EX NAVODAYAN FOUNDATION
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25/01/2023
Morning

Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. A message signal of frequency 5 kHz is used to modulate a carrier signal of frequency 2 MHz. The bandwidth for amplitude modulation is:

(1) 5 kHz (2) 2.5 kHz

(3) 10 kHz (4) 20 kHz

Answer (3)

Sol. Frequency of modulating wave = 5 kHz

Bandwidth = Twice the frequency of modulating signal

= 2 × 5 kHz

= 10 kHz

2. Electron beam used in an electron microscope, when accelerated by a voltage of 20 kV, has a de-Broglie wavelength of λ_0. If the voltage is increased to 40 kV, then the de-Broglie wavelength associated with the electron beam would be:

(1) $9\lambda_0$ (2) $\frac{\lambda_0}{2}$

(3) $\frac{\lambda_0}{\sqrt{2}}$ (4) $3\lambda_0$

Answer (3)

Sol. $\lambda_0 = \frac{h}{\sqrt{2m[e(20\times10^3)]}}$

$$\lambda_{\text{new}} = \frac{h}{\sqrt{2m[e(40\times10^{-3})]}} = \frac{\lambda_0}{\sqrt{2}}$$

3. The root mean square velocity of molecules of gas is

(1) Proportional to square root of temperature $(\sqrt{T})$

(2) Inversely proportional to square root of temperature $\left(\sqrt{\frac{1}{T}}\right)$

(3) Proportional to temperature (T)

(4) Proportional to square of temperature (T^2)

Answer (1)

Sol. $\because \quad v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$\therefore \quad v_{\text{rms}} \propto \sqrt{T}$

4. In Young's double slits experiment, the position of 5th bright fringe from the central maximum is 5 cm. The distance between slits and screen is 1 m and wavelength of used monochromatic light is 600 nm. The separation between the slits is:

(1) 12 μm

(2) 60 μm

(3) 48 μm

(4) 36 μm

Answer (2)

Sol. y_5 = 5 cm, D = 1 m, λ = 600 nm

$$\because \quad \frac{5\lambda D}{d} = \frac{5}{100}$$

$$\therefore \quad d = \frac{5\times600\times10^{-9}\times1\times100}{5}$$

$= 6 \times 10^{-5}$ m

= 60 μm

5. Match **List I** with **List II**

List I		List II	
A.	Surface tension	I.	kg $m^{-1}s^{-1}$
B.	Pressure	II.	kg ms^{-1}
C.	Viscosity	III.	kg $m^{-1}s^{-2}$
D.	Impulse	IV.	kg s^{-2}

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II

(2) A-II, B-I, C-III, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-IV, B-III, C-II, D-I

Answer (3)

Sol. (A) Surface tension : kg s^{-2} (IV)

(B) Pressure : kg $m^{-1}s^{-2}$ (III)

(C) Viscosity : kg $m^{-1}s^{-1}$ (I)

(D) Impulse : kg ms^{-1} (II)

6. A bowl filled with very hot soup cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C. How long it will take to cool from 75°C to 69°C?

(1) 2 minutes (2) 1.4 minutes

(3) 0.5 minute (4) 1 minute

Answer (2)

Sol. From Newton's law of cooling.

$$\frac{dT}{dt} = -k(T - T_s)$$

Case I : dT = 12°C, dt = 2 min

$$\frac{12}{2} = -k[92 - 22°] = -k\ 70 \quad ...(1)$$

Case II : dT = 6°C

$$\frac{6}{dt} = -k[72 - 22] = -k50 \quad ...(2)$$

From (1) and (2)

dt = 1.4 min

7. T is the time period of simple pendulum on the earth's surface. Its time period becomes xT when taken to a height R (equal to earth's radius) above the earth's surface. Then, the value of x will be:

(1) 4 (2) $\frac{1}{2}$

(3) 2 (4) $\frac{1}{4}$

Answer (3)

Sol. $T = 2x\sqrt{\frac{l}{g}}$

g = acceleration due to gravity

On earth's surface $g = \frac{Gm}{R^2}$

On height R, $g_R = \frac{Gm}{4R^2}$

$$g_R = \frac{g}{4}$$

Time period at height $R = 2\pi\sqrt{\frac{l}{g_R}}$

= 2T

8. A Camot engine with efficiency 50% takes heat from a source at 600 K. In order to increase the efficiency to 70%, keeping the temperature of sink same, the new temperature of the source will be :

(1) 360 K (2) 300 K

(3) 900 K (4) 1000 K

Answer (4)

Sol. $\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

50% efficiency $\Rightarrow \frac{1}{2} = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

$$\frac{1}{2} = 1 - \frac{T_{\text{sink}}}{600} \Rightarrow T_{\text{sink}} = 300$$

Now, 70% efficiency $\Rightarrow \frac{7}{10} = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

$$\frac{300}{T_{\text{source}}} = \frac{3}{10}$$

T_{source} = 1000 K

9. The ratio of the density of oxygen nucleus $({}^{16}_{8}O)$ and helium nucleus $({}^{4}_{2}He)$ is

(1) 2 : 1 (2) 8 : 1

(3) 1 : 1 (4) 4 : 1

Answer (3)

Sol. Nuclear density is constant.

$$\frac{\rho_{\text{oxygen}}}{\rho_{\text{Helium}}} = 1$$

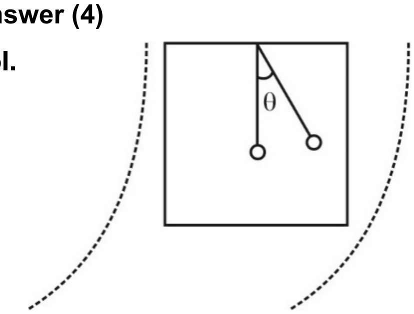
10. A car is moving with a constant speed of 20 m/s in a circular horizontal track of radius 40 m. A bob is suspended from the roof of the car by a massless string. The angle made by the string with the vertical will be : (Take g = 10 m/s^2)

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$

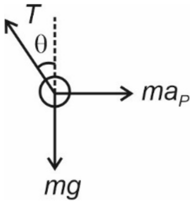
(3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Answer (4)

Sol.



In car's frame, FBD of bob

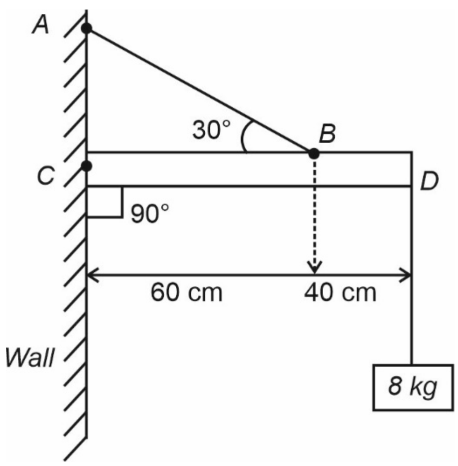


where a_P = Pseudoforce or centrifugal force

$$\theta = \tan^{-1}\left(\frac{a_P}{g}\right) = \tan^{-1}\left(\frac{v^2}{Rg}\right) = \tan^{-1}\left(\frac{400}{40 \times 10}\right)$$

$$= 45°$$

11. An object of mass 8 kg is hanging from one end of a uniform rod CD of mass 2 kg and length 1 m pivoted at its end C on a vertical wall as shown in figure. It is supported by a cable AB such that the system is in equilibrium. The tension in the cable is : (Take g = 10 m/s^2)

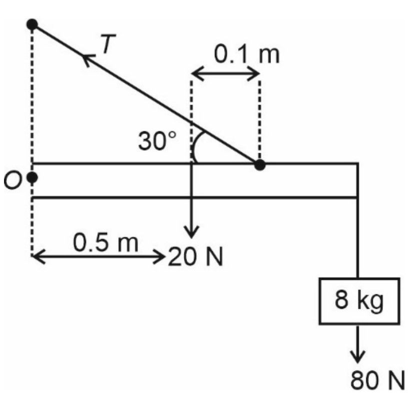


(1) 240 N (2) 90 N

(3) 300 N (4) 30 N

Answer (3)

Sol.



Torque balance about '*O*'

$$\frac{T}{2} \times 0.6 = 20 \times 0.5 + 80 \times 1$$

$$T \times 0.3 = 10 + 80 = 90$$

$$T = \frac{900}{3} = 300 \text{ N}$$

12. A car travels a distance of '*x*' with speed v_1 and then same distance '*x*' with speed v_2 in the same direction. The average speed of the car is

(1) $\frac{v_1 + v_2}{2}$

(2) $\frac{v_1 v_2}{2(v_1 + v_2)}$

(3) $\frac{2 v_1 v_2}{v_1 + v_2}$

(4) $\frac{2x}{v_1 + v_2}$

Answer (3)

Sol. $v_{avg} = \dfrac{2x}{\left(\dfrac{x}{v_1} + \dfrac{x}{v_2}\right)} = \left(\dfrac{2v_1v_2}{v_1 + v_2}\right)$

13. An electromagnetic wave is transporting energy in the negative *z* direction. At a certain point and certain time the direction of electric field of the wave is along positive *y* direction. What will be the direction of the magnetic field of the wave at that point and instant?

(1) Negative direction of *x*

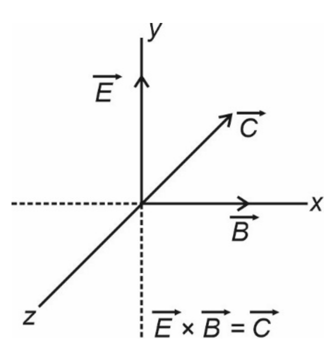
(2) Negative direction of *y*

(3) Positive direction of *z*

(4) Positive direction of *x*

Answer 4)

Sol.



So, $\vec{B}$ should be in *x* direction

14. A uniform metallic wire carries a current 2 A. When 3.4 V battery is connected across it. The mass of uniform metallic wire is 8.92×10^{-3} kg, density is 8.92×10^{3} kg/m^3 and resistivity is $1.7 \times 10^{-8}\ \Omega$ –m. The length of wire is:

(1) $l = 100$ m

(2) $l = 6.8$ m

(3) $l = 10$ m

(4) $l = 5$ m

Answer (3)

Sol. $m = 8.92 \times 10^{-3}$ kg

Density = 8.92×10^{3} kg/m^3

Volume $= \dfrac{8.92\times10^{-3}}{8.92\times10^{3}} = (10^{-6})\text{m}^3$

Resistance $= \dfrac{3.4}{2} = 1.7\ \Omega = \left(\dfrac{\rho l}{A}\right)$

$$1.7 = \frac{\rho l^2}{(Al)}$$

$$\Rightarrow\ 1.7 = \frac{1.7\times10^{-8}\times l^2}{10^{-6}}$$

$l^2 = 100$

$\boxed{l = 10\text{ m}}$

15. In an LC oscillator, if values of inductance and capacitance become twice and eight times, respectively, then the resonant frequency of oscillator becomes x times its initial resonant frequency ω_0. The value of x is:

(1) $\dfrac{1}{16}$

(2) $\dfrac{1}{4}$

(3) 4

(4) 16

Answer (2)

Sol. $\omega_0 = \dfrac{1}{\sqrt{LC}}$

If inductance becomes $2L$ and capacitance becomes $8C$

$$\omega = \frac{1}{\sqrt{2L\times 8C}} = \frac{1}{4\sqrt{LC}}$$

$$\boxed{\omega = \left(\frac{\omega_0}{4}\right)}$$

16. A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2 m long and 0.2 m in diameter. The magnetic intensity at the center of the solenoid when a current of 2 A flows through it is:

(1) A m^{-1} (2) 2.4×10^{-3} A m^{-1}

(3) 1.2×10^{3} A m^{-1} (4) 2.4×10^{3} A m^{-1}

Answer (3)

Sol. Number of turns per unit length = $\dfrac{1200}{2} = 600$

So, Magnetic Intensity $H = nI$

$= 600 \times 2$ Am^{-1}

$= 1200$ Am^{-1}

17. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: Photodiodes are used in forward bias usually for measuring the light intensity.

