wedge (\bullet Q \vee P)$

76. Let Ω be the sample space and $A \subseteq \Omega$ be an event. Given below are two statements:

(S1) : If $P(A) = 0$, then $A = \varnothing$

(S2) : If $P(A) = 1$, then $A = \Omega$

Then

(1) Both (S1) and (S2) are false

(2) Only (S1) is true

(3) Only (S2) is true

(4) Both (S1) and (S2) are true

Answer (4)

Sol. Both statements are correct

77. Let α be a root of the equation $(a - c)x^2 + (b - a)x + (c - b) = 0$ where a, b, c are distinct real numbers such that the matrix $\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$ is singular. Then, the value of

$$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)} \text{ is}$$

(1) 12 (2) 6

(3) 9 (4) 3

Answer (4)

Sol. $$\frac{(a-c)^3 + (b-a)^3 + (c-a)^3}{(a-c)(c-b)(b-a)}$$

$$= \frac{3(a-c)(b-a)(c-b)}{(a-c)(b-a)(c-b)}$$

$= 3$

78. Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Then at $x = 0$

(1) f is continuous but not differentiable

(2) f and f' both are continuous

(3) f' is continuous but not differentiable

(4) f is continuous but f' is not continuous

Answer (4)

Sol. $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^-} f(x) = f(0) = 0$

$\therefore$ f is continuous at $x = 0$

Now, $R.H.D = \lim\limits_{h \to 0} \frac{f(h) - f(0)}{h} = \frac{h^2 \sin\frac{1}{h}}{h} = 0$

at $x = 0$

and LHD

$$= \lim_{h \to 0} \frac{f(-h) - f(0)}{h} = \frac{-h^2 \sin\left(\frac{1}{h}\right)}{-h} = 0$$

$\therefore$ RHD = LHD $\therefore$ f is differentiable at $x = 0$

$\therefore$ $f'(x) = \begin{cases} 2x \sin\left(-\right) - \cos\left(\frac{1}{x}\right), & \neq 0 \\ 0, & x = 0 \end{cases}$

$\lim\limits_{x \to 0^+} f'(x)$ is oscillatory

$\therefore$ f is continuous but f' is not at $x = 0$

79. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8 \quad 4\sqrt{3}}{6+3\sqrt{3}}}\right)$ is equal to:

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

Answer (1)

Sol. $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{}{6}$

and $\sec^{1}\left(\sqrt{\frac{4\left(2+\sqrt{3}\right)}{3\left(2+\sqrt{3}\right)}}\right) = \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$

$\therefore$ $\tan^{-1}\left(\frac{1+\sqrt{3}}{3+\sqrt{3}}\right) + \sec^{-1}\left(\sqrt{\frac{8+4\sqrt{3}}{6+3\sqrt{3}}}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

80. Let $p, q \in \mathbb{R}$ and

$\left(1-\sqrt{3}i\right)^{200} = 2^{199}\left(p+iq\right), i = \sqrt{-1}$, then $p + q + q^2$ and $p - q + q^2$ are roots of the equation.

(1) $x^2 - 4x + 1 = 0$ (2) $x^2 - 4x - 1 = 0$

(3) $x^2 + 4x - 1 = 0$ (4) $x^2 + 4x + 1 = 0$

Answer (1)

Sol. Given $\left(1-\sqrt{3}i\right)^{200} = 2^{199}\left(p+iq\right) \ldots(1)$

L.H.S = $2^{200}\left[\cos\left(\frac{5\pi}{3}\right)+i\sin\frac{5\pi}{3}\right]^{200}$

$2^{200}\left[\cos\frac{1000\pi}{3}+i\sin\frac{1000\pi}{3}\right]$

$= 2^{200}\left[-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right]$

So, by (1)

$P = -1, \quad q = -\sqrt{3}$

$\therefore \quad p+q+q^2 = -1-\sqrt{3}+3 = 2-\sqrt{3} = \alpha$

and $p-q+q^2 = -1+\sqrt{3}+3 = 2+\sqrt{3} = \beta$

$\therefore$ quadratic equation whose roots are α and β

$x^2 - 4x + 1 = 0$

Option (1) is correct.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Suppose $\sum_{r=0}^{2023} r^2\ {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is _____

Answer (1012)