Reason R: For a p-n junction diode, at applied voltage V the current in the forward bias is more than the current in the reverse bias for $|V_z| > \pm V \geq |V_0|$ where V_O is the threshold voltage and V_z is the breakdown voltage.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) A is false but R is true

(2) Both A and R are true and R is correct explanation A

(3) Both A and R are true but R is NOT the correct explanation A

(4) A is true but R is false

Answer (1)

Sol. Photodiodes are used in reverse bias therefore the assertion is incorrect.

18. Assume that the earth is a solid sphere of uniform density and a tunnel is dug along its diameter throughout the earth. It is found that when a particle is released in this tunnel, it executes a simple harmonic motion. The mass of the particle is 100 g. The time period of the motion of the particle will be (approximately)

(Take g = 10 m s^{-2}, radius of earth = 6400 km)

(1) 1 hour 40 minutes

(2) 12 hours

(3) 24 hours

(4) 1 hour 24 minutes

Answer (4)

Sol. Gravitational acceleration at a distance of r from centre of earth is given by

$$g' = \frac{g}{R} r$$

Where R is the radius of earth

So, $\frac{d^2 r}{dt^2} = -\frac{g}{R} r$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{6400000}{10}}$

$= 2\pi \times 800$ sec

$= 5024$ sec

$= 1$ hour 24 minutes (approx.)

19. A parallel plate capacitor has plate area 40 cm^2 and plates separation 2 mm. The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5. The capacitance of the system is :

(1) $\frac{3}{10}\varepsilon_0$ F (2) $\frac{10}{3}\varepsilon_0$ F

(3) $10\varepsilon_0$ F (4) $24\varepsilon_0$ F

Answer (2)

Sol. $c = \frac{\varepsilon_0 A}{(d-t) + \frac{t}{K}}$

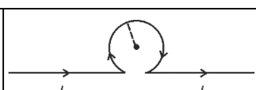
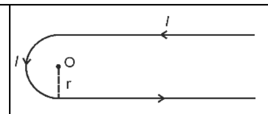
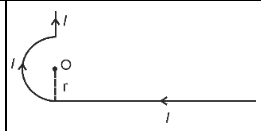
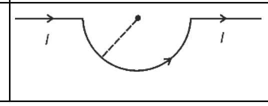
$$= \frac{K\,\varepsilon_0 A}{Kd - t + (K-1)}$$

$$= \frac{5\,\varepsilon_0 \times 40 \times 10^{-4}}{5 \times 2 \times 10^{-3} - 1 \times 10^{-3}(5-1)}$$

$$= \frac{20\,\varepsilon_0}{6}$$

$$= \frac{10\,\varepsilon_0}{3}$$

20. Match List I with List II

List I (Current configuration)		List II (Magnitude of Magnetic Field at point O)	
A.		I.	$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 2]$
B.		II.	$B_0 = \frac{\mu_0}{4}\frac{I}{r}$
C.		III.	$B_0 = \frac{\mu_0 I}{2\pi r}[\pi - 1]$
D.		IV.	$B_0 = \frac{\mu_0 I}{4\pi r}[\pi + 1]$

Choose the correct answer from the options given below :

(1) A–III, B–IV, C–I, D–II (2) A–I, B–III, C–IV, D–II

(3) A–III, B–I, C–IV, D–II (4) A–II, B–I, C–IV, D–III

Answer (3)

Sol. A $\rightarrow$ $B_0 = \frac{-\mu_0 I}{4\pi r} + \frac{\mu_0 I}{2r} - \frac{\mu_0 I}{4\pi r}$

$B_0 = \frac{\mu_0 I}{2\pi r}(\pi - 1)$ $\qquad$ A $\rightarrow$ III

B $\rightarrow$ $B_0 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r} + \frac{\mu_0 I}{4\pi r}$

$B_0 = \frac{\mu_0 I}{4\pi r}(\pi + 2)$ $\qquad$ B $\rightarrow$ I

C $\rightarrow$ $B_0 = \frac{\mu_0 I}{4\pi r} + \frac{\mu_0 I}{4r} + 0$

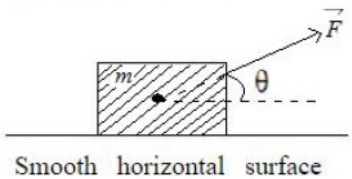
$B_0 = \frac{\mu_0 I}{4\pi r}(\pi + 1)$ $\qquad$ C $\rightarrow$ IV

D. $\rightarrow$ $B_0 = \frac{\mu_0 I}{4r}$ $\qquad$ D $\rightarrow$ II

SECTION - B

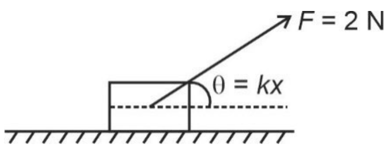
Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. An object of mass 'm' initially at rest on a smooth horizontal plane starts moving under the action of force F = 2N. In the process of its linear motion, the angle θ (as shown in figure) between the direction of force and horizontal varies as $\theta = kx$, where k is a constant and x is the distance covered by the object from its initial position. The expression of kinetic energy of the object will be $E = \frac{n}{k}\sin\theta$, the value of n is ______.



Answer (2)

Sol.



Work done = ΔK.E

$$\therefore \quad \int F \cdot dx = \frac{1}{2}mv^2 = E$$

$$\therefore \quad E = \int_0^x 2\cos(kx)dx$$

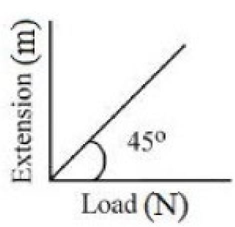
$$E = \frac{2}{k}[\sin kx]_0^x$$

$$= \frac{2}{k}\sin kx$$

$$= \frac{2\sin\theta}{k}$$

22. As shown in the figure, in an experiment to determine Young's modulus of a wire, the extension-load curve is plotted. The curve is a straight line passing through the origin and makes an angle of 45° with the load axis. The length of wire is 62.8 cm and its diameter is 4 mm. The Young's modulus is found to be $x \times 10^4$ Nm^{-2}.

The value of x is _____.



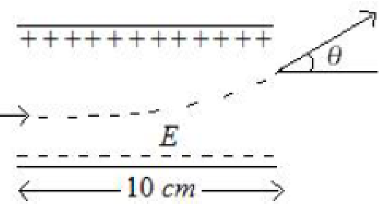
Answer (5)

Sol. $Y = \frac{F}{\Delta l} \times \left(\frac{l(4)}{\pi d^2}\right)$

$$= (\text{slope})\frac{(62.8 \times 10^{-2})}{\pi(4 \times 10^{-3})^2}$$

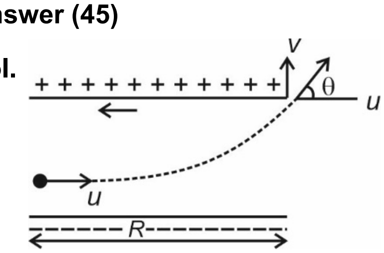
$= (1) \times 5 \times 10^4$ N/m^2

23. A uniform electric field of 10 N/C is created between two parallel charged plates (as shown in figure). An electron enters the field symmetrically between the plates with a kinetic energy 0.5 eV. The length of each plate is 10 cm. The angle (θ) of deviation of the path of electron as it comes out of the field is ____ (in degree).



Answer (45)

Sol.



Let R is the range and T be the time of motion inside the plate.

$\therefore \quad R = vT$

and, $\tan\theta = \frac{v}{u}$

$$= \frac{\left(\frac{eE}{m}\right)T}{u}$$

$$= \frac{\frac{eE}{m}\left(\frac{R}{u}\right)}{u}$$

$$= \frac{eER}{mu^2}$$

$$= \frac{eER}{2(\text{K.E.})}$$

$$= \frac{(e) \times (10) \times (10 \times 10^{-2})}{2 \times (0.5 \text{ eV})}$$

= 1

$\therefore$ $\tan\theta = 1$

$\theta = 45°$

24. An LCR series circuit of capacitance 62.5 nF and resistance of 50 Ω, is connected to an A.C. source of frequency 2.0 kHz. For maximum value of amplitude of current in circuit, the value of inductance is _____mH.

(Take $\pi^2 = 10$)

Answer (100)

Sol. $\because$ For maximum amplitude of current, circuit should be at resonance.

$\therefore \quad X_L = X_C$

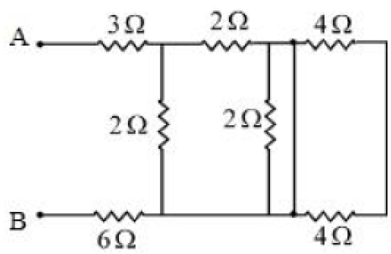
$$\omega L = \frac{1}{\omega C}$$

$$L = \frac{1}{\omega^2 C}$$

$$= \frac{1}{(2\pi \times 2 \times 10^3)^2 \times 62.5 \times 10^{-9}}$$

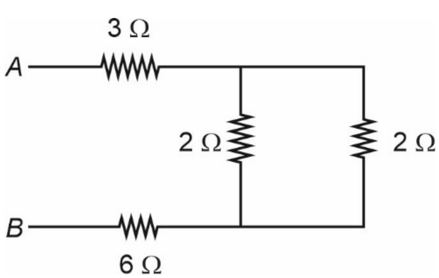
= 100 mH

25. In the given circuit, the equivalent resistance between the terminal A and B is ______ Ω.



Answer (10)

Sol. Equivalent circuit can be redrawn as

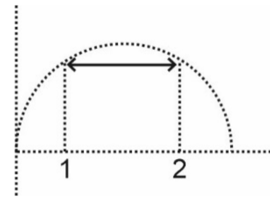


$\therefore R_{AB} = 10\ \Omega$

26. The distance between two consecutive points with phase difference of 60° in a wave of frequency 500 Hz is 6.0 m. The velocity with which wave is traveling is _____ km/s

Answer (18)

Sol.



$$\Delta x = \frac{\lambda}{2\pi} \times \left(\frac{\pi}{3}\right) = \left(\frac{\lambda}{6}\right)$$

$$\Rightarrow \quad \frac{\lambda}{6} = 6\text{m}$$

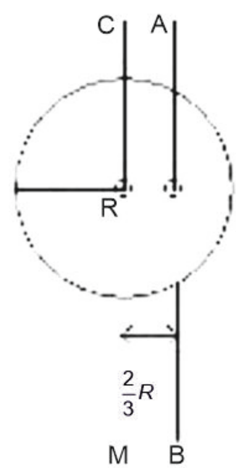
λ = 36 m

$U = f\lambda$ = 500 Hz × 36

= 18000 m/s

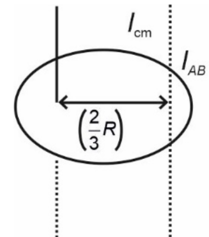
= 18 km/s

27. I_{CM} is the moment of inertia of a circular disc about an axis (CM) passing through its center and perpendicular to the plane of disc. I_{AB} is it's moment of inertia about an axis AB perpendicular to plane and parallel to axis CM at a distance $\frac{2}{3}$R from center. Where R is the radius of the disc. The ratio of I_{AB} and I_{CM} is x : 9. The value of x is______.