Sol. $\because \quad (1+x)^{2023} = \sum_{r=0}^{2023} {}^{2023}C_r x^r$

$\Rightarrow \quad (2023)(1+x)^{2022} = \sum_{r=0}^{2023} {}^{2023}C_r r x^{r-1}$

$\Rightarrow \quad (2023)x(1+x)^{2022} = \sum_{r=0}^{2023} r\ {}^{2023}C_r x^r$

$\Rightarrow \quad (2023)\left[x2022(1+x)^{2021}+(1+x)^{2022}\right]$

$= \sum_{r=0}^{2023} r^2\,{}^{2023}C_r x^{r-1}$

Put $x = 1$

$\Rightarrow \quad 2023\left[2022\cdot 2^{2021}+2^{2022}\right] = \sum_{r=0}^{2023} r^2\,{}^{2023}C_r$

$\therefore \quad \sum_{r=0}^{2023} r^2\,{}^{2023}C_r = 2023\cdot 2^{2022}(1012)$

$\therefore \quad \alpha = 1012$

82. Let C be the largest circle centred at (2, 0) and inscribed in the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$.

If (1, α) lies on C, then $10\alpha^2$ is equal to _____

Answer (118.00)

Sol. $\frac{x^2}{36}+\frac{y^2}{16}=1$

$r^2 = (x-2)^2 + y^2$

Solving simultaneously

$-5x^2 + 36x + (9r^2 - 180) = 0$

$D = 0$

$r^2 = \frac{128}{10}$

Distance between (1, α) and (2, 0) should be r

$1+\alpha^2 = \frac{128}{10}$

$\alpha^2 = \frac{118}{10}$

$= 118.00$

83. The number of 9 digit numbers, that can be formed using all the digits of the number 123412341 so that the even digits occupy only even places, is _____

Answer (60)

Sol. Given number 123412341

$\underset{\wedge}{-}\underset{\wedge}{-}\underset{\wedge}{-}\underset{\wedge}{-}-\underset{\wedge}{-}-\underset{\wedge}{-}-$

Total number $= \frac{4!}{2!2!}\times\frac{5!}{3!2!} = 6\times 10 = 60$

84. Let a tangent to the curve $9x^2 + 16y^2 = 144$ intersect the coordinate axes at the points A and B. Then, the minimum length of the line segment AB is _____

Answer (07)

Sol. Given curve : $9x^2 + 16y^2 = 144$

$$\Rightarrow \quad \frac{x^2}{16}+\frac{y^2}{9}=1$$

Let $P(4\cos\theta, 3\sin\theta)$ be any point on it.

Now tangent at P

$$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{3}=1$$

$\therefore \quad A \equiv (4\sec\theta, 0)\ B \equiv (0, 3\operatorname{cosec}\theta)$

$$AB=\sqrt{16\sec^2\theta+9\operatorname{cosec}^2\theta}$$

$$=\sqrt{16+9+16\tan^2\theta+9\cot^2\theta}$$

$$AB_{\min}=\sqrt{25+2\times12}$$

$$= 7$$

85. A boy needs to select five courses from 12 available courses, out of which 5 courses are language courses. If he can choose at most two language courses, then the number of ways he can choose five courses is _____

Answer (546)

Sol. **Case 1** If no language course is selected.

$$= {}^7C_5 \cdot$$

Case 2 If one language course is selected.

$${}^7C_4 \cdot {}^5C_1 \cdot$$

Case 3 If two language course is selected.

$${}^7C_3 \cdot {}^5C_2 \cdot$$

Total = $${}^7C_5 + {}^7C_4 \cdot {}^5C_1 + {}^7C_3 \cdot {}^5C_2$$

$$= 21 + 175 + 350$$

$$= 546$$

86. The value of $12\int_0^3 \left|x^2-3x+2\right|dx$ is _____.

Answer (22)