Answer (17)

Sol.



$$I_{cm} = \frac{1}{2}MR^2$$

$$I_{AB} = I_{cm} + M \times \left(\frac{2}{3}R\right)^2$$

$$= \frac{1}{2}MR^2 + \frac{4}{9}MR^2$$

$$= \frac{(9+8)MR^2}{18} = \left(\frac{17}{18}\right)MR^2$$

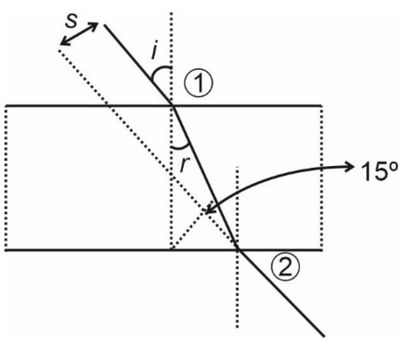
$$\frac{I_{AB}}{I_{cm}} = \frac{17/18}{1/2} = \left(\frac{17}{9}\right)$$

Value of x = 17

28. A ray of light is incident from air on a glass plate having thickness $\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence of a ray is equal to the critical angle for glass-air interface. The lateral displacement of the ray when it passes through the plate is ______ × 10^{-2} cm. (given sin 15° = 0.26)

Answer (52)

Sol.



$$\sin i = \frac{1}{\sqrt{2}} = 45^\circ$$

⇒ at point (1)

$$\mu \sin r = \sin i = \frac{1}{\sqrt{2}}$$

$$\sin r = \frac{1}{2} \qquad \Rightarrow \qquad \boxed{r = 30^\circ}$$

Lateral displacement

$$= \frac{t}{\cos r}\sin(15^\circ) = \frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)} \times 0.26$$

= 2 × 0.26

= 0.52 cm

= 52 × 10^{-2} cm

29. If $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$ and $\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$ then, the unit vector in the direction of $\vec{P} \times \vec{Q}$ is $\frac{1}{x}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$. The value of x is

Answer (4)

Sol. $\vec{P} = 3\hat{i} + \sqrt{3}\hat{j} + 2\hat{k}$

$\vec{Q} = 4\hat{i} + \sqrt{3}\hat{j} + 2.5\hat{k}$

$$\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & \sqrt{3} & 2 \\ 4 & \sqrt{3} & 2.5 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\sqrt{3}}{2}\right) - \hat{j}\left(-\frac{1}{2}\right) + \hat{k}\left(-\sqrt{3}\right)$$

$$= \frac{\sqrt{3}}{2}\hat{i} + \frac{\hat{j}}{2} - \sqrt{3}\hat{k}$$

$$|\vec{P} \times \vec{Q}| = \sqrt{\frac{3}{4} + \frac{1}{4} + 3} = 2$$

Unit vector along $\vec{P} \times \vec{Q} = \frac{1}{4}\left(\sqrt{3}\hat{i} + \hat{j} - 2\sqrt{3}\hat{k}\right)$

$x = 4$

30. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $\frac{20}{x}\lambda_0$. The value of x is _______.

Answer (27)

Sol. Transition, n = 3 to n = 2

$$\frac{1}{\lambda_0} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \left(\frac{5R}{36}\right) \qquad \ldots(1)$$

For transition from, n = 4 to n = 2

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \left(\frac{3}{16}R\right) \qquad \ldots(2)$$

Taking ratio of (1) and (2)

$$\frac{\lambda}{\lambda_0} = \frac{5}{36} \times \frac{16}{3} = \left(\frac{20}{27}\right)$$

$$\boxed{\lambda = \frac{20}{27}\lambda_0}$$

$x = 27$

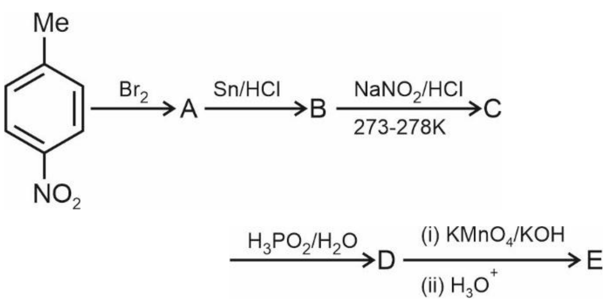
CHEMISTRY

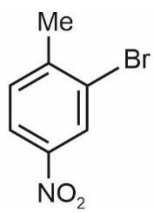
SECTION - A

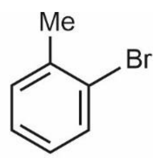
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

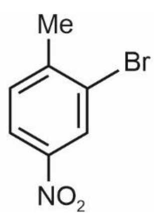
Choose the correct answer :

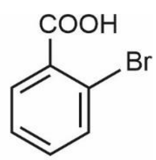
31. Identify the product formed (A and E)

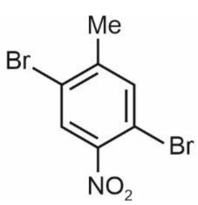


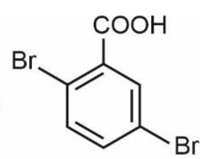
(1) A = 

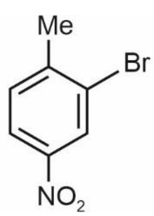
, E = 

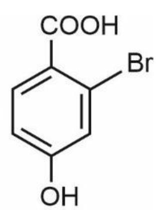
(2) A = 

, E = 

(3) A = 

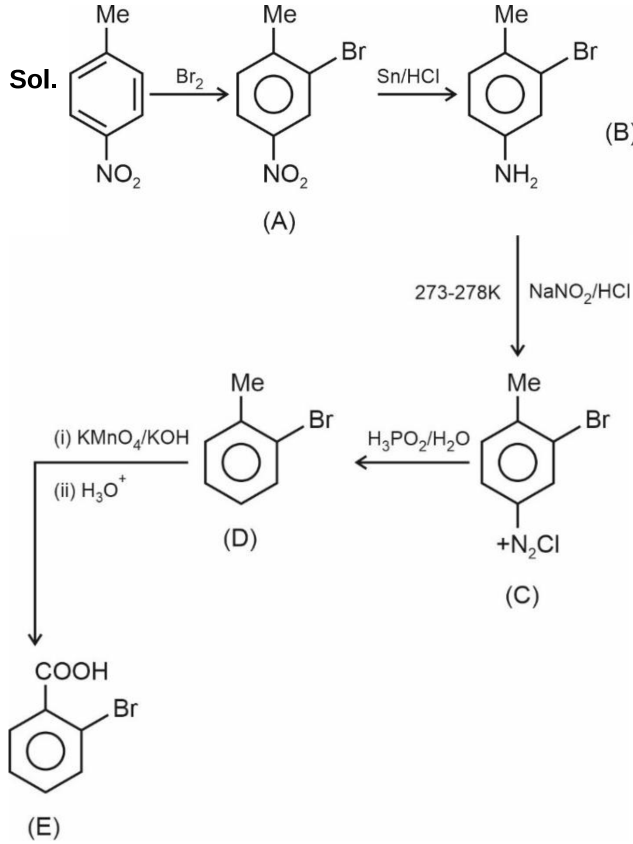
, E = 

(4) A = 

, E = 

Answer (2)

Sol.



32. A cubic solid is made up of two elements X and Y. Atoms of X are present on every alternate corner and one at the center of cube. Y is at $\frac{1}{3}$rd of the total faces. The empirical formula of the compound is

(1) $XY_{2.5}$ (2) $X_{1.5}Y_2$

(3) $X_2Y_{1.5}$ (4) $X_{2.5}Y$

Answer (No answer is correct)

Sol. Number of X particles = $4\times\frac{1}{8}+1$ = 1.5

Number of Y particles = $6\times\frac{1}{3}\times\frac{1}{2}$ = 1

$\therefore$ Empirical formula = $X_{1.5}Y_1 = X_3Y_2$

No answer is correct

33. Reaction of thionyl chloride with white phosphorus forms a compound [A], which on hydrolysis gives [B], a dibasic acid. [A] and [B] are respectively

(1) $POCl_3$ and H_3PO_4 (2) PCl_3 and H_3PO_3

(3) PCl_5 and H_3PO_4 (4) P_4O_6 and H_3PO_3

Answer (2)

Sol. $P_4 + 8SOCl_2 \longrightarrow \underset{(A)}{4PCl_3} + 4SO_2 + 2S_2Cl_2$

$PCl_3 \xrightarrow{\text{Hydrolysis}} \underset{(B)}{H_3PO_3}$

Dibasic acid

34. '25 volume' hydrogen peroxide means

(1) 1 L marketed solution contains 250 g of H_2O_2.

(2) 100 mL marketed solution contains 25 g of H_2O_2.

(3) 1 L marketed solution contains 25 g of H_2O_2.

(4) 1 L marketed solution contains 75 g of H_2O_2.

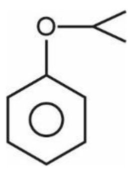
Answer (4)

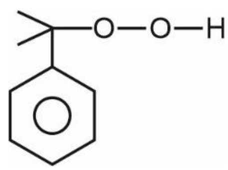
Sol. Molarity of H_2O_2 sol^n = $\frac{\text{volume strength}}{11.2}$

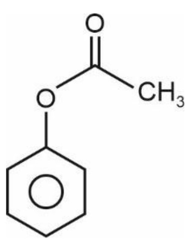
$= \frac{25}{11.2} = 2.23\text{ M}$

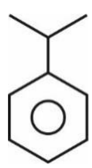
$\therefore$ amount of H_2O_2 in one litre = 2.23 × 34 = 75 gm

35. In the cumene to phenol preparation in presence of air, the intermediate is

(1) 

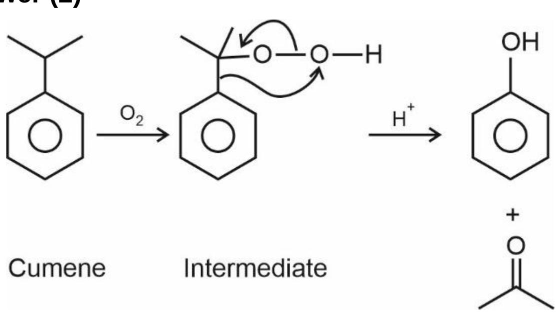
(2) 

(3) 

(4) 

Answer (2)

Sol.