Sol. $12\int_0^3 \left|x^2-3x+2\right|dx$

Let $I=\int_0^3 |(x-2)(x-1)dx$

$$=\int_0^1 (x-1)(x-2)dx-\int_1^2 (x-1)(x-2)dx+\int_2^3 (x-1)(x-2)dx$$

$$=\left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_0^1-\left[\frac{x^3}{3}-\frac{3x^2}{2}+2x\right]_1^2$$

$$\left[\frac{x^3}{3}-\frac{3x^2}{2}+x\right]_2^3$$

$$=\left|\frac{1}{3}-\frac{3}{2}+2\right|-\left\{\left(\frac{8}{3}-6+4\right)-\left(\frac{1}{3}-\frac{3}{2}+2\right)\right|$$

$$+\left\{\left(9-\frac{27}{2}+6\right)-\left(\frac{8}{3}-6+4\right)\right|$$

$$=\frac{11}{6}$$

$$12I=\frac{11}{6}\times12$$

$$= 22$$

87. The 4^{th} term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in \mathbb{N}$ Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is

Answer (12)

Sol. $T_4 = 500$

$$ar^3 = 500 \Rightarrow a=\frac{500}{r^3}$$

Now,

$S_6 > S_5 + 1$

$$\frac{a(1-r^6)}{1-r}-\frac{a(1-r^5)}{1-r}>$$

$ar^5 > 1$

Now, $r=\frac{1}{m}$ and $a=\frac{500}{r^3}$

$$\Rightarrow \quad m^2 < 500$$

$$\because \quad m > 0 \Rightarrow m \in \left(0, 10\sqrt{5}\right) \quad ...(i)$$

$$S_7 < S_6 + \frac{1}{2}$$

$$\frac{a(1-r\ )}{1-r} < \frac{a(1-r^6)}{1-r}+\frac{1}{2}$$

$$ar^6 < \frac{1}{2}$$

$\because \quad r = \frac{1}{m}$ and $a = \frac{500}{r^5}$

$\frac{1}{m^3} < \frac{1}{1000}$

$\Rightarrow \quad m \quad (10, \infty)$ ...(ii)

Possible values of m is {11, 12, ...22}

$\because \quad m \in N$

Total 12 values

88. The shortest distance between the lines $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-6}{2}$ and $\frac{x-6}{3} = \frac{1-y}{2} = \frac{z+8}{0}$ is equal to

Answer (14)

Sol. $\vec{a}_1 = 2\hat{i} - \hat{j} + 6\hat{k}$

$\vec{a}_2 = 6\hat{i} + \hat{j} - 8\hat{k}$

$\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b} = 3\hat{i} - 2\hat{j} + 0\hat{k}$

$S.D = \frac{\left|\left[\vec{a}_2 - \vec{a}_1 \quad \vec{a} \quad \vec{b}\right]\right|}{\left|\vec{a} \times \vec{b}\right|}$

$\left[\vec{a}_2 - \vec{a}_1 \; \vec{a} \quad \vec{b}\right] = \begin{vmatrix} 4 & 2 & -14 \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix}$

$= 4\times (4) - 2 (-6) - 14 (-12)$

$= 16 + 12 + 168 = 196$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & -2 & 0 \end{vmatrix} = 4\hat{i} + 6\hat{j} - 12\hat{k}$

$\left|\vec{a} \times \vec{b}\right| = \sqrt{16 + 36 + 144} = \sqrt{196} = 14$

$S.D = \frac{196}{14} = 14$

89. Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda: x \text{ is an integer solution of } E\}$ is _____.

Answer (05)

Sol. $D \geq 0 \Rightarrow 4 - 4|\lambda - 3| \geq 0$

$|\lambda - 3| \leq 1$

$-1 \leq \lambda - 3 \leq 1$

$2 \leq \lambda \leq 4$

$|x| = \frac{2 \pm \sqrt{4 - 4|\lambda - 3|}}{2}$

$= 1 \pm \sqrt{1 - |\lambda - 3|}$

$x_{\text{largest}} = 1 + 1 = 2$, when $\lambda = 3$

Largest element of $S = 2 + 3 = 5$

90. The value of $\frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos\ )^{2023}}\, dx$ is _____.

Answer (02)

Sol. $I = \frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\cos x)^{2023}}{(\sin x)^{2023} + (\cos\ )^{2023}} dx$ ...(i)

$I = \frac{8}{\pi}\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{2023}}{(\sin x)^{2023} + (\cos\ )^{2023}} dx$ ...(ii)

$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

(i) + (ii) $\Rightarrow 2I = \frac{8}{\pi}\int_0^{\frac{\pi}{2}} 1 dx = \frac{8}{\pi} \times \frac{\pi}{2} = 4$

$I = 2$

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