36. The radius of the 2nd orbit of Li^{2+} is x. The expected radius of the 3rd orbit of Be^{3+} is

(1) $\frac{9}{4}x$ (2) $\frac{16}{27}x$

(3) $\frac{27}{16}x$ (4) $\frac{4}{9}x$

Answer (3)

Sol. $r_{Li^{2+}} = r_0 \times \frac{2^2}{3} = x \Rightarrow r_0 = \frac{3x}{4}$

$r_{Be^{3+}} = r_0 \times \frac{3^2}{4}$

$r_{Be^{3+}} = \frac{3x}{4} \times \frac{3^2}{4} = \frac{27x}{16}$

37. Which one of the following reactions does **not** occur during extraction of copper?

(1) $CaO + SiO_2 \rightarrow CaSiO_3$

(2) $FeO + SiO_2 \rightarrow FeSiO_3$

(3) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(4) $2FeS + 3O_2 \rightarrow 2\ FeO + 2SO_2$

Answer (1)

Sol. In the extraction of copper FeO is removed as slag $FeSiO_3$

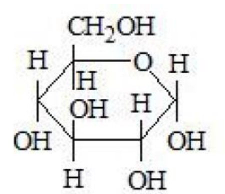
Hence the reaction

$CaO + SiO_2 \rightarrow CaSiO_3$

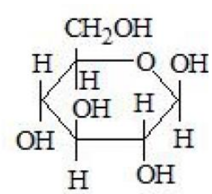
does not occur during extraction of copper

38. Match items of Row I with those of Row II.

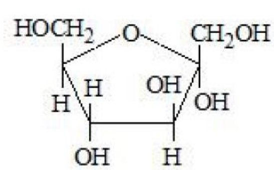
Row I :



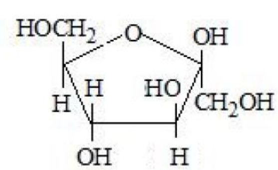
A



B



C



D

Row II :

(i) α-D-(–)-Fructofuranose

(ii) β-D-(–)-Fructofuranose

(iii) α-D-(–) Glucopyranose,

(iv) β-D-(–)-Glucopyranose

Correct match is

(1) A → iii, B → iv, C → i, D → ii

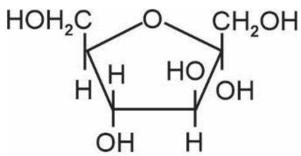
(2) A → i, B → ii, C → iii, D → iv

(3) A → iii, B → iv, C → ii, D → i

(4) A → iv, B → iii, C → i, D → ii

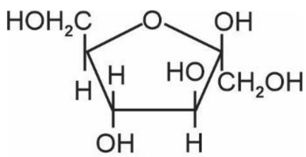
Answer (1)

Sol. (i) α-D-(–)-Fructofuranose –



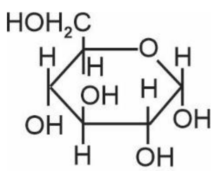
(C)

(ii) β-D-(–)-Fructofuranose –



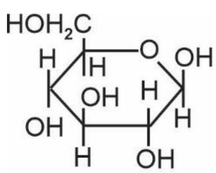
(D)

(iii) α-D-(–)-Glucopyranose –



(A)

(iv) β-D-(–)-Glucopyranose –



(B)

39. Which of the following statements is incorrect for antibiotics?

(1) An antibiotic should promote the growth or survival of microorganisms.

(2) An antibiotic is a synthetic substance produced as a structural analogue of naturally occurring antibiotic.

(3) An antibiotic should be effective in low concentrations.

(4) An antibiotic must be a product of metabolism.

Answer (1)

Sol. An antibiotic inhibit the growth or survival of microorganism.

Except (1) all the statement are correct

40. Match List I with List II

LIST I Elements		LIST II Colour imparted to the flame	
A.	K	I.	Brick Red
B.	Ca	II.	Violet
C.	Sr	III.	Apple Green
D.	Ba	IV.	Crimson Red

Choose the correct answer from the options given below :

(1) A-II, B-I, C-IV, D-III (2) A-IV, B-III, C-II, D-I

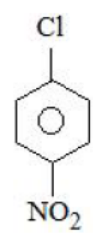
(3) A-II, B-IV, C-I, D-III (4) A-II, B-I, C-III, D-IV

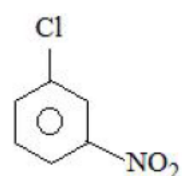
Answer (1)

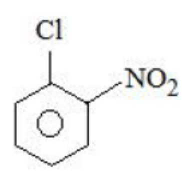
Sol.

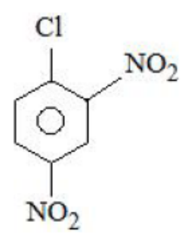
	Element	**Colour imparted to the flame**
(A)	K	Violet
(B)	Ca	Brick red
(C)	Sr	Crimson red
(D)	Ba	Apple green

41. The compound which will have the lowest rate towards nucleophilic aromatic substitution on treatment with OH^- is

(1) 

(2) 

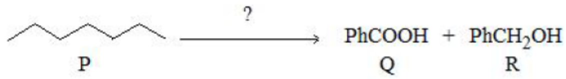
(3) 

(4) 

Answer (2)

Sol. Aryl halides having E.W.G at O-or P-position have greater rate than the m-isomers towards nucleophilic aromatic substitution.

Hence the correct answer is (2)

42. 

The correct sequence of reagents for the preparation of Q and R is:

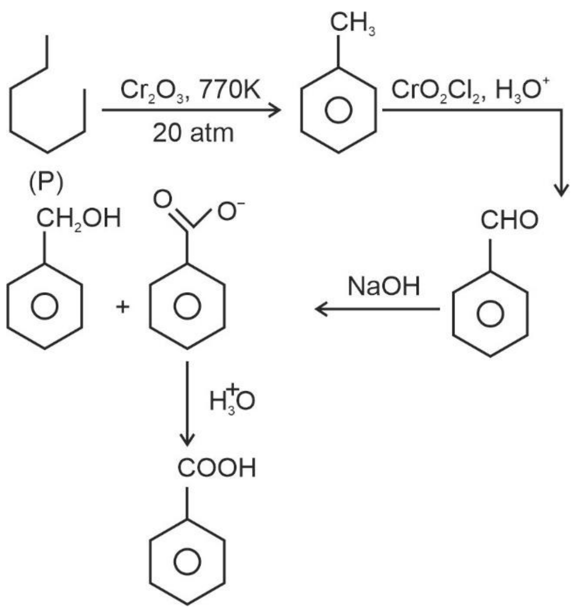
(1) (i) $KMnO_4$, OH^-; (ii) Mo_2O_3, Δ; (iii) NaOH; (iv) H_3O^+

(2) (i) Cr_2O_3, 770 K, 20 atm; (ii) CrO_2Cl_2, H_3O^+; (iii) NaOH; (iv) H_3O^+

(3) (i) CrO_2Cl_2, H_3O^+; (ii) Cr_2O_3, 770 K, 20 atm; (iii) NaOH; (iv) H_3O^+

(4) (i) Mo_2O_3, Δ; (ii) Mo_2O_3, Δ; (iii) NaOH; (iv) H_3O^+

Answer (2)

Sol. 

43. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: Acetal/Ketal is stable in basic medium.

Reason R: The high leaving tendency of alkoxide ion gives the stability to acetal/ketal in basic medium.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) Both A and R are true and R is the correct explanation of A

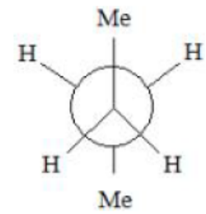
(3) A is false but R is true

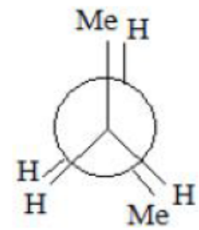
(4) A is true but R is false

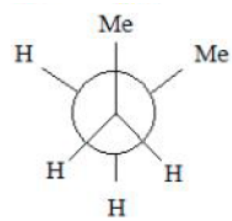
Answer (4)

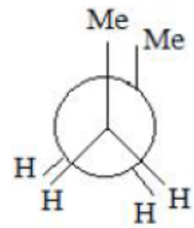
Sol. Acetal/Ketal are known to be quite stable under basic conditions but readily hydrolyse to the corresponding carbonyl compound (aldehyde/keton) and alcohol under acidic condition

44. Which of the following conformations will be the most stable?

(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. Correct stability order of butane is

Anti > Gauche > Partially eclipsed > Fully eclipsed

Hence the correct answer is (1)

45. The correct order in aqueous medium of basic strength in case of methyl substituted amines is:

(1) $Me_2NH > MeNH_2 > Me_3N > NH_3$

(2) $Me_3N > Me_2NH > MeNH_2 > NH_3$

(3) $NH_3 > Me_3N > MeNH_2 > Me_2NH$

(4) $Me_2NH > Me_3N > MeNH_2 > NH_3$

Answer (1)

Sol. The correct order of basic strength in aqueous medium is

$Me_2NH > MeNH_2 > Me_3N > NH_3$

46. Compound A reacts with NH_4Cl and forms a compound B. Compound B reacts with H_2O and excess of CO_2 to form compound C which on passing through or reaction with saturated NaCl solution forms sodium hydrogen carbonate. Compound A, B and C, are respectively

(1) $Ca(OH)_2, NH_4^{\oplus}, (NH_4)_2CO_3$

(2) $Ca(OH)_2$, NH_3, NH_4HCO_3

(3) $CaCl_2, NH_4^{\oplus}, (NH_4)_2CO_3$

(4) $CaCl_2, NH_3, NH_4HCO_3$

Answer (2)

Sol. $\underset{(A)}{Ca(OH)_2} + NH_4Cl \longrightarrow CaCl_2 + \underset{(B)}{NH_3} + H_2O$

$NH_3 + H_2O + \underset{\text{Excess}}{CO_2} \longrightarrow \underset{(C)}{NH_4HCO_3}$

$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + NaHCO_3$

47. Match the **List-I** with **List-II** :

List-I		**List-II**
Cations		**Group reagents**
A. Pb^{2+}, Cu^{2+}	(i)	H_2S gas in presence of dilute HCl
B. Al^{3+}, Fe^{3+}	(ii)	$(NH_4)_2CO_3$ in presence of NH_4OH
C. Co^{2+}, Ni^{2+}	(iii)	NH_4OH in presence of NH_4Cl
D. Ba^{2+}, Ca^{2+}	(iv)	H_2S in presence of NH_4OH

Correct match is

(1) A $\rightarrow$ i; B $\rightarrow$ iii; C $\rightarrow$ iv; D $\rightarrow$ ii

(2) A $\rightarrow$ i; B $\rightarrow$ iii; C $\rightarrow$ ii; D $\rightarrow$ iv

(3) A $\rightarrow$ iv; B $\rightarrow$ ii; C $\rightarrow$ iii; D $\rightarrow$ i

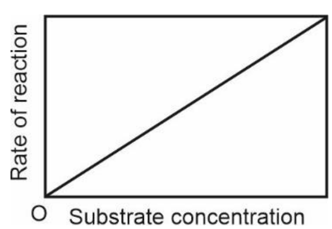
(4) A $\rightarrow$ iii; B $\rightarrow$ i; C $\rightarrow$ iv; D $\rightarrow$ ii

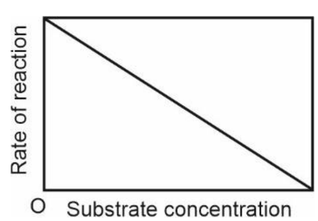
Answer (1)

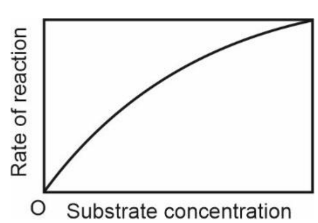
Sol.

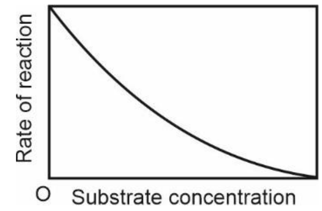
Cations		Group reagents
A. Pb^{2+}, Cu^{2+}	(i)	H_2S gas in presence of dilute HCl
B. Al^{3+}, Fe^{3+}	(iii)	NH_4OH in presence of NH_4Cl
C. Co^{2+}, Ni^{2+}	(iv)	H_2S in presence of NH_4OH
D. Ba^{2+}, Ca^{2+}	(ii)	$(NH_4)_2CO_3$ in presence of NH_4OH

48. The variation of the rate of an enzyme catalyzed with substrate concentration is correctly represented by graph

(a)


(b)


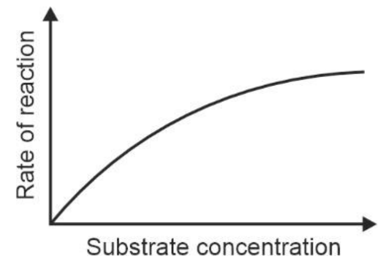
(c)


(d)


(1) (b)
(2) (a)
(3) (d)
(4) (c)

Answer (4)

Sol. The correct plot for enzyme catalysed reaction is



Hence, correct answer is option (4).

49. Some reactions of NO_2 relevant to photochemical smog formation are

$$NO_2 \xrightarrow{\text{sunlight}} X + Y, \quad X \xrightarrow{A} B$$

Identify A, B, X and Y.

(1) X = NO, Y = [O], A = O_2, B = N_2O_3

(2) X = [O], Y = NO, A = O_2, B = O_3

(3) $X = \frac{1}{2}O_2,\ Y = NO_2,\ A = O_3,\ B = O_2$

(4) X = N_2O, Y = [O], A = O_3, B = NO

Answer (2)

Sol. $NO_2 \xrightarrow{h\nu} \underset{(X)}{O} + \underset{(Y)}{NO}$

$$\underset{(X)}{O} \xrightarrow{O_2\ (A)} \underset{(B)}{O_3}$$

50. Inert gases have positive electron gain enthalpy. Its correct order is

(1) He < Xe < Kr < Ne
(2) Xe < Kr < Ne < He
(3) He < Kr < Xe < Ne
(4) He < Ne < Kr < Xe

Answer (1)

Sol.

Electron gain	He	Ne	Ar	Kr	Xe
Enthalpy/ kJ mol^{-1}	48	116	96	96	77

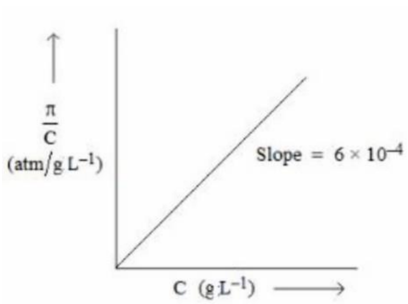
Hence, correct order of positive electron gain enthalpy is

He < Xe < Kr < Ne

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse andw the on-screen virtual numeric keypad in the place designated to enter the answer.

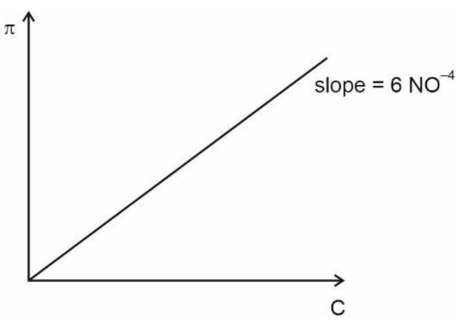
51. The osmotic pressure of solutions of PVC in cyclohexanone at 300 K are plotted on the graph. The molar mass of PVC is _______ g mol^{-1} (Nearest integer)



(Given : R = 0.083 L atm K^{-1} mol^{-1})

Answer (41500)

Sol.



π = CRT

$$\pi = \frac{\text{mole}}{\text{volume}} \times RT$$

$$\pi = \frac{\text{mole}}{\text{volume}} \times \frac{\text{mw}}{\text{mw}} \times RT$$

$$\pi = \frac{\text{mass}}{\text{volume}} \times \frac{RT}{\text{mw}}$$

$$\pi(\text{atm}) = \frac{RT}{\text{mw}} \times C(\text{gm lit}^{-}\ )$$

$$\text{slope} = \frac{RT}{\text{mw}} = 6 \times 10^{-4}$$

mw = 41500

52. The density of a monobasic strong acid (Molar mass 24.2 g/mol) is 1.21 kg/L. The volume of its solution required for the complete neutralization of 25 mL of 0.24 M NaOH is _______ × 10^{-2} mL (Nearest integer)

Answer (12)

Sol. m.eq of NaOH = m.eq of monobasic acid

25 × 0.24 × 1 = 1 × V × molarity

$$\text{Molarity} = \frac{1.21 \times 10^3}{24.2} = 50\text{ M}$$

$$\therefore V = \frac{25 \times 0.24}{50} = 0.12\text{ mL}$$

= 12 × 10^{-2} mL

53. A litre of buffer solution contains 0.1 mole of each of NH_3 and NH_4Cl. On the addition of 0.02 mole of HCl by dissolving gaseous HCl, the pH of the solution is found to be _______ × 10^{-3} (Nearest integer)

[Given : $pK_b(NH_3)$ = 4.745

log 2 = 0.301

log 3 = 0.477

T = 298 K]

Answer (9079)

Sol.

	NH_3	+	HCl	$\rightarrow$	NH_4Cl
At initial	0.1		0		0.1
At time t	0.1 – 0.02				0.1 + 0.02

$$pOH = pK_b + \log\left[\frac{0.1+0.02}{0.1-0.02}\right]$$

$$= 4.745 + \log\left(\frac{3}{2}\right) = 4.745 + [0.477 - 0.301]$$

$$= 4.745 + 0.176$$

pOH = 4.921

pH = 14 – pOH

= 14 – 4.921 = 9.079

pH = 9079 × 10^{-3}

54. The number of paramagnetic species from the following is _______.

$[Ni(CN)_4]^{2-}$, $[Ni(CO)_4]$, $[NiCl_4]^{2-}$,

$[Fe(CN)_6]^{4-}$, $[Cu(NH_3)_4]^{2+}$

$[Fe(CN)_6]^{3-}$ and $[Fe(H_2O)_6]^{2+}$

Answer (04)

Sol.

Species	Magnetic property
$[Ni(CN)_4]^{2-}$	Diamagnetic
$[Ni(CO)_4]$	Diamagnetic
$[NiCl_4]^{2-}$	Paramagnetic
$[FeCN)_6]^{4-}$	Diamagnetic
$[Fe(CN)_6]^{3-}$	Paramagnetic
$Fe(H_2O)_6]^{2+}$	Paramagnetic
$[Cu(NH_3)_4]^{2+}$	Paramagnetic

55. In sulphur estimation, 0.471 g of an organic compound gave 1.4439 g of barium sulphate. The percentage of sulphur in the compound is ______ (Nearest Integer)

(Given: Atomic mass Ba: 137 u, S: 32 u, O:16 u)

Answer (42)

Sol. S% = $\frac{32}{233}\times\frac{1.4439}{0.471}\times 100$ = 42%

56. For the first order reaction A $\rightarrow$ B, the half life is 30 min. The time taken for 75% completion of the reaction is______ min. (Nearest integer)

Given : log 2 = 0.3010

log 3 = 0.4771

log 5 = 0.6989

Answer (60)

Sol. Time taken for 75% completion

= 2 × $t_{1/2}$

= 2 × 30

= 60 min

57. How many of the following metal ions have similar value of spin only magnetic moment in gaseous state? ______

(Given : Atomic number : V, 23; Cr, 24; Fe, 26; Ni, 28)

V^{3+}, Cr^{3+}, Fe^{2+}, Ni^{3+}

Answer (02)

Sol.

Ion	Spin only magnetic moment
V^{3+}	$\sqrt{8}$
Cr^{3+}	$\sqrt{15}$
Fe^{2+}	$\sqrt{24}$
Ni^{3+}	$\sqrt{15}$

58. Consider the cell

Pt(s) | H_2(g) (1 atm) | H^+ (aq, $[H^+]$ = 1)|| Fe^{3+} (aq), Fe^{2+}(aq) | Pt(s)

Given $E^o_{Fe^{3+}/Fe^{2+}} = 0.771\text{ V}$ and $E^{o}_{H^+/1/2\,H_2} = 0\text{ V}$,

T = 298 K

If the potential of the cell is 0.712 V, the ratio of concentration of Fe^{2+} to Fe^{3+} is ______ (Nearest integer)

Answer (10)

Sol. Reaction at anode $\frac{1}{2}H_2 \longrightarrow H^+ + e^-$

Reaction at Cathode $Fe^{3+}_{(aq)} + e^- \rightarrow Fe^{2+}_{(aq)}$

$$E_{cell} = E^o_{cell} - \frac{0.0591}{1}\log\left[\frac{[H^+]\,[Fe^{2+}]}{[Fe^{3+}]\,[pH_2]^{1/2}}\right]$$

$$0.712 = 0.771 - \frac{0.0591}{1}\log\left(\frac{[Fe^{2+}]}{[Fe^{3+}]}\right)$$

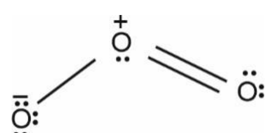
$$-0.059 = -0.0591\log\left(\frac{[Fe^{2+}]}{[Fe^{3+}]}\right)$$

$$\therefore\ \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10^1 = 10$$

59. The total number of lone pairs of electrons on oxygen atoms of ozone is________

Answer (06)

Sol.



60. An athlete is given 100 g of glucose ($C_6H_{12}O_6$) for energy. This is equivalent to 1800kJ of energy. The 50% of this energy gained is utilized by the athlete for sports activities at the event. In order to avoid storage of energy, the weight of extra water he would need to perspire is______ g (Nearest integer)

Assume that there is no other way of consuming stored energy.

Given : The enthalpy of evaporation of water is 45 kJ mol^{-1}

Molar mass of C, H & O are 12, 1 and 16 g mol^{-1}

Answer (360)

Sol. wt of extra water he would need to perspire

$= \frac{1800}{2}\times\frac{18}{45}$

= 20 × 18 = 360 gm

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

61. The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12. If the new mean of the marks is 10.2, then their new variance is equal to :

(1) 4.08

(2) 3.92

(3) 3.96

(4) 4.04

Answer (3)

Sol. $\vec{x} = 10$ & $\sigma^2 = 4$, No. of students = N (let)

$$\therefore \quad \frac{\sum x_i}{N} = 10 \ \& \ \frac{\sum x_i^2}{N} - (10)^2 = 4$$

Now if one of x_i is changed from 8 to 12 we have

New mean $\dfrac{\sum x_i + 4}{N} = 10 + \dfrac{4}{N} = 10.2$

$\Rightarrow$ $N = 20$

and $\sigma^2_{\text{new}} = \dfrac{\sum x_i^2 - (8)^2 + (12)^2}{20} - (10 \cdot 2)^2$

$$= \frac{\sum x_i^2}{20} + \frac{144 - 64}{20} - (10 \cdot 2)^2$$

$= 104 + 4 - (10 \cdot 2)^2$

$= 108 - 104.04 = 3.96$

62. The statement $(p \wedge (\sim q)) \Rightarrow (p \Rightarrow (\sim q))$ is

(1) a contradiction

(2) equivalent to $p \vee q$

(3) equivalent to $(\sim p) \vee (\sim q)$

(4) a tautology

Answer (4)

Sol. Making truth table (Let $(p \wedge \sim q) \Rightarrow (p \Rightarrow \sim q) = E$)

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$p \Rightarrow \sim q$	E
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$\therefore$ E is a tautology

63. The points of intersection of the line $ax + by = 0$, $(a \neq b)$ and the circle $x^2 + y^2 - 2x = 0$ are $A(\alpha, 0)$ and $B(1, \beta)$. The image of the circle with AB as a diameter in the line $x + y + 2 = 0$ is:

(1) $x^2 + y^2 + 5x + 5y + 12 = 0$

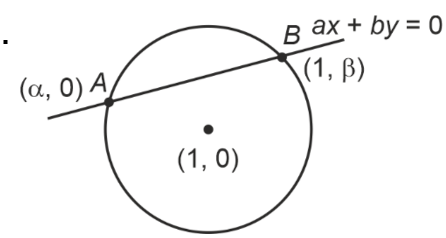
(2) $x^2 + y^2 + 3x + 5y + 8 = 0$

(3) $x^2 + y^2 - 5x - 5y + 12 = 0$

(4) $x^2 + y^2 + 3x + 3y + 4 = 0$

Answer (1)

Sol.



As A and B satisfy both line and circle we have

$\alpha = 0 \Rightarrow A(0, 0)$ and $\beta = 1$ i.e. $B(1, 1)$

Centre of circle as AB diameter is $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ and

radius $= \dfrac{1}{\sqrt{2}}$

$\therefore$ For image of $\left(\dfrac{1}{2}; \dfrac{1}{2}\right)$ in $x + y + z$ we get

$$\frac{x - \frac{1}{2}}{1} = \frac{y - \frac{1}{2}}{1} = \frac{-2(3)}{2}$$

$\Rightarrow$ Image $\left(-\dfrac{5}{2}, -\dfrac{5}{2}\right)$

$\therefore$ Equation of required circle

$$\left(x+\frac{5}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{1}{2}$$

$$\Rightarrow\quad x^2+y^2+5x+5y+\frac{50}{4}-\frac{1}{2}=0$$

$\Rightarrow$ $x^2 + y^2 + 5x + 5y + 12 = 0$

64. Consider the lines L_1 and L_2 given by

$$L_1:\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-2}{2}$$

$$L_2:\frac{x-2}{1}=\frac{y-2}{2}=\frac{z-3}{3}.$$

A line L_3 having direction ratios, 1, –1, –2, intersects L_1 and L_2 at the points P and Q respectively. Then the length of line segment PQ is

(1) $3\sqrt{2}$ (2) $2\sqrt{6}$

(3) $4\sqrt{3}$ (4) 4

Answer (2)

Sol. Let,

$P \equiv (2\lambda+1, \lambda+3, 2\lambda+2)$ and $Q(\mu+2, 2\mu+2, 3\mu+3)$

d.r's of $PQ \equiv <2\lambda - \mu - 1, \lambda - 2\mu + 1, 2\lambda - 3\mu - 1>$

$$\therefore\quad \frac{2\lambda-\mu-1}{1}-\frac{\lambda-2\mu-1}{-1}=\frac{2\lambda-3\mu-1}{-2}$$

$\therefore$ $-2\lambda + \mu + 1 = \lambda - 2\mu + 1$ and $-2\lambda + 4\mu - 2 = -2\lambda + 3\mu + 1$

$\Rightarrow$ $3\lambda - 3\mu = 0$ and $\mu = 3$

$\therefore$ $\lambda = \pm 3$ and $\mu = 3$

$\therefore$ $P \equiv (7, 6, 8)$ and $Q(5, 8, 12)$

$$\therefore\quad |PO|=\sqrt{2^2+2^2+4^2}=\sqrt{24}=2\sqrt{6}$$

65. Let $f : (0, 1) \to R$ be a function defined by $f(x)=\frac{1}{1-e^{-x}}$, and $g(x) = (f(-x) - f(x))$. Consider two statements

(I) g is an increasing function in (0, 1)

(II) g is one-one in (0, 1)

Then,

(1) Only (I) is true

(2) Both (I) and (II) are true

(3) Only (II) is true

(4) Neither (I) nor (II) is true

Answer (2)

Sol. $g(x) = f(-x) - f(x)$

$$=\frac{1}{1-e^x}-\frac{1}{1-e^{-x}}$$

$$=\frac{1}{1-e^x}-\frac{e^x}{e^x-1}$$

$$=\frac{1+e^x}{1-e^x}$$

$$g'(x)=\frac{(1-e^x)e^x-(1+e^x)(-e^x)}{(1-e^x)^2}$$

$$=\frac{e^x-2e^x+e^x+2e^x}{(1-e^x)^2}>0$$

So both statements are correct

66. Let S_1 and S_2 be respectively the sets of $a \in \mathbb{R}-\{0\}$ for which the system of linear equations

$ax + 2ay - 3az = 1$

$(2a + 1) x + (2a + 3) y + (a + 1) z = 2$

$(3a + 5) x + (a + 5) y + (a + 2) z = 3$

has unique solution and infinitely many solutions. Then

(1) $S_1 = \Phi$ and $S_2 = \mathbb{R}-\{0\}$

(2) $S_1 = \mathbb{R}-\{0\}$ and $S_2 = \Phi$

(3) S_1 is an infinite set and $n(S_2) = 2$

(4) $n(S_1) = 2$ and S_2 is an infinite set

Answer (2)

Sol. Given system of equations

$ax + 2ay - 3az = 1$

$(2a + 1)x + (2a + 3)y + (a + 1) z = 2$

$(3a + 5)x + (a + 5)y + (a + 2)z = 3$

$$\text{Let } A=\begin{vmatrix} a & 2a & -3a \\ 2a+1 & 2a+3 & a+1 \\ 3a+5 & a+5 & a+2 \end{vmatrix}$$

$$=a\begin{vmatrix} 1 & 0 & 0 \\ 2a+1 & 1-2a & 7a+4 \\ 3a+5 & -5a-5 & 10a+17 \end{vmatrix}$$

$= a(15a^2 + 31a + 37)$

Now $A = 0$

$\Rightarrow$ $\boxed{a=0}$

So, $S_1 = R - \{0\}$ and at $a = 0$

System has infinite solution but $a \in R - \{0\}$

$\therefore$ $S_2 = \Phi$

67. The minimum value of the function $f(x)\int_0^2 e^{|x-t|}\,dt$ is

(1) $e(e-1)$

(2) $2e-1$

(3) $2(e-1)$

(4) 2

Answer (3)

Sol. $f(x)=\int_0^2 e^{|x-t|}dt$

For $x > 2$

$f(x)=\int_0^2 e^{x-t}dt = e^x\left(1-e^{-2}\right)$

For $x < 0$

$f(x)=\int_0^2 e^{t-x}dt = e^{-x}\left(e^2-1\right)$

For $x\in[0,2]$

$f(x)=\int_0^x e^{x-t}dt \in \int_x^2 e^{t-x}dt$

$= e^{2-x}+e^x-2$

For $x > 2$

$f(x)\big|_{\min}=e^2-1$

For x < 0

$f(x)\big|_{\min}=e^2-1$

For $x\in[0,2]$

$f(x)\big|_{\min}=2(e-1)$

68. Let $y(x) = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)(1 + x^{16})$. Then $y' - y''$ at $x = -1$ is equal to :

(1) 496 (2) 944

(3) 976 (4) 464

Answer (1)

Sol. $y=\dfrac{1-x^{32}}{1-x}=1+x+x^2+x^3+...+x^{31}$

$y' = 1 + 2x + 3x^2 + \ldots + 31x^{30}$

$y'(-1) = 1 - 2 + 3 - 4 + \ldots + 31 = 16$

$y''(x) = 2 + 6x + 12x^2 + \ldots + 31.30\, x^{29}$

$y''(-1) = 2 - 6 + 12 \ldots 31.30 = 480$

$y''(-1) - y'(-1) = -496$

69. Let $x = 2$ be a local minima of the function $f(x) = 2x^4 - 18x^2 + 8x + 12$, $x\in(-4,4)$. If M is local maximum value of the function f in (-4, 4), then M =

(1) $12\sqrt{6}-\dfrac{33}{2}$ (2) $12\sqrt{6}-\dfrac{31}{2}$

(3) $18\sqrt{6}-\dfrac{31}{2}$ (4) $18\sqrt{6}-\dfrac{33}{2}$

Answer (1)

Sol. $f'(x) = 8x^3 - 36x + 8$

$= 4(2x^3 - 9x + 2)$

$= 4(x - 2)(2x^2 + 4x - 1)$

$=4(x-2)\left(x-\dfrac{-2+\sqrt{6}}{2}\right)\left(x-\dfrac{-2\sqrt{6}}{2}\right)$

Local maxima occurs at $x=\dfrac{-2+\sqrt{6}}{2}=x_0$

$f(x_0)=12\sqrt{6}-\dfrac{33}{2}$

70. The value of

$$\lim_{n\to\infty}\frac{1+2-3+4+5-6+...+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

is :

(1) $3\left(\sqrt{2}+1\right)$ (2) $\dfrac{3}{2}\left(\sqrt{2}+1\right)$

(3) $\dfrac{\sqrt{2}+1}{2}$ (4) $\dfrac{3}{2\sqrt{2}}$

Answer (2)

Sol. $I=\lim_{n\to\infty}\dfrac{(1+2+3+...+3n)-2(3+6+9+..+3n)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$

$$=\lim_{n\to\infty}\frac{\dfrac{3n(3n+1)}{2}-6\dfrac{n(n+1)}{2}}{\left(\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}\right)}$$

$$=\lim_{n\to\infty}\frac{3n(n-1)\left\lfloor\sqrt{2n^4+4n+3}+\sqrt{n^4+5n+4}\right\rfloor}{2\cdot\left[\left(2n^4+4n-3\right)-\left(n^4+5n+4\right)\right]}$$

$$=\lim_{n\to\infty}\frac{3\cdot1\cdot\left(1-\dfrac{1}{n}\right)\left\lfloor\sqrt{2+\dfrac{4}{n^3}+\dfrac{3}{n^4}}+\sqrt{1+\dfrac{5}{n^3}+\dfrac{4}{n^4}}\right\rfloor}{2\left[1-\dfrac{1}{n^3}-\dfrac{7}{n^4}\right]}$$

$$=\frac{3\left(\sqrt{2}+1\right)}{2}$$

71. The distance of the point $\left(6, -2\sqrt{2}\right)$ from the common tangent $y = mx + c$, $m > 0$, of the curves $x = 2y^2$ and $x = 1 + y^2$ is

(1) $\frac{14}{3}$ (2) $\frac{1}{3}$

(3) $5\sqrt{3}$ (4) 5

Answer (4)

Sol. $y^2 = \frac{x}{2} \Rightarrow$ tangent $y = mx + \frac{1}{8m}$

$y^2 = x - 1 \Rightarrow$ tangent $y = m(x-1) + \frac{1}{4m}$

For common tangent $\frac{1}{8m} = -m + \frac{1}{4m}$

$\Rightarrow$ $1 = -8m^2 + 2$

$\because\ m > 0 \Rightarrow m = \frac{1}{2\sqrt{2}}$

$\Rightarrow$ Common tangent is $y = \frac{x}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\Rightarrow x - 2\sqrt{2}y + 1 = 0$

Distance of point $(6, -2\sqrt{2})$ from common tangent = 5

72. Let x, y, $z > 1$ and $A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}$.

Then $\left|\text{adj}(\text{adj } A^2)\right|$ is equal to

(1) 2^4 (2) 6^4

(3) 2^8 (4) 4^8

Answer (3)

Sol. $|A| = \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{vmatrix} = 2$

$\Rightarrow \left|\text{adj}(\text{adj } A^2)\right| = \left|\text{adj}(A^2)\right| = \left(\left|A^2\right|^2\right)^2 = |A|^8 = 2^8$

73. Let $f(x) = \int \frac{2x}{\left(x^2+1\right)\left(x^2+3\right)}dx$. If $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$, then $f(4)$ is equal to

(1) $\log_e 17 - \log_e 18$ (2) $\log_e 19 - \log_e 20$

(3) $\frac{1}{2}\left(\log_e 17 - \log_e 19\right)$ (4) $\frac{1}{2}\left(\log_e 19 - \log_e 17\right)$

Answer (3)

Sol. $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}dx$

Put $x^2 = t \Rightarrow 2xdx = dt$

$f(x) = \int \frac{dt}{(t+1)(t+3)} = \int \frac{dt}{(t+2)^2 - 1}$

$= \frac{1}{2}\log_e\left|\frac{t+1}{t+3}\right| + C$

$f(x) = \frac{1}{2}\log_e\left(\frac{x^2+1}{x^2+3}\right) + C \Rightarrow$

$f(3) = \frac{1}{2}\log_e\left(\frac{10}{12}\right) + C$

$\because\ f(3) + \frac{1}{2}\left(\log_e 5 - \log_e 6\right) \Rightarrow C = 0$

$f(x) = \frac{-}{2}\log_e\left(\frac{x^2+1}{x^2+3}\right) \Rightarrow$

$f(4) = \frac{1}{2}\left(\log_e 17 - \log_e 19\right)$

74. The distance of the point P (4. 6, –2) from the line passing through the point (–3, 2, 3) and parallel to a line with direction ratios 3, 3, –1 is equal to

(1) 3

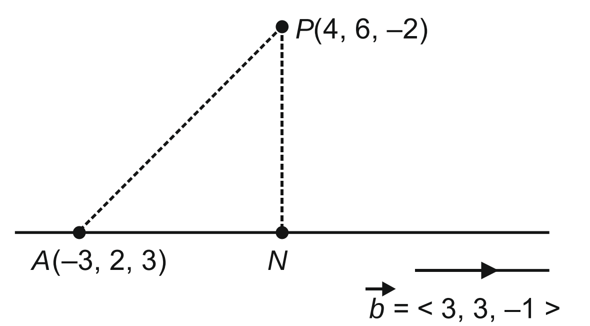
(2) $2\sqrt{3}$

(3) $\sqrt{6}$

(4) $\sqrt{14}$

Answer (4)

Sol.



$\overrightarrow{AP} = 7\hat{i} + 4\hat{j} - 5\hat{k} \Rightarrow \left|\overrightarrow{AP}\right| = \sqrt{49+16+25} = \sqrt{90}$ AN

= projection of $\overrightarrow{AP}$ on $\vec{b} = \overrightarrow{AP}\cdot\vec{b} = \frac{21+12+5}{\sqrt{19}} = \frac{38}{\sqrt{19}}$

$(PN)^2 = (AP)^2 - (AN)^2 = 90 - 76 = 14 \Rightarrow PN = \sqrt{14}$

75. Let M be the maximum value of the product of two positive integers when their sum is 66. Let the sample space $S = \left\{x \in \mathbb{Z} : x(66-x) \geq \frac{5}{9}M\right\}$ and the event $A = \{x \in S : x$ is a multiple of 3$\}$. Then $P(A)$ is equal to

(1) $\frac{7}{22}$ (2) $\frac{1}{3}$

(3) $\frac{1}{5}$ (4) $\frac{15}{44}$

Answer (2)

Sol. $x + y = 66$

$\frac{x+y}{2} \geq \sqrt{xy}$

$\Rightarrow \quad 33 \geq \sqrt{xy}$

$\Rightarrow \quad xy \leq 1089$

$\therefore \quad M = 1089$

$S: \ x(66-x) \geq \frac{5}{9} \cdot 1089$

$66x - x^2 \geq 605$

$\Rightarrow \quad x^2 - 66x + 605 \leq 0$

$\Rightarrow \quad (x-61)(x-5) \leq 0$

$x \in [5, 61]$

$A = \{6, 9, 12, \ldots\ldots 60\}$

$x(A) = 19$

$x(S) = 57$

$\therefore \ P(A) = \frac{1}{3}$

76. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b} \cdot \vec{c} = 0$ and $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{\vec{b} - \vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{d}\right)$ is equal to

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$

(3) $-\frac{1}{4}$ (4) $\frac{3}{4}$

Answer (1)

Sol. $\vec{b}\left(\vec{a} \cdot \vec{c}\right) - \vec{c}\left(\vec{a} \cdot \vec{b}\right) = \frac{\vec{b} - \vec{c}}{2}$

$\vec{a} \cdot \vec{c} = \frac{1}{2}, \quad \vec{a} \cdot \vec{b} = \frac{1}{2}$

$\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{d}\right) = \left(\vec{b} \cdot \vec{d}\right)\left(\vec{a} \cdot \vec{c}\right) - \left(\vec{a} \cdot \vec{d}\right)\left(\vec{b} \cdot \vec{c}\right)$

$= \left(\vec{a} \cdot \vec{b}\right)\left(\vec{a} \cdot \vec{c}\right)$

$= \frac{1}{4}$

77. If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1 + x)^{10}$, then $\sum_{r-1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to

(1) 1210 (2) 5445

(3) 3025 (4) 4895

Answer (1)

Sol. $T_r = {}^{10}C_r x^r$

Coefficient of $x^{10-r} = {}^{10}C_{10-r} = {}^{10}C_r$

$\sum_{r=1}^{10} r^3 \left(\frac{{}^{10}C_r}{{}^{10}C_{r-1}}\right)^2$

$= \sum_{r=1}^{10} r^3 \left(\frac{11-r}{r}\right)^2 \Rightarrow \sum r(11-r)^2$

$\Rightarrow \quad \sum r\left(121 + r^2 - 22r\right)$

$\Rightarrow \quad \sum 121r + \sum r^3 - 22\sum r^2$

$\Rightarrow \quad 121 \times \frac{10 \times 11}{2} + \left(\frac{10 \times 11}{2}\right)^2 - 22 \times \left(\frac{10 \times 11 \times 21}{6}\right)$

$= 6655 + 3025 - 8470$

$= 1210$

78. Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} = \frac{y}{x}\left(1 + xy^2\left(1 + \log_e x\right)\right), x > 0, y(1) = 3$. Then $\frac{y^2(x)}{9}$ is equal to

(1) $\frac{x^2}{7 - 3x^3\left(2 + \log_e x^2\right)}$

(2) $\frac{x^2}{2x^3\left(2 + \log_e x^3\right) - 3}$

(3) $\frac{x^2}{5 - 2x^3\left(2 + \log_e x^3\right)}$

(4) $\frac{x^2}{3x^3\left(1 + \log_e x^2\right) - 2}$

Answer (3)

Sol. $\frac{dy}{dx} = \frac{y}{x}\left(1 + xy^2\left(1 + \log_e x\right)\right), \quad y(1) = 3$

$\Rightarrow \frac{1}{y^3}\frac{dy}{dx} - \frac{1}{x}\cdot\frac{1}{y^2} = (1 + \ln x)$

$-\frac{1}{y^2} = t \Rightarrow \frac{2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{1}{2}\frac{dt}{dx} + \frac{t}{x} = 1 + \ln x$

$\Rightarrow \frac{dt}{dx} + \frac{2t}{x} = 2(1 + \ln x)$

IF = x^2

$t \cdot x^2 = \int (1 + \ln x)x^2 dx$

$\Rightarrow -\frac{1}{y^2}\cdot x^2 = 2\left[\frac{x^3}{3}(1 + \ln x) - \frac{x^3}{9}\right] + c$

$y(1) = 3$

$\Rightarrow c = -\frac{5}{9}$

$\therefore \frac{x^2}{y^2} = -2\left(\frac{x^3}{3}(1 + \ln x) - \frac{x^3}{9}\right] + \frac{5}{9}$

$\Rightarrow \frac{y^2}{9} = \frac{x^2}{5 - 2x^3\left(2 + \ln x^3\right)}$

79. Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set

$S = \left\{z \in \mathbb{C} : |z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2\right\}$

represents a

(1) straight line with the sum of its intercepts on the coordinate axes equals – 18

(2) hyperbola with eccentricity 2

(3) straight line with the sum of its intercepts on the coordinate axes equals 14

(4) hyperbola with the length of the transverse axis 7

Answer (3)

Sol. $|z - z_1|^2 - |z - z_2|^2 = |z_1 - z_2|^2$

$\Rightarrow (x - 2)^2 + (y - 3)^2 - (x - 3)^2 - (y - 4)^2 = 1 + 1$

$\Rightarrow -4x + 4 + 9 - 6y - 9 + 6x - 16 + 8y = 2$

$\Rightarrow 2x + 2y = 14$

$\Rightarrow x + y = 7$

80. The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is

(1) $\sqrt{6}$ (2) $2\sqrt{3}$

(3) 1 (4) $3\sqrt{2}$

Answer (4)

Sol. Let $\vec{b} = \mu\vec{a} + \lambda\hat{j}$

Now $\vec{b}\cdot\vec{a} = 0$

$\Rightarrow \left(\mu\vec{a} + \lambda\hat{j}\right)\cdot\vec{a} = 0$

$\Rightarrow \mu|\vec{a}|^2 + 2\lambda = 0 \Rightarrow 6\mu + 2\lambda = 0$ …(i)

$\Rightarrow \vec{b} = \lambda\left(\vec{a} - 3\hat{j}\right) = \lambda\left(-\hat{i} - \hat{j} + \hat{k}\right)$

$\Rightarrow |\vec{b}| = |\vec{a}| \Rightarrow \lambda = \pm\sqrt{2}$

$\therefore \vec{b} = -\sqrt{2}\left(-\hat{i} - \hat{j} + \hat{k}\right)$

$\therefore 3\vec{a} + \sqrt{2}\vec{b} = 3\left(-\hat{i} + 2\hat{j} + \hat{k}\right) - 2\left(-\hat{i} - \hat{j} + \hat{k}\right)$

$= -\hat{i} + 8\hat{j} + \hat{k}$

$\therefore$ projection $3\sqrt{2}$

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

81. Let the equation of the plane passing through the line $x - 2y - z - 5 = 0 = x + y + 3z - 5$ and parallel to the line $x + y + 2z - 7 = 0 = 2x + 3y + z - 2$ be $ax + by + cz = 65$. Then the distance of the point (a, b, c) from the plane $2x + 2y - z + 16 = 0$ is ____________

Answer (09)

Sol. Let the equation of the plane is

$(x - 2y - z - 5) + \lambda (x + y + 3z - 5) = 0$ …(i)

$\because$ it's parallel to the line

$x + y + 2z - 7 = 0 = 2x + 3y + z - 2$

So, vector along the line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}$

$= -5\hat{i} + 3\hat{j} + \hat{k}$

$\because$ Plane is parallel to line

$\therefore$ $-5(1 + \lambda) + 3(-2 + \lambda) + 1(-1 + 3\lambda) = 0$

$\boxed{\lambda = 12}$

So, by (i)

$13x + 10y + 35z = 65$

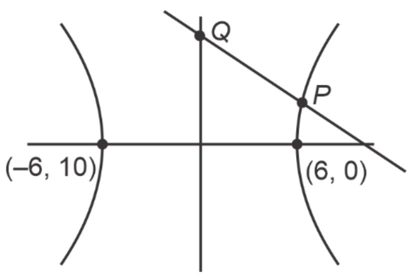
$\therefore$ $a = 13, b = 10, c = 35$

and $d = \dfrac{26 + 20 - 35 + 16}{\sqrt{9}} = 9$

82. The vertices of a hyperbola H are $(\pm 6, 0)$ and its eccentricity is $\dfrac{\sqrt{5}}{2}$. Let N be the normal to H at point in the first quadrant and parallel to the line $\sqrt{2}x + y = 2\sqrt{2}$. If d is the length of the line segment of N between H and the y-axis then d^2 is equal to ____________.

Answer (216)

Sol.



$a = 6,\ e = \dfrac{\sqrt{5}}{2}$

$\therefore$ $\dfrac{5}{4} = 1 + \dfrac{b^2}{36} \Rightarrow b^2 = 36 \times \dfrac{1}{4} = 9$

$\therefore$ $H : \dfrac{x^2}{36} - \dfrac{y^2}{9} = 1$

$P(6\sec\theta, 3\tan\theta)$

Slope of tangent at $P = \dfrac{6\sec\theta}{4 \times 3\tan\theta}$

So, $\dfrac{1}{2\sin\theta} \times -\sqrt{2} = -1 \Rightarrow \sin\theta = \dfrac{1}{\sqrt{2}}$

$\boxed{Q = 45^\circ}$ (for first quad)

$\therefore$ $P \equiv \left(6\sqrt{2}, 3\right)$ and $N : \sqrt{2}x + y = 15$

$\therefore$ $Q(0, 15)$ Now, $PQ^2 = 72 + 144 = 216$

83. Let

$s = \left\{\alpha : \log_2\left(9^{2a-4} + 13\right) - \log_2\left(\dfrac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2\right\}$.

Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{a \in s} \alpha\right)^2 x + \sum_{a \in s} (\alpha + 1)^2 \beta = 0$ has real roots, is ________.

Answer (25)

Sol. $S = \left\{\alpha : \log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(\dfrac{5}{2} \cdot 3^{2\alpha-4} + 1\right) = 2\right\}$

So,

$\dfrac{9^{2\alpha-4} + 13}{\frac{5}{2} 3^{2\alpha-4} + 1} = 4 \Rightarrow 9^{2\alpha-4} + 13 = 103^{2\alpha-4} + 4$

Let $3^{2\alpha-4} = t$ then $t^2 - 10t + 9 = 0$

$(t - 9)(t - 1) = 0$

$\therefore$ $3^{2\alpha-4} = 3^2$ or $3^{2\alpha-4} = 3^\circ$

$\therefore$ $\alpha = 3, 2$

Now equation

$x^2 - 50x + 25\beta = 0$

$D \geq 0 \Rightarrow (50)^2 - 4 \times 25\beta \geq 0$

$\beta \leq 25$

$\therefore$ Max. $\beta = 25$

84. For some $a, b, c \in N$, let $f(x) = ax - 3$ and $g(x) = x^b + c$, $x \in R$. If $(fog)^{-1}(x) = \left(\dfrac{x-7}{2}\right)^{\frac{1}{3}}$, then $(fog)(ac) + (gof)(b)$ is equal to ___________.

Answer (2039)

Sol. $f(x) = ax - 3$

$g(x) = x^b + c$

$(fog)^{-1} = \left(\dfrac{x-7}{2}\right)^{\frac{1}{3}}$

$(fog)^{-1}(x) = \left(\dfrac{x + 3 - ca}{a}\right)^{\frac{1}{b}} = \left(\dfrac{x-7}{2}\right)^{\frac{1}{3}}$

$\Rightarrow$ $a = 2, b = 3, c = 5$

$fog\,(ac) + gof(b)$

$\because$ $f(x) = 2x - 3$

$g(x) = x^3 + 5$

$fog(10) + gof(3)$

$= 2007 + 32$

$= 2039$

85. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5, is ________.

Answer (120)

Sol.

Type	Numbers
$5k$	5, 10, 15, 20, 25
$5k + 1$	1, 6, 11, 16, 21
$5k + 2$	2, 7, 12, 17, 22
$5k + 3$	3, 8, 13, 18, 23
$5k + 4$	4, 9, 14, 19, 24

To select x and y.

Case I : 1 of $(5k + 1)$ and 1 of $(5k + 4) = 5 \times 5 = 25$

Case II : 1 of $(5k + 2)$ and 1 of $(5k + 3) = 5 \times 5 = 25$

Case III : Both of type $5k$ (both cannot be same) = $5 \times 4 = 20$

Total = 120

86. The constant term in the expansion of

$$\left(2x + \frac{1}{x^7} + 3x^2\right)^5$$ is __________.

Answer (1080)

Sol. Constant term in the expansion of

$$\left(2x + \frac{1}{x^7} + 3x^2\right)^5$$

$$\frac{1}{x^{35}}(2x^8 + 1 + 3x^9)^5$$

$$\frac{1}{x^{35}}\left(1 + x^8(3x + 2)\right)^5$$

Term independent of x = coefficient of x^{35} in

$$^5C_4\left(x^8(3x + 2)\right)^4$$

$= {}^5C_4$ coefficient of x^3 in $(2 + 3x)^4$

$= {}^5C_4 \times {}^4C_3 (2)^1 (3)^3$

$= 5 \times 4 \times 2 \times 27$

$= 1080$

87. Let $S = \{1, 2, 3, 5, 7, 10, 11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is

Answer (43)

Sol. Out of the given numbers one is $(3k)$ type and 3 of $(3k + 1)$ type and remaining 3 are $(3k + 2)$ type

Number of subsets of 1 element = 1

(1 of $3k$ type)

Number of subsets of 2 elements

1 of $(3k + 1)$ type + 1 of $(3k + 2)$ type = 9

Number of subsets of 3 elements

1 of $3k$ type + 1 of $(3k + 1)$ type + 1 of $(3k + 2)$ type = 9

3 of $(3k + 1)$ type = 1

3 of $(3k + 2)$ type = 1

Number of subsets of 4 elements

1 of $3k$ type + 3 of $(3k + 1)$ type = 1

1 of $3k$ type + 3 of $(3k + 2)$ type = 1

2 of $(3k + 1)$ type + 2 of $(3k + 2)$ type = 9

Number of subsets of 5 elements

1 of $3k$ + 2 of $(3k + 1)$ type + 2 of $(3k + 2)$ type = 9

Number of subsets of 6 elements

3 of $(3k + 1)$ type + 3 of $(3k + 2)$ type = 1

The set itself = 1

Total = 43

88. If the sum of all the solutions of

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}, -1 < x < 1, x \neq 0,$$

is $\alpha - \frac{4}{\sqrt{3}}$, then α is equal to _________.

Answer (02)

Sol. Case-I

$-1 < x < 0$

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \pi + \tan^{-}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$$

$$\tan^{-1}\frac{2x}{1-x^2} = \frac{-\pi}{3}$$

$$2\tan^{-1}x = \frac{-\pi}{3}$$

$$\tan^{-1}x = \frac{-\pi}{6}$$

$$x = \frac{-1}{\sqrt{3}}$$

Case-II

$0 < x < 1$

$$\tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

$$\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{6}$$

$$2\tan^{-1}x=\frac{\pi}{6}$$

$$\tan^{-1}x=\frac{\pi}{12}$$

$$x=2-\sqrt{3}$$

$$\text{Sum }=\frac{-1}{\sqrt{3}}+2-\sqrt{3}=2-\frac{4}{\sqrt{3}}$$

$\Rightarrow\ \alpha = 2$

89. Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as A, $A + 1$, $A + 2$, respectively. Let a, b, c be the 7th, 9th, 17th terms of A_1, A_2, A_3, respectively such that

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix}+70=0.$$

If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal to _______.

Answer (495)

Sol. $a = A + 6d$

$b = A + 8d + 1$

$c = A + 16d + 2$

$$\begin{vmatrix} a & 7 & 1 \\ 26 & 17 & 1 \\ c & 17 & 1 \end{vmatrix}=-70$$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ 2A+16d+2 & 17 & 1 \\ A+16d+2 & 17 & 1 \end{vmatrix}=-70$$

$R_3 \to R_3 - R_2,\quad R_2 \to R_2 - R_1$

$$\Rightarrow \begin{vmatrix} A+6d & 7 & 1 \\ A+10d+2 & 10 & 0 \\ -A & 0 & 0 \end{vmatrix}=-70$$

$\Rightarrow\ A = -7$

$a = A + 6d = 29 \Rightarrow d = 6$

$b = -7 + 48 + 1 = 42$

$c = -7 + 96 + 2 = 91$

$c - a - b = 91 - 29 - 42 = 20$

$$\text{Sum }=\frac{20}{2}\left[2\times 20+19\times\frac{6}{12}\right]=10\left[40+\frac{19}{2}\right]=495$$

90. In the area enclosed by the parabolas $P_1 : 2y = 5x^2$ and $P_2 : x^2 - y + 6 = 0$ is equal to the area enclosed by P_1 and $y = ax$, $a > 0$, then a^3 is equal to _______.

Answer (600)

Sol. $x^2+6=\frac{5}{2}x^2 \Rightarrow x=\pm 2$

Area between P_1 and P_2 [Say A_1]

$$=\int_{-2}^{2}(x^2+6)-\frac{5}{2}x^2dx$$

$$=2\int_0^2\left(6-\frac{3}{2}x^2\right)dx=2\left[6x-\frac{x^3}{2}\right]_0^2=16$$

$$ax=\frac{5}{2}x^2\Rightarrow x=0,\ \frac{2a}{5}$$

Area between P_1 and $y = ax$ [Say A_2]

$$=\int_0^{\frac{2\alpha}{5}}ax-\frac{5}{2}x^2dx$$

$$=\left.\frac{ax}{2}-\frac{5}{6}x^3\right]_0^{\frac{2a}{5}}:\frac{2a^3}{75}$$

$$A_1 = A_2 \Rightarrow \frac{2a^3}{75}=16$$

$a^3 = 600$

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